JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—Bacon.

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[*The Council of the Institute of Actuaries wish it to be understood that while they consider it their duty to give, from time to time, publicity to certain of the papers presented to the Institute, and to abstracts of the discussions at the Sessional Meetings, they are not responsible for the opinions put forward therein.*]

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INSTITUTE OF ACTUARIES.

Opening Address by the President, FRANK BERTRAND WYATT, Esq.

[Delivered 26 November 1906.]

IN opening the proceedings of the session with an Address, in accordance with the custom which has been followed for so many years, I am anxious to express my profound feeling of gratitude at the honour you have conferred on me in electing me your President, and to tell you how greatly I appreciate such a signal mark of your confidence. With your co-operation and with the help and guidance of the Council, always so loyally and sympathetically given to their Chairman, I shall do all in my power to maintain the prosperity, the usefulness, and the well-deserved prestige of our Institute.

Looking back on a long line of eminent predecessors, with a vivid impression of the great value of their contributions to actuarial knowledge and science, and of their services in the cause of the Institute, I feel guilty of temerity in having accepted the distinguished position in which your goodwill has placed me. I must confess that it is with great diffidence I venture to address you this evening, more especially as I have found it difficult to find new subjects of sufficient interest or importance to warrant me in claiming your attention.

It is most satisfactory to find that the papers contributed to our meetings in recent years have covered a wide range of subjects, of a practical as well as of a theoretical character, and in view of the large membership of the Institute there ought to

be no difficulty in future in obtaining a constant succession of papers of merit. The extending sphere of the actuary's work, and the increase in the number of matters which now come forward for his examination and advice, should yield fresh subjects for fruitful controversy, while old subjects, the consideration of which has by no means been yet exhausted, will present themselves under new conditions for actuarial research and judgment. I am pleased to tell you that the Council have already secured papers for the present session from two of our distinguished Fellows, who have frequently contributed to our instruction and enlightenment, and the Council would be glad to hear that others, and especially some of our younger Members, have in preparation papers on subjects to which they have devoted special attention.

I do not feel able to suggest a list of subjects of a new character on which it would be desirable for the Members of the Institute to write papers, but doubtless the great wealth of information contained in the volumes of data relating to the Mortality Experience of Assured Lives and Annuitants during the 30 years, 1863–1893, collected and arranged by the Institute of Actuaries and the Faculty of Actuaries, will, for many years, form the basis of contributions of an exceedingly interesting and instructive nature. Thus, we may hope to have essays on the "Effect of Selection", following the lines of examination of Dr. Sprague and Mr. Chatham in their great works on this highly important matter; a further endeavour to trace the effect of discontinuance, the recent experience affording far better material than that yielded by the 20 Offices' experience, in which Whole-life Assurances were mingled with other classes; and, further, a complete comparison of the Select Tables deduced from the "old" and "new" experience, 1863–1893, might bring some valuable facts to the surface. We are already indebted to Mr. King and Mr. Ackland for valuable critical examinations of the effect of the New Tables on Valuation Reserves.

I would also mention the intensely important subject of the principles and practice affecting the Finance of a Life Assurance Company. This subject has received little attention from the Institute since Mr. Bailey's paper on the "Principles on which the Funds of Life Assurance Societies should be invested", was written more than 40 years ago, and, in view of the vast extent of the interests entrusted to Life Assurance Companies, it would seem highly desirable that the question should now be submitted

to the fullest examination and discussion by the Members of the Institute. From time to time we have had papers dealing with certain forms of investment and the degree of their suitability for the Funds of a Life Office, but there has been no treatise dealing with this all-important subject in a thoroughly exhaustive and comprehensive manner. The principles laid down by Mr. Bailey are, I believe, acted on to this day, and rightly so, since they are in the nature of axioms. The classes of securities in which British Life Offices invest their funds have, however, greatly increased in recent years, and the proportion of the funds invested in the several classes has undergone a very material change. The Directors of our Offices and their Advising Managers now hold much broader and more enlightened views on the subject of investments, while equally as cautious as their predecessors in the paramount matter of security. They are fully alive to the necessity of seeking new channels of investment, and of considering securities of diverse kinds, within and beyond the limits of the United Kingdom. Thirty years ago about one-half of the Funds of British Ordinary Offices was invested in mortgages on property, while at the present time the proportion is only about one-quarter, and a not insignificant proportion of such mortgages is on property outside the United Kingdom. I should like to see an essay on the subject of "The Finance of Life Offices" dealing exhaustively with the causes which affect the rate of interest, with the appreciation and depreciation of stocks and shares, the reasons of these fluctuations, and the probability of their recurrence from time to time; and, without attempting to assume the *rôle* of a prophet, the essayist might devote his attention to the question of the rate of interest likely to prevail in the future.

A further volume of Monetary Tables founded on the British Offices Select Tables, to be brought out jointly by the Institute and the Faculty, is, I am happy to say, on the point of completion. As you are aware, it will embody the monetary tables contained in the volume of Select Tables already published, and will include, by an arrangement with the authors, the monetary tables, constructed by Messrs. Baker and Raisin, on the $O^{[M]}$ and $O^{[NM]}$ Mortality Tables. Another volume, based on the British Offices' O^{M} Table, which is to be published by the Institute on its own responsibility, will contain the values of policies, and other tables constantly required in our daily practice, following the excellent plan adopted by Mr. R. P. Hardy in his Valuation

Tables on the old H^{M} and $H^{M(5)}$ Mortality Tables. The preparation of these tables is also well advanced, and it is hoped that the work will be available for use very shortly. With these two additional volumes our actuarial armoury based on the British Offices' experience will be well nigh complete. Both these works have been carried out under the supervision of a small Committee consisting of Mr. R. P. Hardy, Mr. Ackland, and Mr. Lidstone, and our gratitude is due to them for their admirable labours.

In accordance with general anticipation, British Life Offices are adopting the new tables in the Valuation of their Liabilities, as the opportunity for change arises, and it is probable that in a very few years these tables will be universally used, and will become the recognized standard basis for the Valuation of Liabilities. Such a condition of things will be in the highest degree gratifying to the Institute and the Faculty, who jointly undertook the immense work of collating the mortality experience of 60 British Offices and of constructing mortality and monetary tables therefrom.

You will be interested to know that Mr. W. Palin Elderton, F.I.A., responding in a most public-spirited manner to the request of the Council, has completed his work on "Frequency Curves and Correlation", as an extension of a paper he originally proposed to read at a sessional meeting. His work may be regarded mainly as a detailed description of the basis and practical application of the modern statistical methods which are associated with the name of Professor Karl Pearson. You will, I know, join with me in offering congratulations to Mr. Elderton, and in thanking him for his labours in the direction of advancing our actuarial science.

In recent years the Institute, in its corporate capacity, has had opportunities of assisting the State in matters of national importance, such, for instance, as advising the Royal Commissioners of the Patriotic Fund as to the financial state of the various Funds under their administration. Following the precedent that has been wisely adopted in similar cases, my predecessor, Mr. Henry Cockburn, recently gave his valuable services on a Departmental Committee of the Board of Trade, respecting Bond Investment Companies and their methods of finance and procedure. As a result of this Report, there is now before Parliament a Bill to provide for the better regulation of Bond Investment Companies, and it is not improbable that it may become law during the present session. Clause 3 of the Bill enacts that every Bond Investment Company shall have quinquennial

investigations made into its financial position by an "Actuary", closely following the wording of section 7 of the Life Assurance Companies Act 1870, and I venture to hope that before the Bill becomes law, the word "Actuary" will be properly defined by the addition of the words "being a Fellow of the Institute of Actuaries, or of the Faculty of Actuaries in Scotland", so that it will not be possible for the periodical investigations of these Companies to be made by unqualified persons to the possible detriment of the interests of a large and thrifty section of the Community. Your President and the President of the Faculty of Actuaries have independently communicated their views on this point to the Board of Trade.

In view of the large increase of Annuity Business, both by Life Offices and by the Government, it is greatly to be desired that the extensive statistics relating to the experience of Government Annuitants from 1875 to the present date should be collected and published, and I venture to express the hope that such a valuable work will be undertaken before long. As is well known, the British Offices' Tables show a greater vitality in both sexes than the Government Annuity Tables (1883), and as each successive publication of annuity experiences has indicated an increasing vitality in both sexes, it is very necessary that our Life Offices should have the latest possible material at their disposal from time to time to enable them to consider the sufficiency of their rates. It is also, of course, important that the Government should be satisfied from time to time that they are not conducting their annuity business at a loss to the country.

The desirability of the Census of this country being taken more frequently, whereby the value of the vital statistics of the country would be greatly enhanced, was, as you are aware, considered by the Council not long since, and a memorial on the subject was addressed to the President of the Local Government Board, but so far we do not seem to be any nearer our goal. The "Case for Census Reform" was fully and admirably laid before us five years ago by Mr. G. H. Ryan, and I think I am right in saying that we are unanimous in our opinion as to the desirability of a Quinquennial Census. When we remember that the Census is taken every five years in France, Germany, and Sweden, and in some of our Australian Colonies, and also in the Administrative County of London, and, further, that a Committee appointed by Government, in 1890, for the purpose of considering the question, strongly recommended that the Census of this

country should be taken every five years, I cannot help thinking that the Council would be fully justified in again approaching the Local Government Board and in expressing their views on a subject of such national importance.

In relation to the varied character and expansion of Actuarial work we observe with much satisfaction the general admission that it is not only in the domain of Life Contingencies that the functions of the Actuary may be usefully extended and recognized. His training fits him to disentangle many knotty problems into which life-probabilities do not enter at all. I believe that one of our Fellows was recently consulted by the Directors of a public Company to advise them upon the terms upon which certain Founders' Shares should be bought out and cancelled; and there must be an infinite number of similar questions coming up for consideration before the Boards of Joint Stock Companies in regard to which our Fellows might be consulted with great advantage. In this direction I hope and firmly believe that the scope of the Actuary's work will continually increase.

And now let me throw out the suggestion that our Fellows, more especially those holding prominent positions, might bear in mind, perhaps more frequently than they now do, the financial basis of much of their work, and enter into the discussion of purely financial questions when suitable opportunities arise. I should like to see as much weight attached to the criticisms or proposals of an Actuary when financial discussions are taking place as would be ungrudgingly accorded to them if they dealt with insurance problems; but to bring this about our Fellows must identify themselves more freely with the matters I have mentioned, and I do not think anyone will call in question their capacity to do so.

The Fifth International Congress of Actuaries was held in September last in Berlin under the auspices of the German Federation for Insurance Science, and the proceedings passed off with great éclat and every mark of success, the attendance of Members from all parts of the world having been upwards of five hundred and fifty. The German Society consists, I understand, of three sections, Legal, Medical, and Mathematical or Actuarial, and has, I believe, about 135 Corporate Members, including the leading private and public institutions for all branches of insurance, and about 1,000 personal Members. It was, no doubt, in consequence of this wide constitution and

extensive membership that the Society succeeded in obtaining for the Congress greater State recognition, attention and encouragement than that accorded to any previous Congress of Actuaries. By the consideration and courtesy of the President of the Reichstag, the magnificent building, in which that Assembly meets, was placed at the disposal of the Congress during the week's sitting, and the large hall was full to overflowing on the opening day, when telegrams of welcome were received from the Emperor, and also from the Imperial Chancellor, Prince von Bülow, and from His Excellency, the Graf von Posadowsky, Minister of the Interior, who also accepted the position of Honorary President of the Congress. In the regrettable absence of Monsieur Lepreux, President of the Permanent Congress Committee in Brussels, through ill-health, the Congress was opened by Monsieur A. Bégault, Secretary of that Committee, and Dr. Hahn, of Magdeburg, unanimously elected President of the Congress, delivered a highly interesting and instructive Inaugural Address, dealing, amongst many other matters, with the development of all branches of insurance in the German Empire, and the formation, constitution and aims of the "German Federation for Insurance Science." It was recognized by everyone that the smooth and successful way in which the proceedings passed off during five days of real hard work,—the meetings lasting from half-past nine in the morning to five in the evening, with an interval of an hour and a half for lunch—was largely due to the constant courtesy and invariable attention shown by the respected President of the Congress. It is also with much pleasure that I add my own personal tribute to those of others to the indefatigable labours of the energetic, versatile and courteous Organising Secretary, Professor Alfred Manes, on whom fell the whole brunt of the arrangements of a Congress on a much larger scale than any of those yet held, as well as of the various social functions and entertainments. He also undertook the responsibility of editing the two volumes of "Reports, Memoirs and Proceedings", containing upwards of 1,500 pages of matter, which, by an excellent arrangement, were placed in the hands of all the subscribers some weeks before the opening of the Congress. Members, therefore, who intended to be present had the opportunity of a leisurely study of the numerous papers and essays, and by way of further assisting them, a brief *précis* of every paper was given in two other languages, the only three languages employed being German, French and English.

As the volumes containing the papers, &c., have been in the hands of so many of our Members, and have doubtless been passed on to others, and as a further volume will soon be published, containing a full report of the proceedings and discussions, I shall not weary you with an attempted description of the numerous essays submitted. The subjects were well chosen, and included many that were not only of much interest to us in this country, but of great and growing importance to several foreign countries. About sixty essays, of which nine were contributed by our own Fellows, were sent in on the following principal subjects:

(1) Industrial Insurance, and particularly the Insurance of Children.

(2) The methods of calculating and determining extra premiums for hazardous risks.

(3) Mortality Tables for Annuitants.

(4) Methods of insuring Abstainers, and persons whose occupations connect them with the manufacture or sale of alcoholic beverages.

(5) Insurance on the lives of women.

(6) The question of Taxes imposed upon Insurance Companies.

(7) Limits within which Insurance is possible.

Those who were there to read their papers, devoted a short time, limited to ten minutes, to giving a *précis* of their paper with any additional information which occurred to them, and this was followed by translations into two other languages. I am inclined to think that this practice of translating the author's *précis* of his own paper into two other languages has a tendency to diminish the interest in the proceedings, while at the same time curtailing the very limited time at the disposal of those who wish to discuss a particular subject after the several papers on it have been read in extract, or taken as read. I would venture to suggest to those who may have the organization of the next Congress that they should take this matter into consideration, for, the papers being already printed, with translations, it would seem unnecessary for the author to read an extract from his paper, though, of course, he should be at liberty to add any further information; and, if this practice were altered, only very brief translations would be required, and more time would be available for the discussion of the papers, and for the necessary translations of the remarks of the debaters.

The English Members were much indebted to Mr. G. W. Richmond, F.I.A., who fulfilled, most satisfactorily, the trying duty of translating the comments and criticisms into English, and we have to thank our friend, Mr. Ernest Woods, F.I.A., for once more acting as Correspondent for England.

In addition to the large number of papers to which I have referred, there were about fifty memoirs of an interesting character submitted on the methods of conducting mortality investigations, the teaching of actuarial science in schools and colleges, the progress of Insurance legislation, aids to actuarial calculation, and (of special interest only to the German Society) on the uniformity of legal requirements, especially as regards reports to be made to the Insurance Authorities. It is very satisfactory to find that the British and Colonial Actuaries took a great interest in the Meetings, there having been about 30 Members of the Institute, and 17 of the Faculty, present, and, in spite of the distance and loss of time, several Actuaries from our own Colonies.

I am sure that all of us who went over to Berlin have come away with strong impressions as to the great thoroughness with which Insurance business in Germany is carried on, and of the admirable manner in which the beneficent laws of the German Empire relating to Insurance against Invalidity and Old Age, and to Workmen's Compensation, are carried out, under the supervision of a Government keenly alive to the paramount importance of these in their influence on the welfare of the nation, and that we realize the immense advantage which has accrued to their country by the great attention given to these subjects by the Government. I would further say that we have all come away with the liveliest sense of pleasure at the excellent arrangements made for the comfort of ourselves and of the ladies who accompanied us, and at the warm-hearted and lavish hospitality shewn to us.

Although I have spared you any comments or criticisms on the papers read before the Congress, I must call your attention to an important proposition which was made by Dr. Klang, of Vienna, which arose out of the discussion on the papers on the assessment of premiums for hazardous risks, a discussion in which several of the Medical Members of the Congress took a great interest. The proposition was to the effect that it was desirable that an endeavour should be made to collect comprehensive statistics as to the mortality of under-average lives, and of the various methods of dealing with them, in the several

Continental Countries and in the United Kingdom; and that possibly the statistics might be aggregated so as to form tables of universal applicability. The Congress itself being debarred from passing any resolutions, it was referred to a meeting of the Permanent Congress Committee, who will, doubtless, take the matter up and communicate with the various Actuarial Societies. As the Council will soon have an opportunity of considering the matter, it would be premature of me to indicate any views I may have as to the feasibility of the scheme.

With regard to these periodical meetings of Actuaries from all parts of the world I hold that, exceedingly pleasant in themselves, they are of great utility to our profession from many points of view. We gain by them a greater knowledge of the way in which the great economical questions of life assurance and kindred matters present themselves to other nations, and we profit greatly from professional and social intercourse with our brother Actuaries in other parts of the world. They also enable us to see more clearly and to appreciate the growing importance and increasing influence of the various Actuarial Societies which have been established in recent years in other countries, more or less closely following the constitution and the aims and objects of our own Institute.

The next Congress has been arranged to meet in Vienna in the month of June 1909. The fact that the last three Congresses have been held during the usual holiday period has, I am afraid, militated against large attendances, and, this consideration evidently weighing with them, our Austrian friends have selected the month of June in the height of the Vienna season, and, if possible, the very week in which the "Grand Prix" is to be run. I venture to express the hope that many of us will find it convenient to take an extra week's holiday at that time, so that the attendance of English Actuaries may be greater than on previous occasions.

Passing from the pleasant contemplation of the work and influence of the recent Actuarial Congress I am led to a consideration of the important events in the domain of Life Assurance which have happened in America during the last two years. The revelations and disclosures of unsound practice and of a vicious system of financing the immense funds entrusted to their care, which have been brought home to the head officials of some of the largest Life Companies in the United States, have caused a surging wave of alarm and discontent to sweep over that

country. I have had considerable doubt whether I could with propriety refer to this subject, which has so engrossed public attention here, and on which so much has been written and said. However, in view of the intense interest which the actuarial profession naturally takes in anything affecting the beneficent system of Life Assurance and its vast importance as an economic factor of life, I think I may fairly ask your attention to the few remarks I have to make on the subject.

In the first place we may, without offence, extend our sympathy to the American public whose faith in their Life Insurance Institutions has been so severely tried, and we may express the confident hope that it will gradually but surely revive under improved official administration.

It is not my intention to dilate on the situation or to add to the flood of criticism, which has appeared in the press in the United States and in our own country, but I think it may be useful to record briefly what has been done to prevent a repetition of the evils exposed, and to consider the important results which have followed in our own country as a consequence of the agitation. In the first place it will be observed with profound gratification that the sound actuarial principles on which Life Insurance is based and carried on have been in no way found wanting, and that it is not out of the application of these scientific principles that the recent difficulties in America have sprung. The troubles have arisen in another direction, from the failure of those in high places invariably to conform to sound principles in the administration of the immense funds entrusted to their charge. No reflections have been cast on the Actuary's capability or probity. Indeed, it is not improbable that if the Actuary in America had held a more dominant official position, and had been heard more in the councils of those responsible for the administration of the Funds, the troubles might never have reached an acute stage.

The intense importance to the American public of the questions at issue will be gathered from the fact that the total Assets of their ordinary and industrial Companies amount to upwards of 500 million pounds, and that, approximately, one half of this amount is held by the three leviathan Companies, whose financial methods have been the cause of so much discussion and disturbance.

The American nation, with its customary business energy and directness of purpose, soon proceeded to put its house in order. A powerful Committee, known as the Armstrong Committee,

held an exhaustive examination and their report of seven volumes comprising 7,000 pages, recommended further legislation with the object of placing the practice of life assurance on a thoroughly safe basis.

What reflections are borne in on our minds by a consideration of the salient features of this Report, which doubtless many of us have examined in abstract? Firstly, it has been demonstrated that the Insurance Laws of America, notwithstanding their drastic regulations and the requirement of Standard Reserves for Liabilities, have not proved effective in attaining their paramount object—the full protection of the policyholder. It has also brought into prominence the great danger and general inexpediency of power over the administration of a Life Office,—and more specially in the Investment of the Funds,—being practically concentrated in the hands of one man, usually called the President, whose influence largely controls the Board of Directors.

May we not venture to hope that one of the results of the investigations now proceeding will be that the Life Assurance Companies in America will ere long adopt the same sound principles as British Life Offices, which protect so efficiently the interests of the policyholders, and under which the sole and undivided responsibility for good administration and the protection of the funds rests with a Board of Directors, who never delegate their powers to anyone else, though they would, of course, be guided by the advice and judgment of their Manager or Chief Officer, in all matters of finance and investment?

Here I may digress so far as to state my entire concurrence with the view expressed by several of my predecessors that it is most desirable for the efficient administration of a Life Assurance Institution that the Chief Officer or Manager should himself be a fully qualified Actuary. On account of his special training and his ingrained habit of analysis and of looking into the future, the Actuary is specially qualified to deal with all the questions of finance and investment which come before the Board of Directors, and it is recognized that his judgment in these matters is of great service. It is highly satisfactory to find that this view now meets with almost universal acceptance, and that, with a very few exceptions, it has been acted upon by the Directors of our Life Offices.

Arising out of what I may call the "American situation" is one very interesting point, to which I should like to draw

your attention. The recently issued Report of the Insurance Commissioner of Massachusetts contains the announcement that the State of New York has passed legislation, as a result of the findings of the Armstrong Committee, which will certainly have a very far-reaching effect. It is there enacted in effect that the standard of Valuation is the Combined Experience Table (17 Offices), and 4 per-cent interest for policies issued prior to 1 January 1901, and the American Experience, with $3\frac{1}{2}$ per-cent interest for subsequent issues, the select and ultimate valuation method to be used for all policies written after the year 1906. Each Company must report the excess of its policy reserve over the reserve computed by the above method. Preliminary term contracts, as that designation is usually understood, are prohibited.

It may be remarked that at the present moment a Canadian Commission is considering the same questions as affecting the position and conduct of life assurance business in the Dominion. I do not therefore propose to make any personal comments on the new laws of the State of New York; but I may at least call your special attention to the fact that a new Standard of Reserve has received legislative sanction, in substitution for the net premium method previously in force in the States. The "Select and Ultimate" method of valuation is associated in this country with the name of Mr. Miles M. Dawson, a Fellow of our Institute.* I do not think I could make any more useful suggestion to any member in search of a suitable subject for investigation than that he should examine the new method critically, pointing out its assumptions, its results, and its merits and demerits generally. Any method of valuation embodied in the laws of a great country may well receive our most serious consideration.

What may be the result of new legislative measures in America and elsewhere we must leave to the future historian, and we shall be more interested for the moment in considering what has happened in our own country as a result of the disclosures. It is not surprising that the British public, so many of whom are policyholders in the American Offices transacting business here, should have been much agitated and that there should even have been a call on the State to afford them protection by requiring deposits from the American Companies commensurate with their liabilities in this country. At the same time it is most satisfactory to find that the stability and sound administration of our own Life Assurance Institutions has not been seriously called in question,

* *Transactions of the Actuarial Society of America*, vol. vii, page 418.

and that the confidence of the British public in them is in no way abated.

In the early part of the year, a question was asked in the House of Lords as to whether the Government proposed to take any steps to protect the interests of British policyholders in American Offices, and Lord Onslow, Chairman of Committees in the House of Lords, promised the appointment of a Select Committee of the House of Lords to consider the question. A Select Committee was duly appointed "to enquire and report "what steps should be taken, by deposit of funds or otherwise, "to provide adequate security for British policyholders in Life "Assurance Companies which have their chief offices outside the "United Kingdom, but which carry on business in this country," and it obtained the evidence of several actuaries and of several persons intimately acquainted with life insurance business. It must be very gratifying to us to find that the recommendations of the Committee are in accordance with the evidence of the actuarial witnesses, which was quite unanimous in its character. Perhaps the main interest in the Report of the Committee lies in the fact that they do not recommend that Foreign and Colonial Companies be compelled to deposit funds in this country, but that they suggest that the deposit of £20,000 which the Act of 1870 makes obligatory on any new company, whether Foreign or British, should be maintained permanently, and not be liable to be withdrawn under the specific conditions mentioned in that Act.

Another important recommendation made by the Committee is to the effect that all Insurance Companies, whether British or Foreign, should be required to furnish the Board of Trade with full Revenue Accounts, Balance Sheets, and Valuation Statements of their business, showing at the same time the expenses of management, and, further, that the Returns to the Board of Trade should distinguish between British and other business. At the end of their Report the Committee state their opinion that it would be "very desirable to provide for a statement in the returns "made by all insurance companies, both British and Foreign, of "the market value of the securities held by them"; but it does not seem clear what is intended, and we await with considerable interest an elucidation of this point.

While in a general way we should welcome any reasonable amendment in the form of the Balance Sheet appended to the Act of 1870 which would have the effect of affording further

useful information to the public, I am disposed to think that it would be most undesirable that any amendment should be made which might lead to the adoption in any form of a legal standard for the Valuation of a Company's Assets.

We may unhesitatingly affirm that Life Assurance has progressed and greatly prospered under the Life Assurance Companies Act, 1870, which statute, while reserving the fullest liberty to the Companies in the conduct of their business, has made it incumbent on them to give to the public the fullest information respecting their transactions and the provision made for the fulfilment of their contracts. The healthy emulation and rivalry, which the Companies of the United Kingdom have been free to embark in, have resulted in their accumulating, for the security of the immense interests of their policyholders, Reserves of a level of strength greater than at any former period and unrivalled in any other country of the world. It is with intense gratification that we can point to this desirable consummation, and also to the improvement in the financial condition of Friendly Societies and other Provident Institutions, being so very largely due to the persistent influence of the actuarial profession in the inculcation of scientific principles and sound practice.

Thirty-six years have elapsed since the passing of the Act of 1870, and, concurrently with a great expansion in the business, the Funds of Ordinary Life Offices have grown from about 100 millions in that year to upwards of 300 millions at the present time. The great changes that have taken place in the circumstances of Life Assurance, and other considerations, would seem to indicate that the time has now arrived when the legislative enactments of 1870 might be amended with advantage.

As a result of the Report of the Select Committee referred to, it is probable that proposals will shortly be brought forward for an amendment of the Act of 1870, but, while it may be found desirable to make several amendments of importance with a view to carrying out the suggestions of the Committee and to improving the value of the Official Returns of Life Assurance Companies, there is not, I should imagine, any intention of departing from the main principles of the existing Acts, which a long experience has shewn to be sound and beneficial.

In 1892, the Board of Trade approached the Institute, and also the Faculty of Actuaries in Scotland and the Life Offices Association, as to an amendment of the Act of 1870, but for some reason the proposals fell through. Should the Board of Trade

follow a similar course on the present occasion, and address a communication to the Institute asking it to express its views regarding the amendments proposed, I feel sure that the Institute through its Council will welcome the communication and will give ungrudgingly its advice and cordial support to the Government in their endeavours to perfect legislation regarding Life Offices. It is, however, not unlikely that the Council, in view of the imminence of the proposed legislation, may be of opinion that it would be better policy for them to give the question their immediate attention and to formulate a plan for the amendment of the Life Assurance Companies' Acts, and then to submit it, with a memorandum explanatory of their views, for the consideration of the Board of Trade. My own opinion is strongly in favour of the Council of the Institute thus taking the initiative.

I venture to hope that among the amendments there will be one defining what is meant by an "Actuary", in the way I have explained before. As having an interesting bearing on this point, I may call your attention to a provision in the new French law, 1905, which directs that there shall be a "Consulting Committee on Life Assurance" in communication with the Minister of Commerce, which shall consist of 21 members, including two Senators and three Deputies, State Officials, and other persons of importance; and also "three duly qualified Fellows of the French Institute of Actuaries." There is no ambiguity here as to what is meant by an "Actuary."

And now I should like to make a few remarks, addressed more especially to the younger Members of the Institute.

It is frequently said that the prospects of advancement in our profession are contracting, but I do not myself think that our profession is as over-crowded as many other of the learned professions, such as Law and Medicine. It is true that, owing to the reduction in the number of Life Offices by amalgamation, the number of positions as Chief Officer is less than formerly, but it must be remembered that, although the number of Companies is less, the amount of business transacted is not only greater than at any former time, but that it is reasonable to assume a great expansion in the future. The public have come to a better understanding and appreciation of the advantages of life assurance. Hence, our Offices will require more actuarial assistance, and there will consequently be a demand for a greater number of Assistant Actuaries, and, therefore, though you cannot all rise to be chief officers, it is not improbable there

will be an increasing number of lucrative posts which can only be filled by qualified Actuaries. Then again, the sphere of actuarial work outside our Life Offices is greatly developing, so that there may be plenty of work for Consulting Actuaries. Moreover, I believe it will come to pass that our great Government Departments will require to have qualified Actuaries permanently attached to them. While I do not, therefore, take a despondent view of your prospects, I may as well state my opinion that the Institute already numbers nearly sufficient Members, and that clerks in our Life Offices should not be indiscriminately advised to join the Institute, but only young men of good education, with fair mathematical knowledge, who are prepared to go through the long course of hard study required of them in order to pass the examinations for the Fellowship.

When I joined the Institute, it provided no direct educational facilities, except of course the broad and liberal education obtainable from the study of the invaluable contributions made to the *Journal,* nor were there any text-books specially adapted for acquiring actuarial knowledge; but for very many years past students have had the great advantage of excellent text-books and of classes for Parts I and II of the examinations, while Associates studying for Parts III and IV receive great help from lectures. The Council, I believe, have every intention of arranging for the continuation of lectures similar to those now being so admirably given by Mr. King.

I would further impress upon you the advantage, both to yourselves and to others, of contributing papers for discussion at our sessional meetings. I am aware that the preparation of a paper of sufficient importance to be read at a meeting involves much study, thought, and application, and that perhaps few of our Members can find leisure after their daily avocations to embark on such work. In order that a greater number of our Members may be able to make themselves and their capabilities better known, I would suggest that one evening at least during each session might be set aside for the discussion of short notes, or memoirs, and I think it would not be impracticable to have three or more in one evening. Such a plan would give a larger number of Members the opportunity of joining in the discussion, and we should probably have a very instructive evening of debate.

In concluding my remarks, I have great pleasure in referring to the very cordial relations which continue to exist between our Institute and the Faculty of Actuaries in Scotland, and also

in assuring our Brother Actuaries abroad of the constant and fraternal interest which we all take in the work and progress of the various Actuarial Societies in Foreign Countries, and of our gratification at their increasing influence for the good of Provident Institutions, the betterment of the economic functions of their respective countries, and the consequent social development of their communities.

Model Office Reserves for Endowment Assurances. *By* JAMES BUCHANAN, M.A., D.Sc., *Fellow of the Institute of Actuaries, and of the Faculty of Actuaries.*

1. WHEN Mr. Manly first put forward the idea of constructing a Model Office for whole of life policies, endowment assurance business was in its infancy. That idea was afterwards improved on by Mr. King, who substituted an actual experience of entrants and withdrawals for the assumptions which Mr. Manly, in the absence of data, was forced to make; and when, twenty-five years later, another Model Office was constructed on the basis of the new experience, the age distribution of the business was found to be so little altered, that it was possible to retain the older figures. In the meantime, however, endowment assurances had ceased to be a minor class; and their growth has been so rapid and continuous that they now form one of the most important sections of the business. It was probably the hope of many that Mr. King would have found time to extend his Model Office to endowment assurances, and it was only because he stated that his other duties would not permit him at present to do so, that I have undertaken to supply the extension.

2. The initial difficulties in the construction of a Model Office for endowment assurances are considerable; for the new experience gives us no information about endowment term, and a knowledge of that is essential. The unadjusted data, however, show that the terminations tend to group themselves round quinquennial maturity ages; and it occurred to me that it might be possible, from the numbers so recorded and the corresponding numbers remaining in force, to work back to the numbers entering at each age for the various endowment terms. The "Old" Assurances form practically a completed experience, but of

ices,

Endowment Term	41	40	39	38	37	36	35	under 35
under 10 years	...	...	...	...	...	...	...	...
10	...	...	..	1	...	1	2	4
11	...	...	...	...	...	...	...	...
12	...	1	...	...	...	...	...	...
13	...	2	...	...	...	...	...	1
14	...	2	...	...	...	...	...	1
15	3	12	3	6	5	3	8	6
16	...	2	...	...	...	...	...	...
17	...	...	...	...	...	...	...	...
18	...	7	...	...	...	...	...	...
19	1	5	...	...	1	...	...	...
20	5	4	2	1	2	2	2	22
21	...	4	...	...	...	...	...	...
22	...	2	...	...	...	...	...	...
23	...	1	...	...	...	...	...	...
24	...	...	...	...	...	...	...	...
25	1	1	1	...	...	...	...	...
26	...	...	...	...	...	1	...	...
27	...	1	...	...	...	...	...	...
28	...	...	...	...	...	..	...	...
29	...	...	...	...	...	...	..	...
30	...	...	...	...	...	...	...	...
31	...	...	...	...	...	...	...	...
32	...	1	...	...	...	...	...	...
33	...	...	...	...	..	...	...	...
34	...	...	...	...	...	...	...	...
35	...	...	...	...	...	...	...	...
36	...	...	...	...	...	...	...	...
37	...	...	...	...	...	...	...	...
38	...	...	...	...	...	...	...	...
39	...	...	...	...	..	...	...	...
40	...	...	...	...	...	...	...	...
41	...	...	...	...	...	...	...	...
42	...	...	...	...	...	...	...	...
43	...	...	...	...	...	...	...	...
44	...	...	...	...	...	...	...	...
45	...	...	...	...	...	...	...	...
46	...	...	...	...	...	...	...	...
47	...	...	...	...	...	...	...	...
48	...	...	...	...	...	...	...	...
49	...	...	...	...	...	...	...	...
Total	10	45	6	8	8	7	12	34

Table I.—*Analysis of one year's ordinary with profit Endowment Assurance Business in six British Offices, showing the number of policies effected for the various Maturity Ages and Endowment Terms.*

Endowment Term	Maturity Age: over 70	70	69	68	67	66	65	64	63	62	61	60	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42	41	40	39	38	37	36	35	under 35
under 10 years	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
10	...	1	...	1	...	1	6	1	1	1	2	3	...	1	1	1	2	3	1	...	1	2	1	1	...	...	1	...	1	1	...	...	...	1	...	1	2	4
11	...	...	...	...	...	...	2	...	...	...	...	...	...	...	...	...	4	...	...	...	...	1	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...
12	...	1	...	...	...	...	1	...	...	...	...	2	1	...	...	1	6	...	...	...	1	2	...	...	...	...	1	...	...	...	...	1	...	...	...	...	...	...
13	1	1	...	...	...	...	2	...	...	...	1	5	1	...	...	...	1	...	...	1	...	1	...	...	...	...	1	...	...	...	...	2	...	...	...	...	...	1
14	...	...	...	...	...	...	3	...	...	1	1	8	1	...	1	1	3	...	1	1	...	3	1	1	...	...	5	...	...	...	...	2	...	...	...	...	...	1
15	3	...	4	2	4	5	22	2	7	5	9	11	6	5	4	9	17	7	6	6	11	19	5	8	11	6	11	4	3	4	3	12	3	6	5	3	8	6
16	...	1	...	...	...	...	13	2	...	...	...	20	3	...	...	1	17	...	...	...	1	6	...	2	...	...	5	1	...	...	...	2	...	...	...	...	...	...
17	...	1	...	...	...	...	15	1	...	...	1	25	1	...	...	...	17	2	...	...	...	25	...	1	...	...	4	...	...	...	...	...	...	...	...	...	...	...
18	1	2	...	...	...	...	15	1	...	...	1	40	2	...	...	1	27	6	...	...	...	26	...	...	...	1	8	...	...	...	...	7	...	...	...	...	...	...
19	...	3	...	...	...	...	10	2	...	4	1	26	...	...	...	1	32	1	...	...	2	33	2	...	...	...	12	...	...	...	1	5	...	...	1	...	...	...
20	1	9	...	7	7	11	48	8	13	17	11	68	10	14	24	20	58	8	28	23	29	60	20	17	12	20	20	7	13	8	5	4	2	1	2	2	2	22
21	...	...	...	...	...	...	18	...	...	2	...	59	2	...	...	2	54	1	2	...	1	48	3	...	1	1	14	...	...	...	...	4	...	...	...	...	...	...
22	...	...	...	...	...	...	19	...	...	...	...	57	2	...	...	...	48	1	...	...	...	39	3	...	...	...	13	1	...	...	...	2	...	...	...	...	...	...
23	...	...	...	...	...	...	17	...	...	...	1	37	1	...	...	1	57	3	...	...	...	61	1	...	...	...	3	...	...	...	...	1	...	...	...	...	...	...
24	...	1	...	...	...	...	17	1	2	...	...	53	3	...	1	...	58	2	...	1	3	46	...	...	...	2	9	...	...	...	...	...	...	...	...	...	...	...
25	1	1	...	...	1	4	21	1	5	3	10	65	6	18	5	15	73	11	13	11	9	82	16	7	8	8	11	5	1	...	1	1	1	...	...	...	...	...
26	...	1	...	...	...	...	17	...	...	...	...	62	1	2	...	1	66	4	...	...	1	54	2	...	...	...	4	...	...	...	...	...	...	...	...	1	...	...
27	...	...	...	...	...	...	20	1	...	...	...	67	3	...	...	...	64	7	...	...	...	55	1	...	...	...	4	...	...	...	...	1	...	...	...	...	...	...
28	...	1	...	...	...	...	13	...	1	...	...	76	3	...	...	...	67	4	...	...	...	32	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
29	...	1	...	...	...	...	12	2	1	...	...	78	2	...	1	...	74	1	...	1	...	54	2	...	...	...	1	1	...	...	...	...	...	...	...	...	...	...
30	1	1	1	...	...	3	21	...	3	8	7	90	9	12	9	12	81	12	1	5	8	29	7	2	2	2	1	...	...	...	...	...	...	...	...	...	...	...
31	...	1	...	...	...	...	14	2	...	...	1	74	5	...	1	1	59	2	...	...	...	17	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
32	...	...	...	...	...	...	14	...	...	...	...	51	7	...	...	...	59	1	...	...	...	11	1	...	...	...	...	...	...	...	...	1	...	...	...	...	...	...
33	...	1	...	...	...	...	10	1	...	...	...	77	7	...	...	...	43	1	...	...	...	8	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
34	...	...	...	...	...	...	10	...	...	...	...	52	1	...	...	...	37	2	...	...	1	6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
35	...	...	...	...	...	...	12	...	1	1	1	65	4	2	...	2	15	...	...	1	...	4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
36	...	...	...	...	...	...	14	1	...	...	...	49	...	...	...	...	9	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
37	...	...	...	...	...	...	8	...	...	...	...	35	1	...	...	...	9	1	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
38	...	2	...	...	...	...	6	...	...	...	...	29	1	...	...	...	2	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
39	...	...	...	...	...	...	7	1	...	...	2	30	1	...	...	1	4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
40	...	...	...	...	...	...	4	1	1	...	...	27	2	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
41	...	...	...	...	...	...	6	...	...	...	...	7	...	...	...	...	...	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
42	...	...	...	...	...	...	4	2	...	...	...	9	1	...	...	...	...	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
43	...	...	...	...	...	...	5	...	...	...	...	5	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
44	...	1	...	...	...	...	5	...	...	...	...	...	1	...	...	...	...	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
45	...	...	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
46	...	...	...	...	...	...	1	1	...	...	...	2	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
47	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
48	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
49	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Total	8	31	5	10	12	24	433	31	35	42	49	1,370	89	54	47	70	1,064	81	52	51	68	728	66	39	34	40	129	19	18	13	10	45	6	8	8	7	12	34

the "New" Assurances only the shorter terms have had time to mature, and for the longer durations the data are far too meagre to form the basis of a Model Office. Probably, too, the class of business transacted at the present time differs as regards endowment term and maturity age from that of the time when the "Old" Assurances were effected: and, after some work spent in this direction, I decided to discard the figures based on the Institute and Faculty experience.

Office Data.

3. It was suggested to me by Mr. Lidstone that it might be possible to get from five or six representative offices, competing for endowment assurances, an analysis of one year's new business. I had some hesitation in approaching offices with this request, but some such data appeared to be necessary; and I am indebted to the actuaries of six leading offices for the statistics which form the basis of the present paper. The data were supplied on cards showing, for each endowment term, the sums assured and corresponding number of policies effected to mature at all ages from 35 to 70, both inclusive, and at ages under 35 and over 70. The treatment of rated-up cases varies in office valuations; but it seemed desirable, while including all the business which would form part of an ordinary grouped valuation of endowment assurances, to adhere as closely as possible to actual facts. These were accordingly included at their real ages, and the analysis was limited to ordinary endowment assurances with profits. Of the six offices, three were mutual and three proprietary—four English and two Scottish—and, as the statistics relate to over 4,800 policies assuring close on a million and three quarters, they may fairly be taken as representative of this class of business, and as sinking the identity of individual offices.

4. It is well known that the great mass of endowment assurance business is effected to mature at quinquennial ages, or at the end of quinquennial periods; but representative statistics have never been published, and a detailed analysis is given in Table I. It will be seen that 60 is the favourite maturity age, about two-sevenths of the total number of policies having been effected to mature at that age. Twenty years appears to be the favourite endowment term, but the average endowment term is somewhat longer. The figures run with great regularity and range themselves along lines crossing at right angles at quinquennial intervals. There is also a slight tendency for business to group itself on each side of the quinquennial maturity

Table II.—*Summary of New Business arranged according to Endowment Term and Maturity Age.*

Endowment Term	Number of Policies	Sum Assured	Average Amount of Policy	Maturity Age	Number of Policies	Sum Assured	Average Amount of Policy
under 10 years	1	2,000	...	over 70	8	10,850	...
10 years	42	12,050	333	70	31	17,220	725
11	8	1,150	...	69	5	2,550	...
12	17	7,450	...	68	10	8,550	...
13	18	11,100	...	67	12	6,950	...
14	34	14,000	...	66	24	16,200	...
15	265	151,350	485	65	433	196,810	458
16	74	23,370	...	64	31	11,900	...
17	93	35,100	...	63	35	13,060	...
18	138	42,500	...	62	42	17,500	...
19	136	37,250	...	61	49	48,600	.
20	631	278,520	384	60	1,370	481,010	373
21	212	85,050	...	59	89	29,300	...
22	185	56,700	...	58	54	21,260	...
23	183	51,150	...	57	47	14,990	...
24	199	67,250	...	56	70	29,530	...
25	414	117,650	311	55	1,064	317,870	315
26	216	77,300	...	54	81	33,250	...
27	223	70,520	...	53	52	18,550	...
28	197	74,100	...	52	51	19,500	...
29	231	63,720	...	51	68	21,550	...
30	327	94,600	311	50	728	208,790	297
31	178	52,450	...	49	66	20,900	...
32	148	50,850	...	48	39	11,950	...
33	148	43,700	...	47	34	11,620	...
34	109	38,950	...	46	40	7,780	...
35	108	33,850	314	45	129	25,850	223
36	74	24,390	...	44	19	3,650	...
37	55	13,980	...	43	18	4,700	...
38	40	12,990	...	42	13	3,000	...
38	46	13,750	...	41	10	2,400	...
40	36	9,150	282	40	45	15,750	281
41	14	3,300	...	39	6	700	...
42	17	3,950	...	38	8	1,200	...
43	11	3,450	...	37	8	1,000	...
44	8	3,300	...	36	7	1,300	...
45	1	300	330	35	12	3,450	461
46	4	1,100	...	under 35	34	22,400	...
47 & over	1	100	...				
...	4,842	1,683,440	348	...	4,842	1,683,440	348

age, which may be due in part to the not uncommon practice of effecting—not necessarily at the same time—two or three policies to mature in successive years of age. The slight preponderance in the numbers effected to mature in the years below the quinquennial ages is accounted for by the fact that in the case of one office, whose policies all mature on the anniversary of the date of issue and which charges premiums by half-years of age, those effected in the first half-year of age were tabulated as maturing at the nearest maturity age.

5. A table showing the distribution of the sums assured presented the same general features; but there were unavoidable irregularities due to the presence of occasional policies of large amount, and it seemed that the special features would be made more apparent by adopting some modes of grouping. This is done in Table II, which shows the sums assured and number of policies effected for each endowment term and each maturity age, with the average amount per policy for quinquennial groups. Generally it will be seen that the higher the rate of premium, the greater is the average amount of the policy. There is a steady increase in amount with increase of age at maturity, and a similar tendency to increase as the endowment term decreases—the irregularities at the ends of the table being due to the presence of a few policies of abnormal amount disturbing the average. This tendency to increase is no doubt due to the fact that policies of short endowment term are effected, largely for investment purposes, at the older ages by men in a settled position in life, who can afford to pay the larger premium.

TABLE III.—*Summary of New Business arranged according to Grouped Ages at Entry.*

Grouped Ages at Entry	Number of Policies	Sum Assured	Average Amount of Policy
Over 52	29	24,800	855
48–52	108	67,020	620
43–47	288	137,090	476
38–42	519	237,370	457
33–37	821	296,040	361
28–32	1,221	412,210	338
23–27	1,236	346,390	280
18–22	534	120,570	226
Under 18	86	41,950	488
...	4,842	1,683,440	348

6. The numbers entering at each age may be obtained by summing the figures of Table I in lines running diagonally from the left upwards. Detailed figures for each age at entry presented no features of special interest; but in Table III are given the numbers entering and the corresponding sums assured for quinquennial age groups,—a few policies effected to mature at ages under 35 or over 70 being assigned approximately to their proper entry age. As might be expected from the figures of the preceding table, the average amount increases steadily with age at entry; and it may be noted that more than half of the total figures, both as regards numbers entering and sums assured, relate to entry ages under 32. It is of interest to compare these figures with the numbers of "New" Assurances effected for corresponding age groups in the endowment assurance experience, and this is done in Table IV. Bearing in mind the fact that in one case the ages are for the most part office ages and in the other nearest ages, the agreement in the percentages seems to show that the office data are a good sampling of present day endowment assurance business, which may fairly be made the basis of the new business of a Model Office.

Table IV.—*Proportionate Numbers entering in each Age Group.*

Grouped Ages at Entry	Endowment Assurance Experience		One Year's Experience of Six Offices	
	Number of Lives	Percentage	Number of Policies	Percentage
Over 52	410	·31	29	·60
48–52	1,600	1·19	108	2·23
43–47	4,959	3·67	288	5·95
38–42	12,189	9·02	519	10·72
33–37	22,284	16·50	821	16·95
28–32	34,955	25·88	1,221	25·22
23–27	38,934	28·81	1,236	25·53
18–22	17,849	13·21	534	11·02
Under 18	1,880	1·39	86	1·78

New Business of Model Office.

7. In Mr. King's Model Office the data are grouped according to central age at entry; but, in endowment assurances, term is of far more importance than age in determining rates of premium and reserves, and this must clearly form the basis of the grouping. A further grouping could be made according to age at entry or age at maturity; but, from the way in which the data of Table I are arranged,

that according to maturity age seemed preferable. The figures there range themselves along lines intersecting at right angles at quinquennial intervals; and, taking all the business transacted for two years on each side of the points of intersection, the totals were assumed to be effected for the central endowment term to mature at the central maturity age. Thus all policies, effected for terms of from 18 to 22 years to mature at ages lying between 53 and 57, were treated as effected for a term of 20 years to mature at 55. This involves a slight mixing up of entry ages, which the arrangement of the data of Table I shows to be quite unimportant. Very little endowment assurance business is transacted at ages under 20 or over 50, and the figures lying outside the dotted lines in Table I were excluded—the practical effect being to include all policies completed at ages lying between 18 and 52 for terms not exceeding 42 years. A few policies of abnormal amount, maturing at very early or very advanced ages—which it would have been desirable to omit from a grouped valuation—were thereby excluded. The figures obtained in this way for the sums assured were rather unwieldy, and the average for the six offices was taken. The Model Office is therefore one which issues each year policies for amounts distributed over the various maturity ages and endowment terms as shown in Table V.

TABLE V.—*New Business of Model Office.*

Maturity Age	Endowment Term							Total
	10 years	15 years	20 years	25 years	30 years	35 years	40 years	
35	200	692	...	...	...	...	...	892
40	117	2,158	1,067	...	...	...	...	3,342
45	242	2,650	4,130	1,445	...	...	...	8,467
50	692	6,408	16,100	16,212	6,387	...	...	45,799
55	967	6,337	19,448	19,780	17,033	4,917	...	68,482
60	742	8,825	25,142	18,308	26,392	15,173	4,638	99,220
65	...	8,833	12,107	7,158	5,017	4,388	1,833	39,336
70	...	...	1,578	592	608	167	67	3,012
Total	2,960	35,903	79,572	63,495	55,437	24,645	6,538	268,550

Rates of Withdrawal.

8. This new business must now be made to pass off the books, making allowance for the influences of both death and discontinuance. Following Mr. King, from the table of distribution of withdrawals, the withdrawals for each of the first

ten years of assurance were assigned to their proper policy years; and from ten years onward by making an adjustment between two years of duration, and the probabilities of surviving death and withdrawal were computed. At this point another difficulty was encountered. The withdrawals tabulated for each age at entry relate to policies of all endowment terms; but the investment element is present much more largely in some than in others, and the rate of withdrawal may be very different in a 10 year endowment assurance and in one effected at the same age for a 40 year term. Some adjustment therefore seemed to be necessary. From the numbers exposed to risk of death, for the same grouped ages at entry, and the numbers of deaths, the probabilities of surviving each year were obtained; and thence were deduced the probabilities of surviving the risk of withdrawal. The average rates of withdrawal for the first ten assurance years, obtained in this way, are set out in Table VI.

TABLE VI.—*Rates of Withdrawal for Grouped Ages at Entry according to the Endowment Assurance Experience.*

Year	Central Age at Entry						
	20	25	30	35	40	45	50
1	4·72	3·55	2·93	2·19	2·19	2·17	1·91
2	10·85	9·39	7·40	5·86	5·34	5·48	5·09
3	6·42	5·66	4·37	3·42	3·49	2·95	2·37
4	5·36	3·99	3·46	2·79	2·82	2·43	1·89
5	4·50	3·21	2·91	2·46	2·43	1·96	1·60
6	3·64	3·17	2·32	2·25	1·65	1·91	1·42
7	2·95	2·37	1·94	1·91	1·94	1·37	·83
8	2·34	2·26	1·98	1·69	1·32	1·32	1·69
9	2·55	2·10	1·89	1·63	1·58	1·30	1·96
10	2·23	1·85	1·53	1·28	1·40	1·07	1·49

9. In a paper read before the Faculty of Actuaries last session (T.F.A., iii, 11), Mr. Chatham drew attention to the fact that rates of withdrawal, deduced from the ultimate data, excluding the first ten years, tend to become approximately constant. Working from the select data for grouped ages at entry, I had observed a similar tendency to constancy; but thought it might be due to meagreness of data. However, as it appeared for each age at entry, and also when using the fuller data of the ultimate table, the feature may be taken to be a real one. It was suggested that this was probably due to the favourable terms on which

endowment assurances may be discontinued during the year or two preceding maturity; but, from a statement made by Mr. Ackland in the "Account of the Principles and Methods adopted in the Compilation of the Data" (p. 46), it appears that maturities paid in advance have, in large numbers, been treated as maturities.*

10. There may have been other influences affecting the rates of withdrawal deduced from the endowment assurance experience. It must be remembered that the endowment assurance experience is a mixed experience, in which the proportion of non-profit business is considerable; and the withdrawals in this section are likely to be more numerous. Again, a very large proportion of endowment assurance business is effected at the younger ages, and a comparison of the rates of withdrawal given above with the corresponding O^M rates of discontinuance given in Mr. King's paper (*J.I.A.*, xxxvii, 463), shows that, for these ages, the endowment assurance rates are generally higher from the very first, when the influence of discounted claims could not be operative. In an interesting letter on the subject of discontinuances (*J.I.A.*, xxxiii, 273), Mr. Todhunter has drawn attention to the fact that the force of withdrawal is essentially different from that of mortality, operating not continuously, but at definite moments of time; and there seems to be little doubt that the more frequent the recurrence of the date when the policyholder has to decide whether to pay or not to pay the premium, the more likely is the policy to be discontinued. This suggested that some light might be thrown on the question by an examination of the tables of distribution of withdrawals for endowment assurances and for whole of life participating policies respectively. The withdrawals are there divided into four groups, the central points of which correspond to the durations of the lapses occurring in the four quarters of the year. Those occurring during the period "6-8 months" would include lapses, under policies subject to half yearly and quarterly premiums, taking place in the middle of the year, with a probably small number of surrenders of policies subject to yearly premiums. Table VII gives the total numbers of withdrawals, occurring in the first ten years in the two experiences, with the numbers taking

* In the discussion which followed the reading of Mr. Chatham's paper, certain office statistics were quoted and were taken as showing that the higher rate of withdrawal in endowment assurances is due to discounted claims. The work of that investigation was carried out by the writer, and discounted claims were treated as maturities.

place in the middle of the year; and, for every age group, the percentage of the latter is greater in the endowment assurance experience. This seems to suggest that a larger proportion of endowment assurances had been effected at half yearly rates of premium; and if, as seems probable, lapses are more numerous in this section, it might account to some extent for the rate of withdrawal being higher and remaining higher to the very end of the assurance.

TABLE VII.—*Proportion of Withdrawals occurring in the middle of the Year in the Endowment Assurance and in the Whole-Life (with Profit) Experience.*

Grouped Ages at date of Assurance	Endowment Assurance Experience			Whole-Life—with Profit—Experience		
	Total Withdrawals during first ten years	Withdrawals during the period '6-8 months'	Percentage	Total Withdrawals during first ten years	Withdrawals during the period '6-8 months'	Percentage
18–22	5,394	986	18·28	15,071	2,147	14·24
23–27	10,177	1,802	17·71	38,200	5,692	14·90
28–32	7,586	1,335	17·60	39,609	5,932	14·97
33–37	4,100	706	17·22	29,932	4,497	15·03
38–42	2,092	403	19·27	19,629	3,031	15·44
43–47	768	146	19·01	11,612	1,821	15·68
48–52	216	43	19·91	6,135	966	15·75
...	30,333	5,421	17·87	160,188	24,086	15·04

11. Following Mr. Chatham, the rate of withdrawal was assumed to remain constant after about 14 years from entry—the constant varying from 1·3 per-cent for central age 20 to 1 per-cent for central age 50; but these rates were made to decrease gradually for the last five years preceding maturity. The initial difficulty was a more serious one, and some more or less satisfactory assumptions had to be made. In one of his papers on Staff Pension Funds, Mr. Manly made use of a spline, or tapering ruler made of lancewood, for the purpose of graduating some rough data; and, where so much had to be left to assumption, it seemed desirable to use some mechanical means of guidance. The rates, given in Table VI, were plotted out on cross ruled paper, and a curve was drawn, so as to run smoothly through them. For the first two assurance years, the assumption was made that the withdrawals amongst ten and fifteen-year policies, which partake more of the investment element, were

20 per-cent less numerous*; and a curve was then drawn, by means of the spline, so as to run nearly parallel to the first. The assumptions here made are arbitrary and suggest criticism; but a series of rates were obtained decreasing gradually to zero about the fifteenth year. Rates of withdrawal vary very much in different offices, and possibly those obtained in this way do not differ more from true rates than do actual rates in different offices. The adjusted rates for central ages 25, 35 and 45 are given in Table VIII, with the corresponding O^M rates for comparison.

TABLE VIII.—*Adjusted Rates of Withdrawal.*

Year	Adjusted O[EM] Rates of Withdrawal			Adjusted Rates for Policies of Short Endowment Term			O[M] Rates of Withdrawal *J.I.A.*, xxxvii, 463		
	25	35	45	25	35	45	25	35	45
1	3·55	2·35	2·17	2·85	1·88	1·74	3·05	2·42	2·25
2	9·5	6·25	5·4	7·6	5·1	4·3	8·59	6·56	5·70
3	6·35	4·25	3·62	4·85	3·37	2·7	5·81	4·44	3·88
4	4·75	3·12	2·6	3·35	2·3	1·75	4·21	3·40	2·92
5	3·67	2·42	1·95	2·4	1·6	1·17	3·27	2·66	2·44
6	2·95	2·	1·6	1·75	1·12	·85	2·78	2·34	2·09
7	2·45	1·75	1·45	1·33	·85	·6	2·3	2·08	1·7
8	2·1	1·55	1·35	1·05	·7	·52	2·17	1·72	1·49
9	1·85	1·4	1·3	·8	·55	·46	1·91	1·67	1·33
10	1·65	1·3	1·27	·68	·42	·4	1·68	1·52	1·21

Model Office for Endowment Assurances.

12. The probabilities of living for each of the first ten years, derived from the select data, were joined on to the ultimate rates, as these ran with greater regularity; and were then combined with the adjusted rates of withdrawal given above so as to obtain a new set of probabilities of surviving death and withdrawal. The sums assured for each age and each endowment term were then multiplied by the proper rates, so as to get the amounts remaining on the books for each year of duration; and for each endowment term the figures for all maturity ages were then thrown together.

* It seems probable that for 10-year endowment assurances, the proportion should be even greater than this. Thus, Mr. Hunter has stated (*J.I.A.*, xxxvi, 80), that in one case he found the lapse rate under the 10-year endowment plan to be about one half that under the ordinary life plan. Lapses in America are more numerous than in this country, and leave room for wider fluctuations; and in any case the proportion of business effected for very short terms is not great.

Endowment term is thus in Table XI made to take the place of age at entry in the Model Office for whole of life policies.

13. The practice of stating the existing endowment assurance business in the Board of Trade returns according to unexpired term is now fairly common, and from the advantages arising from this mode of statement it seems likely that it will become even more general. A summary of the business in force in the Model Office at the end of quinquennial periods is therefore given in this form in Table XII. As new business is effected at the beginning of each year, the figures in the first line opposite "unexpired term 0" represent the amounts of business maturing at the end of quinquennial periods of the history of the office. It will be seen that the rate of increase is very rapid for the first fifteen or twenty years; and in the case of an office which has been doing a steadily increasing new business, this would be even more marked. Table XIII gives a summary of the business in force at the end of quinquennial periods arranged according to maturity age. At all periods the amount effected to mature at age 60 is largely in excess of any other; and, when the office is nearing a stationary state, it amounts to about two-fifths of the total.

Mean Valuation Ages.

14. From the closeness in the values of c, on which Mr. Lidstone's Z's for the H^M and $O^{M(5)}$ tables are based, it is evident that the mean valuation ages derived from these tables can never differ sensibly. However, it seemed desirable to ascertain the direction and extent of the variation; and the mean ages were computed according to each table. The ages in most cases agreed exactly and never differed by more than a tenth of a year; so that H^M Z's might without appreciable error be used for an $O^{M(5)}$ valuation, or *vice versa, provided they be used throughout.* Those derived from the O^M table are, as will be seen from Table XIV, sensibly lower than either; and, as was pointed out by Mr. Lidstone, it would be necessary in passing to an O^M valuation to change the Z's, or to make a small graduated deduction from the H^M or $O^{M(5)}$ mean valuation ages.

15. Policies in the Model Office are supposed to be issued at the beginning of the year, so that the unexpired term is in every case integral and the mean valuation age is obtained by deducting the term from the mean maturity age. Table XIV shows that the mean maturity age fluctuates within very narrow limits. The chief variations occur in the values for the five year office,

and seem to be due mainly to the exclusion of business effected at ages under 18 and over 52. Policies issued at ages over 52 were all for short terms maturing at rather advanced ages; while those taken out below age 18 were generally effected to mature at ages below the average; and their inclusion would have raised the mean maturity age at one end of the table, and lowered it at the other. This appears to confirm the view expressed by Mr. Lidstone that, in any ordinary distribution of business, the mean maturity age is never likely to exceed 61.

Values of Sums Assured and Net Premiums.

16. The net premiums for each of the three principal valuation tables, at three rates of interest, are given in Tables XV-XVII, arranged according to unexpired term. As the premiums are in every case due, the unexpired term is exactly equal to the number of unpaid premiums. The annuities corresponding to the mean valuation age were obtained for each of the valuation tables; their values were differenced and a check applied by placing these differences at three rates of interest side by side. Since, for the whole business,

$$\Sigma(\mathrm{SA})=\Sigma(\mathrm{S})-\Sigma(d\mathrm{S}\mathbf{a})$$

the same mean annuities may be used for finding the values of both sums assured and net premiums, and the trouble of entering conversion tables was thus avoided. As the use of four figure logarithms would probably have led to considerable irregularities in the fifth significant figure, five figure logarithms were employed for this portion of the work. A further check was then applied to both sets of values. Taking, for example, the results of the O^{M} and $\mathrm{O}^{\mathrm{M}(5)}$ valuations, the differences of corresponding values at three rates of interest were placed alongside one another, and any irregularities carefully noted and examined. The values were then summed in quinquennial groups, and the results are given in some detail in Tables XVIII-XX, and XXIII-XXV. It is hoped that the arrangement, according to unexpired term, of the values of sums assured and premiums separately will render the results of sufficient value to justify the additional space which the tables are made to occupy.

Errors in Z-Valuation.

17. A detailed valuation was also made by one of the tables, both as a further test of the accuracy of the work, and as a means of exhibiting in practical shape the extent and direction of the error involved in the Z-method of valuation. The closeness of the approximation afforded by that method is now universally recognized; but, in order to apply as severe a

test as possible, the additional valuation was made at 3 per-cent by the O^M table, which does not follow Makeham's law. The "errors" tabulated in Table IX are the amounts by which the figures of the grouped valuation are in excess (+) or defect (−) of those by the detailed valuation.

TABLE IX.—*Errors in Values of Sums Assured and Net Premiums, and in Reserve, brought out by the Z method of valuation.*

O^M 3 *per-cent.*

Unexpired Term	Age of Office							
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years
	Errors in Values of Sums Assured							
1–4 years	...	0	− 2	− 10	− 12	− 7	− 11	− 11
5–9	+ 3	− 3	− 18	− 21	− 22	− 36	− 35	− 35
10–14	+ 20	+ 30	+ 32	+ 56	+ 50	+ 21	+ 21	+ 21
15–19	+ 90	+120	+121	+101	+144	+144	+144	+144
20–24	+138	+179	+212	+227	+217	+217	+217	+217
25–29	+ 60	+ 79	+ 82	+ 82	+ 82	+ 82	+ 82	+ 82
30–34	+ 43	+ 49	+ 49	+ 49	+ 49	+ 49	+ 49	+ 49
35–39	+ 4	+ 4	+ 4	+ 4	+ 4	+ 4	+ 4	+ 4
Total ...	+358	+458	+480	+488	+512	+474	+471	+471
	Errors in Values of Net Premiums							
1–4 years	...	− 1	+ 9	+ 18	+ 28	+ 22	+ 14	+ 16
5–9	+ 6	+ 45	+112	+162	+134	+136	+115	+115
10–14	+ 78	+212	+338	+267	+169	+163	+163	+163
15–19	+113	+235	+192	+ 90	− 18	− 18	− 18	− 18
20–24	+ 9	− 45	−138	−192	−192	−192	−192	−192
25–29	+ 5	− 35	− 66	− 66	− 66	− 66	− 66	− 66
30–34	+ 2	− 9	− 9	− 9	− 9	− 9	− 9	− 9
35–39	− 2	− 2	− 2	− 2	− 2	− 2	− 2	− 2
Total ...	+211	+400	+436	+268	+ 44	+ 34	+ 5	+ 7
Error in Reserve	+147	+ 58	+ 44	+220	+468	+440	+466	+464

18. In an additional note appended to his last paper on the "Valuation of Endowment Assurances in Groups", Mr. Lidstone has given a table (*J.I.A.*, xxxviii, 49) showing, for certain assumptions as regards the distribution of business, the amounts by which the true mean annuity differs from that corresponding

to the mean valuation age; and the results of that table are entirely confirmed by those of Table IX. For terms of less than 10 years, there is, in the values of the sums assured, a small negative deviation; but after 10 years this changes to a considerable positive deviation, and the deviation for the total business is in excess for all ages of the office.

19. In the values of the net premiums there is at first a considerable positive deviation, changing to a negative deviation after 15 or 20 years. The extent of the deviation for the shorter terms seems to be due to the use of the same mean age, based upon the sums assured, for the valuation of both sums assured and premiums. A separate mean age for the premiums is obtained by weighting the sums assured in proportion to the rates of premium; and as policies of short endowment term, which have a shorter unexpired period to run, are effected for the most part at the older ages, the effect for these periods would have been to raise the mean age and to diminish the annuities to be used in valuing the premiums. While the total value of the premiums appears to be always in excess, the amount of that excess is in every case less than the excess value of the sums assured; so that the reserve brought out by the grouped method of valuation is always on the side of safety. The deviation is so small that the method may for practical purposes be described as exact.

20. For comparative purposes a valuation was also made at 3 per-cent by the combined H^{M} and $H^{M(5)}$ tables. The Z function has not been tabulated for the $H^{M(5)}$ table, as that table does not follow Makeham's law, and the valuation was a detailed one. It seems, however, from the success with which the method may be applied to an O^{M} valuation, that it might also be used with good results for an $H^{M(5)}$ valuation, by making a small graduated addition to the H^{M} mean valuation age. The results are given in Tables XXI and XXII, and for the five year office the figures, corresponding to the H^{M} valuation, may be compared with those of Tables XIX and XXIV, as a further illustration of the accuracy of the method.

Valuations by Select Tables.

21. The question of valuations by select tables has recently excited some interest and several excellent methods have been devised, whereby such a valuation can be made for whole of life policies with a minimum of trouble. It is beyond the scope of this paper to discuss the way in which such a valuation could most conveniently be applied to endowment

assurances; but it is desirable to ascertain the effect which would be produced on the reserves by its adoption. A detailed valuation was therefore made by the $O^{[M]}$ table at 3 per-cent interest; and for this purpose it was necessary to calculate a series of annuities showing the wearing out of the benefit of selection, and checks similar to those previously described were applied. The $O^{[M]}$ 3 per-cent premiums required for the valuation are given at the end of Table XVI, and the values of sums assured and premiums in Tables XXI and XXII.

22. Following on the reading of the papers on Select Valuations before the Institute last session, Mr. D. C. Fraser drew attention to the fact (*J.I.A.*, xl, 122) that, for whole of life policies, about 85 per-cent of the difference in the reserve, brought out by the use of select tables, was due to the difference in the value of the premiums; and that a good approximation could be obtained by making what he called an $O^{[M]}$ and O^{M} valuation, *i.e.*, by scheduling the select net premiums instead of those according to the aggregate table. It seemed to be worth examining how far this would hold good for endowment assurances.

23. Mr. King has shown how with great facility to transform the valuations of his Model Office, when the valuation premiums are different from those of the valuation table. This was possible because the results are arranged according to age at entry. That advantage had to be sacrificed when the arrangement of results according to unexpired term was adopted; but a close approximation to the results of an $O^{[M]}$ and O^{M} valuation may be obtained by the method of the following example:

25 *Year Office.*

Unexpired Term	log ($O^{[M]}$ Net Prem.) (1)	col (O^{M} Net Prem.) (2)	log (Value of O^{M} Net Prem.) (3)	(4) =(1)+(2)+(3)	Estimated Values of $O^{[M]}$ Net Prem. by O^{M} Annuities	By Actual Valuation
1–4 years	4·27761	5·71584	4·66581	4·65926	45,631	45,634
5–9	·47603	·51818	5·26764	5·26185	182,750	182,753
10–14	·52709	·46760	·51098	·50567	320,380	320,379
15–19	·44836	·54766	·54617	·54219	348,490	348,464
20–24	·19362	·80562	·36756	·36680	232,700	232,706
25–29	3·91158	$\bar{4}$·09040	·14038	·14236	138,790	138,796
30–34	·41211	·59431	4·67997	4·68639	48,572	48,571
35–39	2·68485	3·32698	3·98281	3·99464	9,877	9,879

The assumption which is here made is that, for each quinquennial group, the following approximation will hold :

$$\Sigma\begin{pmatrix}\text{Value of } O^{[M]} \text{ premiums} \\ \text{by } O^{M} \text{ annuities}\end{pmatrix}$$

$$= \frac{\Sigma(O^{[M]} \text{ net premiums})}{\Sigma(O^{M} \text{ net premiums})} \times \Sigma\begin{pmatrix}\text{Value of } O^{M} \text{ premiums} \\ \text{by } O^{M} \text{ annuities}\end{pmatrix}$$

The example illustrates the closeness of the approximation in the case of the 25 year office; and, as it was confirmed in others, the results of an $O^{[M]}$ and O^{M} valuation, given in Table X, were obtained in this way. It seems probable that the method may be usefully applied in other similar cases, where the valuation premiums are different from those of the valuation table; or to make the Model Office data conform as closely as possible to actual facts.

TABLE X.—*Reserves of Model Office*
$O^{[M]}$ O^{M} 3 *per-cent.*

Age of Office	Value of Sums Assured O^{M} 3 %	Value of $O^{[M]}$ Net Prem. by O^{M} Annuities 3 %	Reserve $O^{[M]}$ O^{M} 3 %
5 years	677,119	562,997	114,122
10	1,328,008	936,889	391,119
15	1,977,718	1,165,749	811,969
20	2,509,675	1,280,306	1,229,369
25	2,827,272	1,327,190	1,500,082
30	2,993,860	1,342,346	1,651,514
35	3,046,270	1,345,515	1,700,755
40	3,055,745	1,345,889	1,709,856

24. Variations in the valuation age or in the average due date of the premium income are of much less importance in endowment assurances than in whole of life policies; for, whereas in the case of whole of life policies the effect of deferring the average due date is always to increase the reserve, in endowment assurances the reserve for recently effected policies is increased, but that for policies nearing maturity is decreased. Taken separately, however, the values of both sums assured and premiums are affected in the same direction; and it should be borne in mind that in all the figures of this paper the premiums are assumed to be due. With certain assumptions as regards

valuation age and incidence of premium income it is believed that the effect even on reserves may be appreciable; but the object of the tables is comparative, and ratios are what are required for the purpose in view.

25. No account has been taken of bonus additions; but these must bear a relatively smaller proportion to the original sums assured than in the case of whole life policies, owing to the generally shorter period during which bonus additions to endowment assurances accrue. They will, however, be heaviest for the shortest unexpired terms; and, if it be desired to take account of them in estimating the effect of a change of valuation basis, it would probably be sufficient to choose for the sums assured a Model Office older in years than that which is suitable for the premiums, or to modify the data of the Model Office by suitable factors, as is done in the example given above.

Comparative Reserves.

26. Table XXVI gives a summary of the results, with comparative reserves, taking (1) the O^{M} 3 per-cent reserve as 10,000, and (2) the $O^{[M]}$ 3 per-cent reserve as 10,000. A change of a half per-cent in the rate of interest appears to alter the reserve by from $2\frac{1}{2}$ to $4\frac{1}{2}$ per-cent; which is about half that produced in the reserves for whole of life policies. As regards comparative reserves by different tables, the most noticeable features are the close agreement between the H^{M} and $O^{M(5)}$ reserves; and the strength of the O^{M} reserve, which for all durations of the office, is in excess of the combined H^{M} and $H^{M(5)}$ reserve. The larger $H^{M(5)}$ reserve required for bonus would, of course, tend to bring them nearer an equality. The closest approximation to a select reserve is given by the use of the combined O^{M} and $O^{M(5)}$ tables throughout; but this requires an exceptionally stringent reserve for policies of less than 5 years' duration, and perhaps a better approximation is that suggested by Mr. Fraser.

27. It has been remarked that, if the object in view is simply to ascertain the change in reserves due to a change in valuation basis, it would better to make a grouped valuation. In this every one would agree, assuming that all the valuation data have been tabulated. But intermediate valuations are now frequent, and at such a time an actuary might wish to form an estimate of the cost of a change of basis, before going to the trouble of a complete tabulation. Probably, however, the chief use of Model Offices is educational; and this seems to be especially so in the case of endowment assurances, which are at present in a transition

stage. In the last published Board of Trade returns the total amount of endowment assurances in force, both with and without profits, is stated at approximately 209 millions out of 737 millions, or 28·4 per-cent; five years earlier, the corresponding figures were 144 out of 651 millions, or 22·1 per-cent; while, in the returns for 1890, the figures are 36 out of 466 millions, or less than 8 per-cent. The growth of this section of the business has thus taken place chiefly during the last fifteen or twenty years, and twenty years is the shortest endowment term which appeals largely to the public. The next ten years should witness a rapid increase in the amounts of endowment assurances maturing each year, and probably, as was suggested in the *Economist* some time ago, a considerable rearrangement of the ratios of policy claims to the other items of the revenue accounts. For a number of years the figures, relating to ordinary business for the whole term of life, have remained practically stationary; and during the last two or three years there has been an actual decrease. If this is a correct forecast, the effect can only be to intensify that produced by the increasing numbers of claims emerging under maturing endowment assurances. The point is one of great importance to life offices, and it is in the hope that the accompanying tables may throw light on some aspects of the question, that I offer to the Institute another Model Office paper.

TABLE XI.—*Model Office for Endowment Assurances.*
Sums Assured existing at the end of each year.

Duration	Endowment Term						
	10 years	15 years	20 years	25 years	30 years	35 years	40 years
0	2,960	35,903	79,572	63,495	55,437	24,645	6,538
1	2,893	35,075	77,317	61,529	53,524	23,690	6,237
2	2,745	33,196	71,977	56,627	48,805	21,372	5,554
3	2,646	31,985	68,440	53,490	45,826	19,934	5,140
4	2,577	31,115	65,841	51,208	43,660	18,902	4,847
5	2,522	30,434	63,817	49,468	42,038	18,131	4,630
6	2,477	29,899	62,116	48,028	40,749	17,514	4,454
7	2,434	29,335	60,539	46,735	39,595	16,987	4,302
8	2,384	28,781	59,100	45,630	38,640	16,550	4,184
9	2,337	28,223	57,691	44,612	37,770	16,149	4,071
10	2,310	27,786	56,408	43,670	36,985	15,796	3,978
11	...	27,225	55,055	42,737	36,215	15,463	3,888
12	...	26,671	53,710	41,803	35,463	15,157	3,812
13	...	26,209	52,429	40,905	34,722	14,837	3,732
14	...	25,694	51,138	40,005	34,002	14,542	3,658
15	...	25,097	49,767	39,094	33,289	14,259	3,590
16	...	...	48,430	38,204	32,574	13,977	3,523
17	...	...	47,114	37,301	31,861	13,675	3,454
18	...	...	45,973	36,422	31,156	13,383	3,383
19	...	...	44,847	35,529	30,449	13,097	3,314
20	...	...	43,670	34,557	29,702	12,799	3,246
21	...	...	...	33,661	28.989	12,512	3,179
22	...	...	...	32,766	28,285	12,221	3,106
23	...	...	...	32,011	27,584	11,940	3,037
24	...	...	...	31,257	26,870	11,652	2,968
25	...	...	...	30,485	26,129	11,347	2,898
26	...	...	...	...	25,402	11,057	2,829
27	...	...	...	...	24,714	10,769	2,762
28	...	...	...	...	24,071	10,486	2,694
29	...	...	...	...	23,484	10,198	2,627
30	...	...	...	...	22,850	9,893	2,555
31	...	...	...	...	...	9,595	2,486
32	...	...	...	...	...	9,307	2,418
33	...	...	...	...	...	9,059	2,350
34	...	...	...	...	...	8,808	2,283
35	...	...	...	...	...	8,535	2,209
36	...	...	...	...	...	...	2,137
37	...	...	...	...	...	...	2,068
38	...	...	...	...	...	...	2,010
39	...	...	...	...	...	...	1,949
40	...	...	...	...	...	...	1,883

TABLE XII.—*Summary of Business in force in Model Office at the end of Quinquennial Periods arranged according to Unexpired Term.*

Unexpired Term	Age of Office							
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years
0 years	...	2,310	27,407	71,077	101,562	124,412	132,947	134,830
1–4	...	9,632	115,431	301,795	431,490	529,161	565,930	574,094
5–9	13,383	157,407	419,506	601,519	739,376	791,779	803,525	803,525
10–14	161,805	457,659	662,203	817,945	877,617	891,084	891,084	891,084
15–19	347,392	576,067	749,758	816,689	831,877	831,877	831,877	831,877
20–24	272,322	466,061	540,319	557,239	557,239	557,239	557,239	557,239
25–29	233,853	316,849	335,529	335,529	335,529	335,529	335,529	335,529
30–34	102,029	123,018	123,018	123,018	123,018	123,018	123,018	123,018
35–39	26,408	26,408	26,408	26,408	26,408	26,408	26,408	26,408
Total	1,157,192	2,135,411	2,999,579	3,651,219	4,024,116	4,210,507	4,267,557	4,277,604

TABLE XIII.—*Summary of Business in force in Model Office at the end of Quinquennial Periods arranged according to Maturity Age.*

Maturity Age	Age of Office							
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years
35	3,754	6,933	9,219	9,219	9,219	9,219	9,219	9,219
40	14,088	25,976	36,579	39,311	39,311	39,311	39,311	39,311
45	35,707	65,493	91,726	106,725	110,058	110,058	110,058	110,058
50	193,654	355,042	498,136	605,511	659,766	672,869	672,869	672,869
55	293,572	540,749	759,373	933,794	1,037,422	1,084,080	1,092,935	1,092,935
60	429,246	793,132	1,115,700	1,373,253	1,535,738	1,636,402	1,672,784	1,679,953
65	173,833	323,369	454,414	541,118	587,016	611,364	622,786	625,571
70	13,338	24,717	34,432	42,288	45,586	47,204	47,595	47,688
Total	1,157,192	2,135,411	2,999,579	3,651,219	4,024,116	4,210,507	4,267,557	4,277,604

Table XIV.—*Mean Valuation Ages in Model Office, based on Sums Assured.*

Unexpired Term	Age of Office										
	5 years		15 years		20 years			25 years		35 years	
	O^M	$O^{M(5)}$	O^M	$O^{M(5)}$	O^M	$O^{M(5)}$	H^M	O^M	$O^{M(5)}$	O^M	$O^{M(5)}$
2 years	...	...	55·5	55·7	55·8	56·	56·	55·6	55·8	56·	56·2
3	...	...	54·5	54·8	54·8	55·	55·1	54·6	54·8	55·	55·2
4	...	...	53·5	53·8	53·9	54·1	54·1	53·7	53·9	54·1	54·2
5	49·	49·2	52·9	53·1	52·7	53·	53·	52·9	53·1	53·1	53·3
6	48·	48·2	51·9	52·1	51·8	52·	52·	51·9	52·1	52·2	52·3
7	47·	47·2	50·9	51·2	50·8	51·	51·	51·	51·2	51·2	51·4
8	45·9	46·1	50·	50·2	49·8	50·	50·	50·	50·2	50·2	50·4
9	44·9	45·1	49·	49·2	48·8	49·	49·1	49·	49·2	49·2	49·4
10	48·	48·3	47·9	48·1	48·1	48·3	48·3	48·2	48·4	48·3	48·5
11	47·	47·3	46·9	47·1	47·1	47·3	47·3	47·2	47·4	47·3	47·5
12	46·	46·2	45·9	46·1	46·1	46·3	46·3	46·3	46·5	46·3	46·5
13	44·9	45·2	44·9	45·1	45·1	45·3	45·3	45·3	45·5	45·3	45·5
14	43·9	44·2	43·9	44·1	44·1	44·3	44·3	44·3	44·5	44·3	44·5
15	43·2	43·4	43·1	43·3	43·3	43·5	43·5	43·4	43·6	43·4	43·6
16	42·2	42·4	42·1	42·3	42·3	42·5	42·5	42·4	42·6	42·4	42·6
17	41·2	41·4	41·1	41·3	41·3	41·5	41·5	41·4	41·6	41·4	41·6
18	40·1	40·4	40·1	40·3	40·3	40·5	40·5	40·4	40·5	40·4	40·5
19	39·1	39·3	39·1	39·3	39·3	39·5	39·5	39·4	39·5	39·4	39·5
20	37·5	37·7	38·4	38·6	38·5	38·7	38·7	38·5	38·7	38·5	38·7
21	36·4	36·6	37·4	37·6	37·5	37·7	37·7	37·5	37·7	37·5	37·7
22	35·4	35·6	36·4	36·5	36·5	36·7	36·7	36·5	36·7	36·5	36·7
23	34·4	34·6	35·3	35·5	35·5	35·6	35·6	35·5	35·6	35·5	35·6
24	33·3	33·5	34·3	34·4	34·4	34·6	34·6	34·4	34·6	34·4	34·6
25	33·7	33·9	34·4	34·5	34·4	34·5	34·5	34·4	34·5	34·4	34·5
26	32·7	32·8	33·4	33·5	33·4	33·5	33·5	33·4	33·5	33·4	33·5
27	31·7	31·8	32·3	32·5	32·3	32·5	32·5	32·3	32·5	32·3	32·5
28	30·6	30·8	31·3	31·4	31·3	31·4	31·4	31·3	31·4	31·3	31·4
29	29·6	29·7	30·2	30·3	30·2	30·3	30·3	30·2	30·3	30·2	30·3
30	30·5	30·5	30·7	30·8	30·7	30·8	30·8	30·7	30·8	30·7	30·8
31	29·4	29·5	29·7	29·8	29·7	29·8	29·8	29·7	29·8	29·7	29·8
32	28·4	28·5	28·7	28·8	28·7	28·8	28·8	28·7	28·8	28·7	28·8
33	27·4	27·5	27·7	27·7	27·7	27·7	27·7	27·7	27·7	27·7	27·7
34	26·4	26·4	26·6	26·7	26·6	26·7	26·7	26·6	26·7	26·6	26·7
35	26·8	26·9	26·8	26·9	26·8	26·9	26·9	26·8	26·9	26·8	26·9
36	25·8	25·9	25·8	25·9	25·8	25·9	25·8	25·8	25·9	25·8	25·9
37	24·8	24·8	24·8	24·8	24·8	24·8	24·8	24·8	24·8	24·8	24·8
38	23·8	23·8	23·8	23·8	23·8	23·8	23·8	23·8	23·8	23·8	23·8
39	22·7	22·8	22·7	22·8	22·7	22·8	22·8	22·7	22·8	22·7	22·8

TABLE XV.—*Net Premiums corresponding to business in force in Model Office.*

$2\frac{1}{2}$ *per-cent.*

Unexpired Term	Age of Office: 5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years
	O^M							
Years 1–4	...	898	7,387	15,672	20,080	22,758	23,597	23,755
5–9	1,247	10,091	21,758	27,950	31,733	32,930	33,157	33,157
10–14	9,936	23,113	30,076	34,352	35,716	35,976	35,976	35,976
15–19	15,467	23,253	28,022	29,553	29,847	29,847	29,847	29,847
20–24	9,263	14,583	16,281	16,608	16,608	16,608	16,608	16,608
25–29	6,416	8,313	8,675	8,675	8,675	8,675	8,675	8,675
30–34	2,330	2,736	2,736	2,736	2,736	2,736	2,736	2,736
35–39	510	510	510	510	510	510	510	510
Total	45,169	83,497	115,445	136,056	145,905	150,040	151,106	151,264
	$O^{M(5)}$							
Years 1–4	...	901	7,426	15,785	20,263	23,005	23,874	24,039
5–9	1,251	10,144	21,914	28,205	32,078	33,316	33,554	33,554
10–14	9,990	23,284	30,357	34,734	36,144	36,417	36,417	36,417
15–19	15,601	23,510	28,392	29,974	30,283	30,283	30,283	30,283
20–24	9,411	14,857	16,613	16,957	16,957	16,957	16,957	16,957
25–29	6,570	8,532	8,911	8,911	8,911	8,911	8,911	8,911
30–34	2,409	2,835	2,835	2,835	2,835	2,835	2,835	2,835
35–39	535	535	535	535	535	535	535	535
Total	45,767	84,598	116,983	137,936	148,006	152,259	153,366	153,531
	H^M							
Years 1–4	...	903	7,447	15,845	20,348	23,109	23,982	24,149
5–9	1,254	10,172	21,997	28,324	32,223	33,468	33,708	33,708
10–14	10,019	23,375	30,488	34,895	36,314	36,588	36,588	36,588
15–19	15,673	23,627	28,542	30,134	30,444	30,444	30,444	30,444
20–24	9,465	14,948	16,715	17,060	17,060	17,060	17,060	17,060
25–29	6,614	8,588	8,969	8,969	8,969	8,969	8,969	8,969
30–34	2,424	2,852	2,852	2,852	2,852	2,852	2,852	2,852
35–39	538	538	538	538	538	538	538	538
Total	45,987	85,003	117,548	138,617	148,748	153,028	154,141	154,308

Table XVI.—*Net Premiums corresponding to business in force in Model Office.*

3 *per-cent.*

Un-expired Term	Age of Office: 5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years
	O^{M}							
Years 1–4	...	876	7,142	15,065	19,238	21,751	22,531	22,677
5–9	1,216	9,756	20,915	26,776	30,326	31,441	31,651	31,651
10–14	9,594	22,198	28,790	32,802	34,072	34,313	34,313	34,313
15–19	14,793	22,165	26,640	28,064	28,336	28,336	28,336	28,335
20–24	8,770	13,762	15,343	15,645	15,645	15,645	15,645	15,645
25–29	6,021	7,787	8,121	8,121	8,121	8,121	8,121	8,121
30–34	2,170	2,545	2,545	2,545	2,545	2,545	2,545	2,545
35–39	471	471	471	471	471	471	471	471
Total	43,035	79,560	109,967	129,489	138,754	142,623	143,613	143,758
	$O^{M(5)}$							
Years 1–4	...	879	7,183	15,181	19,427	22,007	22,818	22,972
5–9	1,220	9,811	21,075	27,038	30,682	31,839	32,060	32,060
10–14	9,652	22,375	29,081	33,200	34,517	34,771	34,771	34,771
15–19	14,935	22,435	27,029	28,508	28,794	28,794	28,794	28,794
20–24	8,922	14,047	15,688	16,007	16,007	16,007	16,007	16,007
25–29	6,182	8,016	8,367	8,367	8,367	8,367	8,367	8,367
30–34	2,253	2,648	2,648	2,648	2,648	2,648	2,648	2,648
35–39	497	497	497	497	497	497	497	497
Total	43,661	80,708	111,568	131,446	140,939	144,930	145,962	146,116
	H^{M}							
Years 1–4	...	880	7,204	15,241	19,512	22,110	22,927	23,081
5–9	1,222	9,840	21,158	27,157	30,827	31,992	32,214	32,214
10–14	9,682	22,466	29,212	33,360	34,686	34,942	34,942	34,942
15–19	15,007	22,551	27,177	28,665	28,953	28,953	28,953	28,953
20–24	8,975	14,136	15,787	16,108	16,108	16,108	16,108	16,108
25–29	6,225	8,070	8,425	8,425	8,425	8,425	8,425	8,425
30–34	2,268	2,666	2,666	2,666	2,666	2,666	2,666	2,666
35–39	500	500	500	500	500	500	500	500
Total	43,879	81,109	112,129	132,122	141,677	145,696	146,735	146,889
	$O^{[M]}$							
Years 1–4	...	861	7,005	14,806	18,950	21,466	22,257	22,407
5–9	1,196	9,568	20,549	26,371	29,925	31,053	31,268	31,268
10–14	9,408	21,810	28,357	32,374	33,658	33,905	33,905	33,905
15–19	14,559	21,879	26,359	27,799	28,078	28,078	28,078	28,078
20–24	8,710	13,709	15,307	15,618	15,618	15,618	15,618	15,618
25–29	6,029	7,815	8,158	8,158	8,158	8,158	8,158	8,158
30–34	2,198	2,583	2,583	2,583	2,583	2,583	2,583	2,583
35–39	484	484	484	484	484	484	484	484
Total	42,584	78,709	108,802	128,193	137,454	141,345	142,351	142,501

TABLE XVII.—*Net Premiums corresponding to business in force in Model Office.*

$3\frac{1}{2}$ *per-cent.*

Unexpired Term	Age of Office: 5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years
	O^{M}							
Years								
1–4	...	854	6,906	14,483	18,435	20,793	21,521	21,656
5–9	1,185	9,435	20,106	25,658	28,990	30,029	30,223	30,223
10–14	9,267	21,323	27,566	31,332	32,515	32,737	32,737	32,737
15–19	14,150	21,130	25,331	26,659	26,910	26,910	26,910	26,910
20–24	8,305	12,990	14,463	14,742	14,742	14,742	14,742	14,742
25–29	5,651	7,297	7,606	7,606	7,606	7,606	7,606	7,606
30–34	2,022	2,369	2,369	2,369	2,369	2,369	2,369	2,369
35–39	436	436	436	436	436	436	436	436
Total	41,016	75,834	104,783	123,285	132,003	135,622	136,544	136,679
	$O^{M(5)}$							
Years								
1–4	...	857	6,947	14,602	18,628	21,054	21,813	21,956
5–9	1,189	9,490	20,270	25,926	29,355	30,437	30,643	30,643
10–14	9,324	21,501	27,861	31,737	32,970	33,206	33,206	33,206
15–19	14,291	21,403	25,726	27,109	27,376	27,376	27,376	27,376
20–24	8,461	13,284	14,819	15,116	15,116	15,116	15,116	15,116
25–29	5,817	7,532	7,860	7,860	7,860	7,860	7,860	7,860
30–34	2,108	2,476	2,476	2,476	2,476	2,476	2,476	2,476
35–39	464	464	464	464	464	464	464	464
Total	41,654	77,007	106,423	125,290	134,245	137,989	138,954	139,097
	H^{M}							
Years								
1–4	...	858	6,968	14,663	18,716	21,161	21,926	22,070
5–9	1,192	9,519	20,356	26,050	29,506	30,596	30,803	30,803
10–14	9,354	21,595	27,997	31,904	33,146	33,384	33,384	33,384
15–19	14,366	21,525	25,882	27,276	27,544	27,544	27,544	27,544
20–24	8,517	13,378	14,925	15,223	15,223	15,223	15,223	15,223
25–29	5,862	7,590	7,920	7,920	7,920	7,920	7,920	7,920
30–34	2,123	2,493	2,493	2,493	2,493	2,493	2,493	2,493
35–39	467	467	467	467	467	467	467	467
Total	41,881	77,425	107,008	125,996	135,015	138,788	139,760	139,904

TABLE XVIII.—*Values of Sums Assured in Model Office.*
$2\frac{1}{2}$ *per-cent.*

Unexpired Term	Age of Office: 5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years
				O^M				
Years								
0	...	2,310	27,407	71,077	101,562	124,412	132,947	134,830
1–4	...	9,062	108,627	283,994	406,016	497,937	532,535	540,221
5–9	11,334	133,627	356,129	510,931	627,647	672,186	682,177	682,177
10–14	123,208	348,662	504,396	623,204	668,827	679,115	679,115	679,115
15–19	238,299	395,104	514,527	560,727	571,286	571,286	571,286	571,286
20–24	168,322	288,699	335,096	345,727	345,727	345,727	345,727	345,727
25–29	131,227	178,206	188,846	188,846	188,846	188,846	188,846	188,846
30–34	52,247	63,098	63,098	93,098	63,098	63,098	63,098	63,098
35–39	12,353	12,353	12,353	12,353	12,353	12,353	12,353	12,353
Total	736,990	1,431,121	2,110,479	2,659,957	2,985,362	3,154,960	3,208,084	3,217,653
				$O^{M(5)}$				
Years								
0	...	2,310	27,407	71,077	101,562	124,412	132,947	134,830
1–4	...	9,063	108,633	284,003	406,032	497,952	532,546	540,235
5–9	11,338	133,673	356,248	511,102	627,859	672,410	682,400	682,400
10–14	123,336	349,038	504,865	623,783	669,448	679,756	679,756	679,756
15–19	238,783	395,894	515,556	561,843	572,364	572,364	572,364	572,364
20–24	169,016	289,816	336,374	347,052	347,052	347,052	347,052	347,052
25–29	132,198	179,452	190,158	190,158	190,158	190,158	190,158	190,158
30–34	52,878	63,843	63,843	63,843	63,843	63,843	63,843	63,843
35–39	12,596	12,596	12,596	12,596	12,596	12,596	12,596	12,596
Total	740,145	1,435,685	2,115,680	2,665,457	2,990,914	3,160,543	3,213,662	3,223,234
				H^M				
Years								
0	...	2,310	27,407	71,077	101,562	124,412	132,947	134,830
1–4	...	9,063	108,639	284,009	406,035	497,961	532,563	540,246
5–9	11,338	133,694	356,314	511,199	627,958	672,519	682,499	682,499
10–14	123,421	349,257	505,245	624,172	669,890	680,184	680,184	680,184
15–19	239,037	396,324	516,095	562,425	572,988	572,988	572,988	572,988
20–24	169,418	290,493	337,136	347,840	347,840	347,840	347,840	347,840
25–29	132,626	180,046	190,788	190,788	190,788	190,788	190,788	190,788
30–34	53,079	64,091	64,091	64,091	64,091	64,091	64,091	64,091
35–39	12,634	12,634	12,634	12,634	12,634	12,634	12,634	12,634
Total	741,553	1,437,912	2,118,349	2,668,235	2,993,786	3,163,417	3,216,534	3,226,100

TABLE XIX.—*Values of Sums Assured in Model Office.*

3 *per-cent.*

Unexpired Term	Age of Office							
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years
	O^{M}							
Years								
0	...	2,310	27,407	71,077	101,562	124,412	132,947	134,830
1–4	...	8,955	107,349	280,635	401,224	492,051	526,242	533,834
5–9	10,972	129,410	344,895	494,470	607,853	650,995	660,679	660,679
10–14	116,835	330,669	478,354	591,051	634,348	644,117	644,117	644,117
15–19	221,532	367,286	478,367	521,366	531,209	531,209	531,209	531,209
20–24	153,453	263,307	305,716	315,446	315,446	315,446	315,446	315,446
25–29	117,550	159,715	169,274	169,274	169,274	169,274	169,274	169,274
30–34	46,061	55,640	55,640	55,640	55,640	55,640	55,640	55,640
35–39	10,716	10,716	10,716	10,716	10,716	10,716	10,716	10,716
Total	677,119	1,328,008	1,977,718	2,509,675	2,827,272	2,993,860	3,046,270	3,055,745
	$O^{M(5)}$							
Years								
0	...	2,310	27,407	71,077	101,562	124,412	132,947	134,830
1–4	...	8,955	107,354	280,654	401,240	492,073	526,262	533,856
5–9	10,975	129,467	345,038	494,664	608,096	651,257	660,937	660,937
10–14	116,985	331,095	478,878	591,716	635,060	644,844	644,844	644,844
15–19	222,083	368,181	479,521	522,626	532,429	532,429	532,429	532,429
20–24	154,240	264,570	307,145	316,931	316,931	316,931	316,931	316,931
25–29	118,631	161,100	170,734	170,734	170,734	170,734	170,734	170,734
30–34	46,750	56,464	56,464	56,464	56,464	56,464	56,464	56,464
35–39	10,984	10,984	10,984	10,984	10,984	10,984	10,984	10,984
Total	680,648	1,333,126	1,983,525	2,515,850	2,833,500	3,000,128	3,052,532	3,062,009
	H^{M}							
Years								
0	...	2,310	27,407	71,077	101,562	124,412	132,947	134,830
1–4	...	8,955	107,356	280,659	401,249	492,079	526,267	533,870
5–9	10,977	129,497	345,109	494,772	608,216	651,379	661,059	661,059
10–14	117,081	331,340	479,320	592,180	635,597	645,352	645,352	645,352
15–19	222,371	368,670	480,136	523,285	533,159	533,159	533,159	533,159
20–24	154,666	265,333	308,000	317,803	317,803	317,803	317,803	317,803
25–29	119,115	161,755	171,434	171,434	171,434	171,434	171,434	171,434
30–34	46,975	56,732	56,732	56,732	56,732	56,732	56,732	56,732
35–39	11,029	11,029	11,029	11,029	11,029	11,029	11,029	11,029
Total	682,214	1,335,621	1,986,523	2,518,971	2,836,781	3,003,379	3,055,782	3,065,268

TABLE XX.—*Values of Sums Assured in Model Office.*

$3\frac{1}{2}$ *per-cent.*

Unexpired Term	Age of Office: 5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years
				O^{M}				
Years								
0	...	2,310	27,407	71,077	101,562	124,412	132,947	134,830
1–4	...	8,850	106,092	277,345	396,509	486,274	520,063	527,568
5–9	10,623	125,359	334,090	478,976	588,818	630,622	640,012	640,012
10–14	110,847	313,747	453,822	560,803	601,911	611,194	611,194	611,194
15–19	206,113	341,706	445,100	485,161	494,352	494,352	494,352	494,352
20–24	140,074	240,479	279,267	288,183	288,183	288,183	288,183	288,183
25–29	105,505	143,420	152,027	152,027	152,027	152,027	152,027	152,027
30–34	40,715	49,202	49,202	49,202	49,202	49,202	49,202	49,202
35–39	9,339	9,339	9,339	9,339	9,339	9,339	9,339	9,339
Total	623,216	1,234,412	1,856,346	2,372,113	2,681,903	2,845,605	2,897,319	2,906,707
				$O^{M(5)}$				
Years								
0	...	2,310	27,407	71,077	101,562	124,412	132,947	134,830
1–4	...	8,850	106,098	277,360	396,527	486,297	520,086	527,586
5–9	10,627	125,420	334,256	479,190	589,087	630,910	640,294	640,294
10–14	111,011	314,223	454,446	561,562	602,722	612,012	612,012	612,012
15–19	206,722	342,702	446,378	486,563	495,694	495,694	495,694	495,694
20–24	140,941	241,846	280,843	289,805	289,805	289,805	289,805	289,805
25–29	106,672	144,914	153,603	153,603	153,603	153,603	153,603	153,603
30–34	41,455	50,082	50,082	50,082	50,082	50,082	50,082	50,082
35–39	9,619	9,619	9,619	9,619	9,619	9,619	9,619	9,619
Total	627,047	1,239,966	1,862,732	2,378,861	2,688,701	2,852,434	2,904,142	2,913,525
				H^{M}				
Years								
0	...	2,310	27,407	71,077	101,562	124,412	132,947	134,830
1–4	...	8,850	106,102	277,367	396,537	486,311	520,105	527,606
5–9	10,630	125,447	334,332	479,301	589,223	631,059	640,428	640,428
10–14	111,118	314,512	454,941	562,074	603,311	612,571	612,571	612,571
15–19	207,032	343,224	447,056	487,271	496,485	496,485	496,485	496,485
20–24	141,419	242,672	281,764	290,759	290,759	290,759	290,759	290,759
25–29	107,181	145,623	154,379	154,379	154,379	154,379	154,379	154,379
30–34	41,693	50,365	50,365	50,365	50,365	50,365	50,365	50,365
35–39	9,664	9,664	9,664	9,664	9,664	9,664	9,664	9,664
Total	628,737	1,242,667	1,866,010	2,382,257	2,692,285	2,856,005	2,907,703	2,917,087

TABLE XXI.—*Values of Sums Assured in Model Office.*

3 *per-cent.*

Un-expired Term	Age of Office							
	5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years
	$O^{[M]}$							
Years 0	...	2,310	27,407	71,077	101,562	124,412	132,947	134,830
1–4	...	8,955	107,355	280,654	401,247	492,072	526,267	533,859
5–9	10,958	129,408	344,969	494,584	607,998	651,158	660,843	660,843
10–14	116,586	330,563	478,389	591,159	634,495	644,300	644,300	644,300
15–19	221,102	367,132	478,454	521,552	531,369	531,369	531,369	531,369
20–24	153,503	263,793	306,356	316,109	316,109	316,109	316,109	316,109
25–29	118,027	160,506	170,133	170,133	170,133	170,133	170,133	170,133
30–34	46,470	56,171	56,171	56,171	56,171	56,171	56,171	56,171
35–39	10,925	10,925	10,925	10,925	10,925	10,925	10,925	10,925
Total	677,571	1,329,763	1,980,159	2,512,364	2,830,009	2,996,649	3,049,064	3,058,539
	Combined H^{M} and $H^{M(5)}$							
Years 0	...	2,310	27,407	71,077	101,562	124,412	132,947	134,830
1–4	...	8,955	107,366	280,686	401,293	492,129	526,330	533,924
5–9	10,973	129,557	345,309	495,058	608,576	651,780	661,475	661,475
10–14	117,040	331,653	479,862	592,924	636,379	646,202	646,202	646,202
15–19	222,306	369,161	481,041	524,349	534,215	534,215	534,215	534,215
20–24	154,555	265,888	308,811	318,642	318,642	318,642	318,642	318,642
25–29	119,055	162,161	171,915	171,915	171,915	171,915	171,915	171,915
30–34	46,931	56,842	56,842	56,842	56,842	56,842	56,842	56,842
35–39	11,037	11,037	11,037	11,037	11,037	11,037	11,037	11,037
Total	681,900	1,337,564	1,989,590	2,522,530	2,840,461	3,007,174	3,059,605	3,069,082

TABLE XXII.—*Values of Net Premiums corresponding to Business in force in Model Office.*

3 *per-cent.*

Unexpired Term	Age of Office: 5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years
	$O^{[M]}$							
Years 1–4	...	2,078	16,832	35,608	45,587	51,634	53,532	53,890
5–9	7,440	58,451	125,359	160,915	182,540	189,363	190,661	190,661
10–14	90,191	207,922	270,187	308,226	320,293	322,597	322,597	322,597
15–19	181,536	272,279	327,487	345,091	348,471	348,471	348,471	348,471
20–24	130,379	204,243	227,664	232,180	232,180	232,180	232,180	232,180
25–29	102,488	132,423	138,124	138,124	138,124	138,124	138,124	138,124
30–34	41,024	48,130	48,130	48,130	48,130	48,130	48,130	48,130
35–39	9,746	9,746	9,746	9,746	9,746	9,746	9,746	9,746
Total	562,804	935,272	1,163,529	1,278,020	1,325,071	1,340,245	1,343,441	1,343,799
	Combined H^M and $H^{M(5)}$							
Years 1–4	...	2,120	17,277	36,599	46,871	53,115	55,075	55,447
5–9	7,547	59,764	128,424	164,908	187,143	194,160	195,493	195,493
10–14	91,876	212,271	276,001	314,996	327,372	329,736	329,736	329,736
15–19	185,260	277,833	334,324	352,349	355,813	355,813	355,813	355,813
20–24	133,144	208,448	232,368	236,982	236,982	236,982	236,982	236,982
25–29	104,888	135,344	141,153	141,153	141,153	141,153	141,153	141,153
30–34	42,027	49,234	49,234	49,234	49,234	49,234	49,234	49,234
35–39	10,003	10,003	10,003	10,003	10,003	10,003	10,003	10,003
Total	574,745	955,017	1,188,784	1,306,224	1,354,571	1,370,196	1,373,489	1,373,861

Table XXIII.—*Values of Net Premiums corresponding to business in force in Model Office.*

2½ per-cent.

Unexpired Term	Age of Office: 5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years
	O^M							
Years								
1–4	...	2,177	17,841	37,896	48,572	55,045	57,068	57,458
5–9	7,831	62,520	134,775	173,244	196,590	203,909	205,284	205,284
10–14	97,167	225,718	293,894	335,389	348,432	350,927	350,927	350,927
15–19	199,143	299,523	360,530	379,831	383,417	383,417	383,417	383,417
20–24	145,042	227,551	253,562	258,496	258,496	258,496	258,496	258,496
25–29	115,434	149,147	155,494	155,494	155,494	155,494	155,494	155,494
30–34	46,599	54,637	54,637	54,637	54,637	54,637	54,637	54,637
35–39	11,140	11,140	11,140	11,140	11,140	11,140	11,140	11,140
Total	622,356	1,032,413	1,281,873	1,406,127	1,456,778	1,473,065	1,476,463	1,476,853
	$O^{M(5)}$							
Years								
1–4	...	2,185	17,923	38,149	48,986	55,615	57,712	58,112
5–9	7,842	62,718	135,482	174,496	198,351	205,914	207,359	207,359
10–14	97,375	226,600	295,751	338,093	351,554	354,159	354,159	354,159
15–19	199,970	301,506	363,680	383,561	387,406	387,406	387,406	387,406
20–24	146,372	230,368	257,128	262,269	262,269	262,269	262,269	262,269
25–29	117,083	151,684	158,292	158,292	158,292	158,292	158,292	158,292
30–34	47,584	55,907	55,907	55,907	55,907	55,907	55,907	55,907
35–39	11,479	11,479	11,479	11,479	11,479	11,479	11,479	11,479
Total	627,705	1,042,447	1,295,642	1,422,246	1,474,244	1,491,041	1,494,583	1,494,983
	Combined O^M and $O^{M(5)}$ throughout.							
Years								
1–4	...	2,177	17,829	37,876	48,545	55,016	57,045	57,429
5–9	7,818	62,390	134,511	172,919	196,219	203,531	204,905	204,905
10–14	96,845	224,938	293,022	334,391	347,392	349,869	349,869	349,869
15–19	198,255	298,222	358,950	378,173	381,830	381,830	381,830	381,830
20–24	144,074	226,114	251,981	256,881	256,881	256,881	256,881	256,881
25–29	114,342	147,807	154,101	154,101	154,101	154,101	154,101	154,101
30–34	46,011	53,958	53,958	53,958	53,958	53,958	53,958	53,958
35–39	10,948	10,948	10,948	10,948	10,948	10,948	10,948	10,948
Total	618,293	1,026,554	1,275,300	1,399,247	1,449,874	1,466,134	1,469,537	1,469,921
	H^M							
Years								
1–4	...	2,186	17,969	38,281	49,186	55,852	57,956	58,361
5–9	7,849	62,838	135,873	175,062	199,076	206,666	208,144	208,144
10–14	97,453	227,020	296,306	338,957	352,452	355,097	355,097	355,097
15–19	200,424	302,284	364,762	384,726	388,520	388,520	388,520	388,520
20–24	146,626	230,883	257,729	262,877	262,877	262,877	262,877	262,877
25–29	117,370	152,025	158,641	158,641	158,641	158,641	158,641	158,641
30–34	47,676	56,009	56,009	56,009	56,009	56,009	56,009	56,009
35–39	11,506	11,506	11,506	11,506	11,506	11,506	11,506	11,506
Total	628,904	1,044,751	1,298,795	1,426,059	1,478,267	1,495,168	1,498,750	1,499,155

TABLE XXIV.—*Values of Net Premiums corresponding to business in force in Model Office.*

3 *per-cent.*

Un-expired Term	Age of Office: 5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years
	O^M							
Years 1–4	...	2,113	17,169	36,261	46,324	52,368	54,240	54,591
5–9	7,526	59,592	127,711	163,605	185,201	191,923	193,165	193,165
10–14	91,542	211,499	274,470	312,464	324,316	326,558	326,558	326,558
15–19	184,011	275,833	331,140	348,505	351,698	351,698	351,698	351,698
20–24	131,424	205,564	228,749	233,109	233,109	233,109	233,109	233,109
25–29	102,802	132,594	138,156	138,156	138,156	138,156	138,156	138,156
30–34	40,873	47,860	47,860	47,860	47,860	47,860	47,860	47,860
35–39	9,612	9,612	9,612	9,612	9,612	9,612	9,612	9,612
Total	567,790	944,667	1,174,867	1,289,572	1,336,276	1,351,284	1,354,398	1,354,749
	$O^{M(5)}$							
Years 1–4	...	2,119	17,255	36,511	46,757	52,947	54,900	55,270
5–9	7,535	59,802	128,440	164,903	187,027	193,989	195,312	195,312
10–14	91,790	212,477	276,448	315,313	327,593	329,954	329,954	329,954
15–19	184,955	277,992	334,545	352,491	355,940	355,940	355,940	355,940
20–24	132,819	208,518	232,463	237,031	237,031	237,031	237,031	237,031
25–29	104,584	135,286	141,108	141,108	141,108	141,108	141,108	141,108
30–34	41,901	49,193	49,193	49,193	49,193	49,193	49,193	49,193
35–39	9,970	9,970	9,970	9,970	9,970	9,970	9,970	9,970
Total	573,554	955,357	1,189,422	1,306,520	1,354,619	1,370,132	1,373,408	1,373,778
	Combined O^M and $O^{M(5)}$ throughout.							
Years 1–4	...	2,112	17,159	36,234	46,299	52,336	54,213	54,564
5–9	7,513	59,466	127,466	163,309	184,862	191,567	192,819	192,819
10–14	91,238	210,787	273,685	311,545	323,367	325,600	325,600	325,600
15–19	183,203	274,650	329,728	347,017	350,275	350,275	350,275	350,275
20–24	130,560	204,294	227,356	231,675	231,675	231,675	231,675	231,675
25–29	101,844	131,424	136,943	136,943	136,943	136,943	136,943	136,943
30–34	40,365	47,277	47,277	47,277	47,277	47,277	47,277	47,277
35–39	9,450	9,450	9,450	9,450	9,450	9,450	9,450	9,450
Total	564,173	939,460	1,169,064	1,283,450	1,330,148	1,345,123	1,348,252	1,348,603
	H^M							
Years 1–4	...	2,122	17,302	36,647	46,948	53,190	55,152	55,527
5–9	7,540	59,924	128,831	165,459	187,735	194,749	196,080	196,080
10–14	91,890	212,919	277,024	316,179	328,466	330,883	330,883	330,883
15–19	185,428	278,773	335,609	353,641	357,035	357,035	357,035	357,035
20–24	133,128	209,041	233,083	237,669	237,669	237,669	237,669	237,669
25–29	104,855	135,637	141,469	141,469	141,469	141,469	141,469	141,469
30–34	42,009	49,315	49,315	49,315	49,315	49,315	49,315	49,315
35–39	10,001	10,001	10,001	10,001	10,001	10,001	10,001	10,001
Total	574,851	957,732	1,192,634	1,310,380	1,358,638	1,374,311	1,377,604	1,377,979

TABLE XXV.—*Values of Net Premiums corresponding to business in force in Model Office.*

$3\frac{1}{2}$ *per-cent.*

Unexpired Term	Age of Office: 5 years	10 years	15 years	20 years	25 years	30 years	35 years	40 years
	O^M							
Years 1–4	...	2,050	16,526	34,696	44,180	49,825	51,560	51,886
5–9	7,232	56,813	121,050	154,568	174,549	180,721	181,853	181,853
10–14	86,310	198,310	256,537	291,326	302,116	304,128	304,128	304,128
15–19	170,165	254,228	304,431	320,079	322,933	322,933	322,933	322,933
20–24	119,258	185,939	206,659	210,527	210,527	210,527	210,527	210,527
25–29	91,715	118,119	123,012	123,012	123,012	123,012	123,012	123,012
30–34	35,939	42,035	42,035	42,035	42,035	42,035	42,035	42,035
35–39	8,324	8,324	8,324	8,324	8,324	8,324	8,324	8,324
Total	518,943	865,818	1,078,574	1,184,567	1,227,676	1,241,505	1,244,372	1,244,698
	$O^{M(5)}$							
Years 1–4	...	2,056	16,610	34,955	44,619	50,424	52,241	52,586
5–9	7,243	57,037	121,813	155,913	176,436	182,854	184,063	184,063
10–14	86,551	199,294	258,520	294,221	305,439	307,578	307,578	307,578
15–19	171,116	256,417	307,879	324,112	327,219	327,219	327,219	327,219
20–24	120,705	188,990	210,465	214,556	214,556	214,556	214,556	214,556
25–29	93,546	120,872	126,023	126,023	126,023	126,023	126,023	126,023
30–34	37,011	43,408	43,408	43,408	43,408	43,408	43,408	43,408
35–39	8,723	8,723	8,723	8,723	8,723	8,723	8,723	8,723
Total	524,895	876,797	1,093,441	1,201,911	1,246,423	1,260,785	1,263,811	1,264,156
	Combined O^M and $O^{M(5)}$ throughout.							
Years 1–4	...	2,050	16,511	34,674	44,156	49,797	51,540	51,864
5–9	7,218	56,704	120,827	154,301	174,240	180,399	181,533	181,533
10–14	86,027	197,645	255,781	290,467	301,227	303,239	303,239	303,239
15–19	169,431	253,147	303,153	318,727	321,649	321,649	321,649	321,649
20–24	118,479	184,811	205,414	209,257	209,257	209,257	209,257	209,257
25–29	90,882	117,103	121,955	121,955	121,955	121,955	121,955	121,955
30–34	35,506	41,536	41,536	41,536	41,536	41,536	41,536	41,536
35–39	8,191	8,191	8,191	8,191	8,191	8,191	8,191	8,191
Total	515,734	861,187	1,073,368	1,179,108	1,222,211	1,236,023	1,238,900	1,239,224
	H^M							
Years 1–4	...	2,059	16,655	35,092	44,816	50,665	52,488	52,835
5–9	7,249	57,156	122,215	156,510	177,177	183,639	184,875	184,875
10–14	86,643	199,774	259,163	295,182	306,409	308,606	308,606	308,606
15–19	171,641	257,307	309,059	325,397	328,456	328,456	328,456	328,456
20–24	121,072	189,629	211,209	215,310	215,310	215,310	215,310	215,310
25–29	93,898	121,294	126,453	126,453	126,453	126,453	126,453	126,453
30–34	37,127	43,545	43,545	43,545	43,545	43,545	43,545	43,545
35–39	8,752	8,752	8,752	8,752	8,752	8,752	8,752	8,752
Total	526,382	879,516	1,097,051	1,206,241	1,250,918	1,265,426	1,268,485	1,268,832

TABLE XXVI.—*Comparative Reserves of Model Office.*

Age of Office		5 Years			10 Years			15 Years			20 Years		
Tables and Rate of Interest		Actual Reserve	Comparative Reserve O^M 3 % =10,000	Comparative Reserve $O^{[M]}$ 3 % =10,000	Actual Reserve	Comparative Reserve O^M 3 % =10,000	Comparative Reserve $O^{[M]}$ 3 % =10,000	Actual Reserve	Comparative Reserve O^M 3 % =10,000	Comparative Reserve $O^{[M]}$ 3 % =10,000	Actual Reserve	Comparative Reserve O^M 3 % =10,000	Comparative Reserve $O^{[M]}$ 3 % =10,000
$O^{[M]}$	3 %	114,767	10,497	10,000	394,491	10,291	10,000	816,630	10,172	10,000	1,234,344	10,117	10,000
O^M	$2\frac{1}{2}$%	114,634	10,484	9,987	398,708	10,401	10,107	828,606	10,321	10,146	1,253,830	10,276	10,158
	3 %	109,329	10,000	9,526	383,341	10,000	9,717	802,851	10,000	9,831	1,220,103	10,000	9,885
	$3\frac{1}{2}$%	104,273	9,537	9,085	368,594	9,616	9,344	777,772	9,687	9,524	1,187,546	9,732	9,621
$O^{M(5)}$	$2\frac{1}{2}$%	112,440	10,285	9,797	393,238	10,258	9,968	820,038	10,214	10,042	1,243,211	10,189	10,072
	3 %	107,091	9,795	9,331	377,769	9,854	9,576	794,103	9,890	9,724	1,209,330	9,911	9,797
	$3\frac{1}{2}$%	102,152	9,343	8,900	363,169	9,474	9,206	769,291	9,582	9,422	1,176,950	9,646	9,536
H^M	$2\frac{1}{2}$%	112,649	10,304	9,815	393,161	10,256	9,966	819,554	10,208	10,036	1,242,176	10,181	10,064
	3 %	107,363	9,820	9,354	377,889	9,858	9,579	793,889	9,888	9,721	1,208,591	9,906	9,792
	$3\frac{1}{2}$%	102,355	9,362	8,919	363,151	9,473	9,206	768,959	9,578	9,416	1,176,016	9,639	9,527
Combined O^M and $O^{M(5)}$ throughout . .	$2\frac{1}{2}$%	121,852	11,145	10,617	409,131	10,673	10,371	840,380	10,467	10,291	1,266,210	10,378	10,258
	3 %	116,475	10,654	10,149	393,666	10,270	9,979	814,461	10,145	9,973	1,232,400	10,101	9,984
	$3\frac{1}{2}$%	111,313	10,181	9,699	378,779	9,881	9,602	789,364	9,832	9,666	1,199,753	9,834	9,720
Combined O^M and $O^{M(5)}$ (O^M first 5 years)	$2\frac{1}{2}$%	114,634	10,484	9,989	401,913	10,484	10,188	833,162	10,378	10,202	1,258,992	10,319	10,200
	3 %	109,329	10,000	9,526	386,520	10,083	9,798	807,315	10,055	9,886	1,225,254	10,043	9,927
	$3\frac{1}{2}$%	104,273	9,537	9,085	371,739	9,697	9,423	782,324	9,744	9,580	1,192,713	9,775	9,663
Combined H^M and $H^{M(5)}$ (H^M first 5 years)	3 %	107,363	9,820	9,354	382,547	9,979	9,697	800,806	9,974	9,806	1,216,306	9,969	9,854
$O^{[M]}$ and O^M . . .	3 %	114,122	10,439	9,943	391,119	10,203	9,914	811,969	10,114	9,943	1,229,369	10,076	9,960

TABLE XXVI.—*continued.*

Age of Office		25 Years			30 Years			35 Years			40 Years		
Tables and Rate of Interest		Actual Reserve	Comparative Reserve O^M 3 % =10,000	Comparative Reserve $O^{[M]}$ 3 % =10,000	Actual Reserve	Comparative Reserve O^M 3 % =10,000	Comparative Reserve $O^{[M]}$ 3 % =10,000	Actual Reserve	Comparative Reserve O^M 3 % =10,000	Comparative Reserve $O^{[M]}$ 3 % =10,000	Actual Reserve	Comparative Reserve O^M 3 % =10,000	Comparative Reserve $O^{[M]}$ 3 % =10,000
$O^{[M]}$	3 %	1,504,938	10,093	10,000	1,656,404	10,084	10,000	1,705,623	10,081	10,000	1,714,740	10,081	10,000
O^M	2½%	1,528,584	10,252	10,157	1,681,895	10,239	10,154	1,731,621	10,236	10,153	1,740 800	10,234	10,152
	3 %	1,490,996	10,000	9,908	1,642,576	10,000	9,916	1,691,872	10,000	9,920	1,700,996	10,000	9,920
	3½%	1,454,227	9,752	9,663	1,604,100	9,766	9,684	1,652,947	9,770	9,691	1,662,009	9,771	9,693
$O^{M(5)}$	2½%	1,516,670	10,172	10,078	1,669,502	10,165	10,080	1,719,079	10,161	10,080	1,728,251	10,160	10,079
	3 %	1,478,881	9,919	9,827	1,629,996	9,923	9,840	1,679,124	9,924	9,845	1,688,231	9,925	9,845
	3½%	1,442,278	9,673	9,584	1,591,649	9,690	9,609	1,640,331	9,695	9,617	1,649,369	9,697	9,619
H^M	2½%	1,515,519	10,164	10,070	1,668,249	10,156	10,071	1,717,784	10,153	10,071	1,726,945	10,152	10,071
	3 %	1,478,143	9,913	9,822	1,629,068	9,918	9,835	1,678,178	9,919	9,839	1,687,289	9,920	9,840
	3½%	1,441,367	9,667	9,578	1,590,579	9,684	9,603	1,639,218	9,689	9,611	1,648,255	9,690	9,613
Combined O^M and $O^{M(5)}$ throughout .	2½%	1,541,040	10,335	10,240	1,694,409	10,315	10,229	1,744,125	10,309	10,226	1,753,313	10,307	10,225
	3 %	1,503,352	10,083	9,990	1,655,005	10,076	9,992	1,704.280	10,073	9,993	1,713,406	10,073	9,993
	3½%	1,466,490	9,836	9,745	1,616,411	9,840	9,758	1,665,242	9,842	9,763	1,674,301	9,843	9,764
Combined O^M and $O^{M(5)}$ (O^M first 5 years)	2½%	1,533,822	10,287	10,192	1,687,191	10,272	10,186	1,736,907	10,267	10,184	1,746,095	10,265	10,183
	3 %	1,496,206	10,035	9,942	1 647,859	10,032	9,948	1,697,134	10,031	9,950	1,706,260	10,031	9,951
	3½%	1,459,450	9,789	9,698	1,609,371	9,798	9,716	1,658,202	9,801	9,722	1,667,261	9,802	9,724
Combined H^M and $H^{M(5)}$ (H^M first 5 years)	3 %	1,485,890	9,966	9,874	1,636,978	9,966	9,883	1,686,116	9,966	9,886	1,695,221	9,966	9,886
$O^{[M]}$ and O^M . . .	3 %	1,500,082	10,061	9,968	1,651,514	10,054	9,970	1,700,755	10,052	9,972	1,709,856	10,052	9,972

Note on the Report of the Registrar of the Land Registry for the years 1902–1905.

By JAMES ROBERT HART, F.I.A.

A NUMBER of important changes in our English system of registration of title are foreshadowed in the report of the Registrar of the Land Registry for the years 1902–1905. If given effect to by legislation, there can be little doubt that these will go a long way towards removing the chief difficulties and complications that have marked the working of the Acts hitherto, and placing the system on the practical and reasonable basis the public so much desire. One of the most important steps in this direction is the increase in the number of titles registered as Absolute. Hitherto our Acts have failed in this respect. Certain new facilities were indeed introduced in 1903 as a remedy, but these have had but slight effect, only 188 applications for absolute title having been made under them. Fresh legislation is therefore suggested by the Registrar on the following lines. Experience gained with the applications for absolute title already received shows that it would be possible, without increasing the initial fees for registration, to make all registered titles absolute instead of possessory from the beginning. As the Registrar puts it, the period of incubation (so to speak) of a registered title would either be eliminated altogether or reduced to so short a duration as to be practically of no moment; while the additional cost could be defrayed by slightly increasing the fees on subsequent dealings. The method on which it would be proposed to work is based on the extreme rarity of any real defect in a title which has passed through a sale, especially if that sale be a public one; moreover, the Registry's investigation can be practically reduced to a check of the purchaser's investigation, and is greatly facilitated by the fact that a number of titles in one district throw light upon each other. It is further suggested that when this process has advanced some way and a fair proportion of land is on the registry with absolute title, there should be power by Order in Council to provide for the similar registration of the whole of the remainder within a short period. It is obvious that general registration of all titles in a district at once could be conducted on much safer and more economical lines than registration of the same titles one by one according to the accident of sale.

Having regard to the great benefits that would accrue from these improved conditions, little opposition would probably be raised to the Registrar's proposals for the extension of the system

to other counties beyond London. At present compulsory registration is confined to London by a clause in the Act of 1897 by which the Privy Council cannot make the necessary order for extension without the invitation of the County Council of the County to be affected. The main reason for such a provision seems to have been the fear of disturbance of business as the result of rash action in applying registration to a large area. But no such disturbance has occurred in London; and it is, therefore, suggested that the question of extension might be left to the Privy Council. There are a number of weighty reasons why such a course would be preferable to the present arrangement. There would be very obvious advantages in the system being extended by a central authority according to a pre-arranged plan, instead of haphazard according as the invitations of the County Councils come in; and, further, there would be a great advance towards removing the want of uniformity which is one of the drawbacks felt at present. In Middlesex and Yorkshire there are deed registries, in other counties none. Registration of title in London is compulsory, in other counties optional.

What may be regarded as by no means the least interesting and important question referred to in the report relates to the possession of, and dealings with, the "legal estate." In respect of this feature our present system is very unsatisfactory. The Acts practically allow two titles, one on, and the other off, the register; with the result that not only are the benefits of registration in regard to simplicity of dealing almost annulled in many cases, but new difficulties and complications are created which tend to render the new system of conveyancing worse than the old. It is, however, satisfactory to find that the Registrar recommends further legislation to remedy this state of things. After referring to the question as to whether, in view of registration, any useful purpose is served by retaining the time-honoured distinctions between legal and equitable estate, he proposes that the registered proprietor for the time being should have the fee simple (the legal fee simple, if the distinction is not abolished) vested in him, and all powers properly incident to it, and that it should be inalienable except by registered disposition. And it is further suggested that there should be fresh regulations in regard to a creditor under the statutory charge acquiring and disposing of the legal estate. These amendments would get rid of the main reason for deeds off the register; but a practice has grown up among solicitors, even where the statutory transfer or charge has been used, of having a conveyance or mortgage in the old form in addition to the registered instrument. In defence of

this practice it is said that it is more convenient to have a conveyance of the legal estate with the covenants for repairs and any special covenants in a separate deed. The Registrar points out, however, that it is very seldom that one of these documents contains anything that might not just as well have been inserted in the registered instrument, and that it is an inconvenient practice, inasmuch as it has a tendency to obscure the true issue in various legal proceedings, and that, moreover, it is preparing confusion for the future by creating and maintaining a kind of spurious title tending to perpetuate the old-fashioned system of abstracts of title with its attendant expense and uncertainty in a somewhat aggravated form.

In the above remarks mention has been made only of those features of the Registrar's Report in which changes are recommended; but the report itself contains other interesting matter, including the discussion of such questions as the demand for an enquiry and solicitors' charges for dealing with registered land, and is well worth the attention of members of the actuarial profession.

On the Error introduced into Mortality Tables by Summation Formulas of Graduation. *By* GEORGE KING, F.I.A., F.F.A.

[Read before the Institute, 17 December 1906.]

THE controversy which took place a few years ago (*J.I.A.* xxvi, 77 and 420; xxix, 59, 232 and 236) between Dr. Sprague and the late Mr. Woolhouse on Summation Formulas for the graduation of mortality tables, and more particularly on that of Mr. Woolhouse himself, left the question in a very unsatisfactory state, as a few extracts from the writings of these two great, but divergent, authorities will sufficiently show:

Dr. Sprague, xxvi, 111, said, "We must include in one "general condemnation all such graduation formulas as "Mr. Woolhouse's, Mr. Higham's, and Mr. Ansell's. They all "have a tendency to distort the true law of the facts"; and, again, xxix, 61, "I see no sufficient reason for distinguishing "between his (Mr. Woolhouse's) formula, and others of a like "kind; and I include them all in one general condemnation."

In contradiction of this, Mr. Woolhouse, xxvi, 424, said, "The method (his own) stands alone, as systematically based on "rational principles"; and, again, xxix, 241, "I emphatically "repeat that, with practicable data, the application of my system "of adjustment to a mortality table will always give the best "possible results."

In justification of his opinion, Dr. Sprague showed, xxvi, 108, that if Mr. Woolhouse's formula be applied to a series of the fourth degree, which is already perfectly free from irregularities, and if u_x be the original value of a term, and (u) the value after application of the graduation formula, then $(u) = u_x - 5{\cdot}4\Delta^4 u_x$; and on this result remarked that, "if the original series proceeds "by constant third differences, that is to say, if the fourth and "all subsequent differences of u_x vanish, then Mr. Woolhouse's "formula reproduces the original series; but if the fourth "differences are constant, then the formula introduces a constant "error, $-5{\cdot}4\Delta^4 u_x$, into the graduated series."

Mr. Woolhouse replied, xxvi, 425, "This is undoubtedly true, "but it is of no consequence whatever in respect of the method, "as the assumption of constant fourth differences will necessarily "take the series of numbers very far out of the range of all "mortality tables. If Mr. Sprague had carried the expression "a little further, say to fifth or sixth differences, he would have "startled himself with still more alarming coefficients."

Moreover, there appear to have been divergent views as to the function to which the graduation formula may best be applied. The late Mr. J. A. Higham, xxv, 20, graduated the q_x column of A. J. Finlaison's Female Annuitants Table by Woolhouse's formula, and also by a similar formula of his own; and Dr. Sprague, xxvi, 82, made use of that graduation for purpose of comparison with his own graphic method. But on this procedure Mr. Woolhouse, xxvi, 424, remarked, "It should also "be noted that the method was specially designed for the "adjustment of the *Number Living* at each year of age, and was "never put forward by myself for the adjustment of the probability "of dying in the year, a function that is not sufficiently "amenable to interpolation according to the intervals chosen."

To meet this contention of Mr. Woolhouse, Dr. Sprague tested the formula by applying it to graduate the l_x column of the Text-Book Table; and, to a limited extent modifying his condemnation, he remarked on the results, xxix, 234, that, "although Mr. Woolhouse's method slightly disturbs the law "of the series (l_x) to which it is applied, the practical effect of "this disturbance on the probability of dying is so slight as to "be of no practical importance except at extreme old ages. I feel, "therefore, bound to admit that my objection to the method, on the "ground of its tendency to disturb the law of the series, although "theoretically well founded, may be disregarded in practice."

It might be thought from Mr. Woolhouse's emphatic assertion, and, although he has not touched on this part of the question, from the circumstances of Dr. Sprague's partial change of views as shown above, that summation formulas of graduation, or at any rate that of Mr. Woolhouse, give better results when applied to the l_x column than when applied to q_x; yet Mr. J. Spencer, xxxviii, 341, graduated q_x of the O^M Table by a formula, deduced by himself according to Mr. G. F. Hardy's methods, and similar in principle to that of Woolhouse; and produced a table which, as far as he has carried it, is, by every test of good graduation, as nearly perfect as can be. It is intended in the present paper, not only to try to ascertain the nature, and to measure the magnitude, of the error which all these formulas unquestionably do introduce, and so to gauge its importance, but also to enquire as to the functions to which the formulas should be applied in order to keep the error at its minimum.

In a series of remarkable papers, xxiii, 335; xxiv, 44; xxv, 15 and 245; and xxxi, 319, the late Mr. J. A. Higham developed his method of constructing summation formulas of graduation; and interest in the subject was greatly enhanced by the publication, xxxii, 371, by Mr. G. F. Hardy, of a most original alternative plan. The principle of the two systems is the same, but the details differ.

Mr. Higham's method consists in expressing the value of the central term of $n+1$ terms of a series, assumed to progress by third differences, by means of the difference between certain multiples of two sets of summations of the same $n+1$ terms; the multipliers being so chosen that, in the result, the differences of the series, up to and including the third, vanish.

Mr. Hardy, on the other hand, takes two or more unequal sections of the series, but all having the same central term, and expresses the value of that term by means of the sums of, and the differences between, certain multiples of summations of the several sections, the multipliers being again so chosen that the differences of the series, up to and including the third, vanish.

Mr. Higham takes the first term of the series for his point of origin, while Mr. Hardy fixes his attention only on the central term; but, seeing that by both everything is collected into one great central term, this diversity of procedure does not affect the final result. It even happens in a few cases that a formula may be deduced by either method, and therefore belongs to both classes.

Mr. Hardy shows how, by means of a short prepared table

which he gives, to ascertain very easily the proper summations and multipliers; whereas Mr. Higham decides first on the summations, and then, by an algebraical process, finds the proper multipliers. But his way of arriving at the multipliers has the appearance, from his examples, of being empirical, although, xxv, 18, he supplies a general formula. That formula is, perhaps, a little troublesome to apply, and at any rate it gives the impression of being intricate; and hence Mr. Hardy's method has by many been preferred. Which of the two produces the better graduation formulas it might be hazardous to say. Probably there is not much to choose between them; and in order that Mr. Higham's may be used as easily as Mr. Hardy's, opportunity is here taken to demonstrate a more simple way of finding Mr. Higham's multipliers.

Let there be a series, assumed to progress by third differences, of $n+1$ terms, n being even, whereof u_0 is the first term, and consequently $u_{\frac{n}{2}}$ is the central term; and let the series be summed p at a time, then q at a time, and so on, to the number of t summations in all, until the whole is collected into one great central term, which we may write S. Then Mr. Higham has shown that the terms of the original series must appear in S to the number of $p \times q \times$ &c, which we may write P. Also, if $p^2+q^2+\text{\&c}=s_2$, Mr. Higham has shown that

$$\frac{\mathrm{S}}{\mathrm{P}}=u_0+\frac{n}{2}\Delta+\left\{\frac{n(n-2)}{8}+\frac{s_2-t}{24}\right\}\Delta^2 + \left\{\frac{n(n-2)(n-4)}{48}+\frac{(n-2)(s_2-t)}{48}\right\}\Delta^3 \quad . \quad . \quad . \quad (1)$$

Similarly, if the same series be summed v at a time, then w at a time, and so on to the number of τ summations in all, and if the result be written Σ, and if $v \times w \times \text{\&c.} = \Pi$, and $v^2+w^2+\text{\&c.}=\sigma_2$, we shall have

$$\frac{\Sigma}{\Pi}=u_0+\frac{n}{2}\Delta+\left\{\frac{n(n-2)}{8}+\frac{\sigma_2-\tau}{24}\right\}\Delta^2 + \left\{\frac{n(n-2)(n-4)}{48}+\frac{(n-2)(\sigma_2-\tau)}{48}\right\}\Delta^3 \quad . \quad . \quad . \quad (2)$$

And also, by the ordinary rules of Finite Differences,

$$u_{\frac{n}{2}}=u_0+\frac{n}{2}\Delta+\frac{n(n-2)}{8}\Delta^2+\frac{n(n-2)(n-4)}{48}\Delta^3 \quad . \quad . \quad . \quad (3)$$

From equations (1) and (3),

$$u_{\frac{n}{2}}=\frac{\mathrm{S}}{\mathrm{P}}-\frac{s_2-t}{24}\Delta^2-\frac{(n-2)(s_2-t)}{48}\Delta^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From equations (1) and (2),

$$\frac{\mathrm{S}}{\mathrm{P}}-\frac{\Sigma}{\Pi}=\frac{(s_2-t)-(\sigma_2-\tau)}{24}\Delta^2+\frac{(n-2)\{(s_2-t)-(\sigma_2-\tau)\}}{48}\Delta^3 \quad . \; (5)$$

From equations (4) and (5),

$$u_{\frac{n}{2}}=\frac{\mathrm{S}}{\mathrm{P}}-m\left(\frac{\mathrm{S}}{\mathrm{P}}-\frac{\Sigma}{\Pi}\right),\text{ when}$$

$$m\left\{\frac{(s_2-t)-(\sigma_2-\tau)}{24}\Delta^2+\frac{(n-2)\left[(s_2-t)-(\sigma_2-\tau)\right]}{48}\Delta^3\right\}$$

$$=\frac{s_2-t}{24}\Delta^2-\frac{(n-2)(s_2-t)}{48}\Delta^3$$

that is when

$$m\{\Delta^2+\tfrac{1}{2}(n-2)\Delta^3\}\{(s_2-t)-(\sigma_2-\tau)\}=\{\Delta^2+\tfrac{1}{2}(n-2)\Delta^3\}(s_2-t)$$

that is when

$$m=\frac{s_2-t}{(s_2-t)-(\sigma_2-\tau)} \quad . \; . \; . \; . \; . \; . \; . \; (6)$$

Therefore,

$$u_{\frac{n}{2}}=\frac{\mathrm{S}}{\mathrm{P}}-\frac{s_2-t}{(s_2-t)-(\sigma_2-\tau)}\left\{\frac{\mathrm{S}}{\mathrm{P}}-\frac{\Sigma}{\Pi}\right\} \quad . \; (7)$$

We may take as an example of formula (7) that given, xxv, 18, by Mr. Higham, and worked out by his own method. Let the series consist of 17 terms, so that $n=16$. Let S result from summing four times in fives, so that $\mathrm{P}=5^4=625$; $t=4$; and $s_2=4\times5^2=100$. Also let Σ result from summing twice in fives, and then in nines, so that $\Pi=5^2\times9=225$; $\tau=3$; and $\sigma_2=2\times5^2+9^2=131$. Then

$$u_{\frac{n}{2}}=\frac{\mathrm{S}}{625}-\frac{96}{96-128}\left(\frac{\mathrm{S}}{625}-\frac{\Sigma}{225}\right)$$

$$=\frac{4\mathrm{S}}{625}-\frac{3\Sigma}{225}=\frac{4\mathrm{S}}{625}-\frac{4\Sigma}{300}$$

$$=\frac{64\mathrm{S}}{10{,}000}-\frac{4\Sigma}{300}$$

as deduced by Mr. Higham.

This result may be expressed in a form more convenient for purposes of numerical application. If (u) be the graduated value of $u_{\frac{n}{2}}$, and if, following Mr. Hardy, $[r]$ represent a summation by r's, then

$$30{,}000(u)=192\mathrm{S}-400\Sigma=4[5]^2\{48[5]^2-100[9]\}$$

This formula, although it produces a good graduation, is, even at its best, somewhat cumbrous to use.

Mr. Woolhouse arrived at his graduation formula by making five interpolations for the graduated value by means of five quinquennial curves, and then by taking the arithmetical mean of the results. There was no suspicion that it was a summation formula, until Mr. Ackland, xxiii, 352, gave it summation form, and Mr. Higham, xxxi, 332, and Mr. Hardy, xxxii, 373, condensed it into a very simple expression, namely

$$125(u)=[5]^3\{10[1]-3[3]\}.$$

It is found to be very similar indeed to several others, and we may, not unreasonably, surmise that all summation formulas might be arrived at in the same way as Mr. Woolhouse found his, if only we could select the right curves to average.* We must therefore agree with Dr. Sprague that there is no sufficient reason for distinguishing between Woolhouse's formula and the others. Consequently, and also because it was over his formula that the controversy arose, it has been used to illustrate this paper.

We have seen that Dr. Sprague showed that, to fourth differences, the error introduced by the formula is $-5{\cdot}4\Delta^4 u_x$; but this expression does not enable us to assign to it a numerical value. We cannot easily, if at all, find the form of the fourth difference of any of the functions that are to be graduated, and it will be much more convenient to employ the calculus, and examine the differential coefficients.

In all summation formulas, it is the central term which is graduated, and terms equidistant on each side are symmetrically involved. The graduated term takes the form

$$k(u)=au_0+b(u_{-1}+u_{+1})+c(u_{-2}+u_{+2})+\&c.$$

where k, a, b, c, &c. are numerical coefficients depending on the particular graduation formula used, and where u_0 is now the central term.

Expanding $(u_{x-h}+u_{x+h})$ by Taylor's Theorem, we have

$$(u_{x-h}+u_{x+h})=2u_x+h^2\cdot\frac{d^2u_x}{dx^2}+\frac{h^4}{12}\cdot\frac{d^4u_x}{dx^4}+\frac{h^6}{360}\cdot\frac{d^6u_x}{dx^6}+\&c.$$

Here all the differential coefficients of odd orders vanish, leaving

* See *J.I.A.*, xxxii, 387, where Mr. Todhunter proves the converse proposition, that the average of the interpolated values given by any odd number of curves can be expressed by a summation formula.

only those of even orders. If we write $(u_{x-h}+u_{x+h})=\gamma_h$, Woolhouse's formula takes the form

$$125(u)=25u_0+24\gamma_1+21\gamma_2+7\gamma_3+3\gamma_4-2\gamma_6-3\gamma_7$$

and it is easy from this to calculate that, to the sixth differential coefficient, the error involved is

$$-5{\cdot}4\frac{d^4u_0}{dx^4}-9{\cdot}5\frac{d^6u_0}{dx^6}.$$

It is legitimate to expand $(u_{x-h}+u_{x+h})$ by Taylor's Theorem, because we shall see that, in the case of all the functions with which we have to deal, the differential coefficients, as we pass to the higher orders, rapidly diminish, and soon become infinitesimal, and, moreover, although at first their numerical multipliers increase, yet very soon these also begin to decrease rapidly, until ultimately they become evanescent.

In order to arrive at the form of the differential coefficients of the functions to be graduated, it is necessary to assume some law of mortality, and that of Makeham lends itself admirably to the purpose. Several of the principal mortality tables in use adhere to it almost throughout; and, to the others, the law can be applied to the short sections we require, of, say, twenty-five terms at the most, without altering the sequence of the values; and especially so at the highest ages, where summation formulas produce the largest errors.

We therefore proceed to find the successive differential coefficients, according to Makeham's law, of the principal mortality functions suitable for graduation, namely, p_x, l_x, colog p_x, and m_x. In doing so we shall, for brevity, use the Greek letter "λ" to denote Napierian logarithms, while "log" and "colog" denote common logarithms.

Differential Coefficients of p_x, or q_x.

Let $\lambda p_x=w$.

$$\frac{dp}{dx}=p\frac{dw}{dx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\frac{d^2p}{dx^2}=p\frac{d^2w}{dx^2}+\frac{dp}{dx}\cdot\frac{dw}{dx}$$

$$=p\left\{\frac{d^2w}{dx^2}+\left(\frac{dw}{dx}\right)^2\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\frac{d^3p}{dx^3}=p\frac{d^3w}{dx^3}+2\frac{dp}{dx}\cdot\frac{d^2w}{dx^2}+\frac{d^2p}{dx^2}\cdot\frac{dw}{dx}$$

$$=p\frac{d^3w}{dx^3}+2p\frac{dw}{dx}\cdot\frac{d^2w}{dx^2}+p\frac{dw}{dx}\left\{\frac{d^2w}{dx^2}+\left(\frac{dw}{dx}\right)^2\right\}$$

$$=p\left\{\frac{d^3w}{dx^3}+3\frac{dw}{dx}\cdot\frac{d^2w}{dx^2}+\left(\frac{dw}{dx}\right)^3\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\frac{d^4p}{dx^4}=p\frac{d^4w}{dx^4}+3\frac{dp}{dx}\cdot\frac{d^3w}{dx^3}+3\frac{d^2p}{dx^2}\cdot\frac{d^2w}{dx^2}+\frac{d^3p}{dx^3}\cdot\frac{dw}{dx}$$

$$=p\frac{d^4w}{dx^4}+3p\frac{dw}{dx}\cdot\frac{d^3w}{dx^3}+3p\left\{\frac{d^2w}{dx^2}+\left(\frac{dw}{dx}\right)^2\right\}\frac{d^2w}{dx^2}$$

$$+p\left\{\frac{d^3w}{dx^3}+3\frac{dw}{dx}\cdot\frac{d^2w}{dx^2}+\left(\frac{dw}{dx}\right)^3\right\}\frac{dw}{dx}$$

$$=p\left\{\frac{d^4w}{dx^4}+4\frac{dw}{dx}\cdot\frac{d^3w}{dx^3}+6\left(\frac{dw}{dx}\right)^2\frac{d^2w}{dx^2}+3\left(\frac{d^2w}{dx^2}\right)^2+\left(\frac{dw}{dx}\right)^4\right\} \quad . \quad (4)$$

To find the successive differential coefficients of w, we have

$$w=\lambda p_x=\lambda s+(c-1)(\lambda g)c^x\,;$$

whence $\qquad (w-\lambda s)=(\lambda p_x-\lambda s)=(c-1)(\lambda g)c^x$

$$\frac{dw}{dx}=(c-1)(\lambda g)(\lambda c)c^x=(\lambda p_x-\lambda s)(\lambda c) \quad . \quad . \quad . \quad (5)$$

$$\frac{d^2w}{dx^2}=\frac{dw}{dx}(\lambda c) \qquad =(\lambda p_x-\lambda s)(\lambda c)^2 \quad . \quad . \quad . \quad (6)$$

$$\frac{d^3w}{dx^3}=\frac{d^2w}{dx^2}(\lambda c) \qquad =(\lambda p_x-\lambda s)(\lambda c)^3 \quad . \quad . \quad . \quad (7)$$

$$\frac{d^4w}{dx^4}=\frac{d^3w}{dx^3}(\lambda c) \qquad =(\lambda p_x-\lambda s)(\lambda c)^4 \quad . \quad . \quad . \quad (8)$$

If now we substitute for the differential coefficients of w in equations 1 to 4, their values as found in equations 5 to 8, we have, after reduction,

$$\frac{dp_x}{dx}=p_x(\lambda p_x-\lambda s)(\lambda c)$$

$$\frac{d^2p_x}{dx^2}=p_x\{(\lambda p_x-\lambda s)+(\lambda p_x-\lambda s)^2\}(\lambda c)^2$$

$$\frac{d^3p_x}{dx^3}=p_x\{(\lambda p_x-\lambda s)+3(\lambda p_x-\lambda s)^2+(\lambda p_x-\lambda s)^3\}(\lambda c)^3$$

$$\frac{d^4p_x}{dx^4}=p_x\{(\lambda p_x-\lambda s)+7(\lambda p_x-\lambda s)^2+6(\lambda p_x-\lambda s)^3+(\lambda p_x-\lambda s)^4\}(\lambda c)^4$$

The differential coefficients of q_x are, of course, of the same numerical values as those of p_x, only of opposite sign.

Differential Coefficients of l_x.

Let $\lambda l_x = u$.

Then $\dfrac{dl_x}{dx} = l_x \dfrac{d\lambda l_x}{dx} = l_x \dfrac{du}{dx}$.

But we had $\dfrac{dp_x}{dx} = p_x \dfrac{dw}{dx}$, which is in the same form. Therefore, substituting l_x for p_x, and u for w, the equations for the differential coefficients of p_x apply. That is,

$$\frac{dl}{dx} = l\frac{du}{dx} \quad \ldots \quad (1a)$$

$$\frac{d^2l}{dx^2} = l\left\{\frac{d^2u}{dx^2} + \left(\frac{du}{dx}\right)^2\right\} \quad \ldots \quad (2a)$$

$$\frac{d^3l}{dx^3} = l\left\{\frac{d^3u}{dx^3} + 3\frac{du}{dx}\cdot\frac{d^2u}{dx^2} + \left(\frac{du}{dx}\right)^3\right\} \quad \ldots \quad (3a)$$

$$\frac{d^4l}{dx^4} = l\left\{\frac{d^4u}{dx^4} + 4\frac{du}{dx}\cdot\frac{d^3u}{dx^3} + 6\left(\frac{du}{dx}\right)^2\frac{d^2u}{dx^2} + 3\left(\frac{d^2u}{dx^2}\right)^2 + \left(\frac{du}{dx}\right)^4\right\} \quad . \quad (4a)$$

To find the differential coefficients of u, we have—

$$\mu_x = (-\lambda s) + (\lambda c)(-\lambda g)c^x.$$

whence— $(\mu_x + \lambda s) = (\lambda c)(-\lambda g)c^x$

$$\frac{d\mu_x}{dx} = (\lambda c)(-\lambda g)c^x(\lambda c)$$

$$= (\mu_x + \lambda s)(\lambda c) \quad \ldots \quad (\alpha)$$

$$\frac{d^2\mu_x}{dx} = (\mu_x + \lambda s)(\lambda c)^2 \quad \ldots \quad (\beta)$$

$$\frac{d^3\mu_x}{dx^3} = (\mu_x + \lambda s)(\lambda c)^3 \quad \ldots \quad (\gamma)$$

$$\frac{d^4\mu_x}{dx^4} = (\mu_x + \lambda s)(\lambda c)^4 \quad \ldots \quad (\delta)$$

But— $u = \lambda l_x$;

therefore—

$$\frac{du}{dx} = -\mu_x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5a)$$

$$\frac{d^2u}{dx^2} = -(\mu_x + \lambda s)(\lambda c) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

$$\frac{d^3u}{dx^3} = -(\mu_x + \lambda s)(\lambda c)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7a)$$

$$\frac{d^4u}{dx^4} = -(\mu_x + \lambda s)(\lambda c)^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8a)$$

Inserting in equations (1*a*) to (4*a*) the values given in equations (5*a*) to (8*a*), we finally have—

$$\frac{dl_x}{dx} = -l_x\mu_x$$

$$\frac{d^2l_x}{dx^2} = -l_x\{(\mu_x + \lambda s)(\lambda c) - (\mu_x)^2\}$$

$$\frac{d^3l_x}{dx^3} = -l_x\{(\mu_x + \lambda s)(\lambda c)^2 - 3\mu_x(\mu_x + \lambda s)(\lambda c) + (\mu_x)^3\}$$

$$\frac{d^4l_x}{dx^4} = -l_x\{(\mu_x + \lambda s)(\lambda c)^3 - 4\mu_x(\mu_x + \lambda s)(\lambda c)^2 - 3(\mu_x + \lambda s)^2(\lambda c)^2 + 6(\mu_x)^2(\mu_x + \lambda s)(\lambda c) - (\mu_x)^4\}$$

Differential Coefficients of colog p_x.

We have $\operatorname{colog} p_x = (-\log s) + (c-1)(-\log g)c^x$.
Whence $(\operatorname{colog} p_x + \log s) = (c-1)(-\log g)c^x$.

And

$$\frac{d \operatorname{colog} p_x}{dx} = (c-1)(-\log g)c^x(\lambda c)$$

$$= (\operatorname{colog} p_x + \log s)(\lambda c)$$

$$\frac{d^2 \operatorname{colog} p_x}{dx^2} = (\operatorname{colog} p_x + \log s)(\lambda c)^2$$

$$\frac{d^3 \operatorname{colog} p_x}{dx^3} = (\operatorname{colog} p_x + \log s)(\lambda c)^3$$

$$\frac{d^4 \operatorname{colog} p_x}{dx^4} = (\operatorname{colog} p_x + \log s)(\lambda c)^4.$$

Differential Coefficients of m_x.

The central death rate may, for present purposes, be considered to be the same as the force of mortality at an age half-a-year older: that is, we may assume that $m_x = \mu_{x+\frac{1}{2}}$. In finding the differential coefficients of l_x we incidentally found in equations α to δ those of μ_x. Therefore,

$$\frac{dm_x}{dx} = (m_x + \lambda s)(\lambda c)$$

$$\frac{d^2m_x}{dx^2} = (m_x + \lambda s)(\lambda c)^2$$

$$\frac{d^3m_x}{dx^3} = (m_x + \lambda s)(\lambda c)^3$$

$$\frac{d^4m_x}{dx^4} = (m_x + \lambda s)(\lambda c)^4$$

To apply the foregoing investigations numerically we must have a mortality table which follows Makeham's law throughout, and the $O^{M(5)}$ Table has been selected. But at the older ages the published table follows the law only nominally, because the values in the column of l_x have been cut down to avoid fractions, and the tabulated values of q_x are derived from these curtailed values. It was therefore necessary to recalculate the portion from age 80 onwards, and the results are given in Table I of the appendix hereto, headed "$O^{M(5)}$ Table Recomputed." *

The table, so recomputed, was graduated four times by Woolhouse's formula, the functions q_x, l_x, d_x, and colog p_x being successively used; but the graduation was limited to ages 80 to 95 inclusive. It was not thought necessary to go through the labour of graduating at the younger ages, because the error introduced is there so small as to be quite unimportant, and, moreover, it can be ascertained with minute accuracy by means of the differential coefficients.

Both l_x and d_x were graduated as a matter of curiosity, but that was not necessary, because exactly the same results are produced by the two processes to the last place of many decimals. In some of his writings, as we have seen above, Mr. Woolhouse was very emphatic that his formula must be applied to l_x, and to nothing else; and an impression has thereby been produced, and in some quarters has been widely current, that to graduate d_x has

* It should be noted that the values of colog$_{10}p_x$, calculated by the constants employed in Mr. G. F. Hardy's graduation of the $O^{M(5)}$ Table, are given, to seven places of decimals, on page 153 of the "Account of Principles and Methods, &c."—[ED. *J.I.A.*]

not the same effect. One actuary of eminence, who is of unusual skill as a mathematician, has even written that "Mr. Woolhouse always held that his formula applied only to "the numbers living, and this is equivalent to assuming second "differences of d_x constant, and not third differences. In other "words, assuming that the values of l_x are to be graduated, we "ought, from Mr. Woolhouse's point of view, to use a fourth "difference summation formula."

Here it is unequivocally asserted that, if we graduate d_x instead of l_x, we gain one order of differences; but that is a fallacy, in so far as summation formulas for graduation are concerned. In the notation of Mr. G. F. Hardy, $d_x = l_x(2[1]-[2])$. This shows that the graduation of d_x instead of l_x is equivalent to introducing the operations represented by $(2[1]-[2])$; and it is immaterial at which stage of the process these operations are performed. They may come first; that is, we may take d_x of the ungraduated table; or they may come last; that is, we may take d_x of the graduated table; and the final result is the same.

Mr. Woolhouse was fully aware of this fact, and, in the first paper he wrote on his graduation formula, he said, xv, 393, that the yearly decrements may with advantage be adjusted, because the numbers are so much smaller; and he went on to explain that, for the Institute of Actuaries' Tables, he adjusted the decrements, and, for a check, adjusted every fifth value of l_x. Precisely the same explanation is also given in the introduction to the Institute of Actuaries' Life Tables, pp. xc and xci.

In Table 2 of the Appendix are given the second and fourth differential coefficients of the several functions to be discussed. In this Table, and in all the others, positive quantities are printed in old style type, and negative quantities in modern type.

Graduation by q_x.

In Table 3 the result of graduating q_x is set forth, with columns showing the actual error, and the error as estimated by means of the fourth differential coefficients. As far as age 82 the error is negative, and the value of q_x is understated by the graduation, but only to a very minute extent, never by more than a unit in the fifth place of decimals. Above age 82 the error becomes positive, and increases a little numerically in amount; but it always remains very small as compared with q_x itself. At age 95 it amounts to ·00013, but, as

regards q_x, that still is less than 1 in 3,000. Such an unimportant deviation cannot affect monetary values, and, therefore, notwithstanding the protests of Mr. Woolhouse, we must admit that we have in the rate of mortality a function well suited for graduation by summation formulas.

The closeness of the actual error to that estimated is remarkable. The sum of the actual errors from age 80 to 95 inclusive is $+\cdot000779-\cdot000009$, or $+\cdot000770$; and the sum of those estimated is $+\cdot000788-\cdot000007$, or $+\cdot000781$; the difference being only ·000011. That is because the whole error is practically concentrated in the fourth differential coefficient, those of higher orders being too small to exercise any appreciable effect. The form of the coefficients is too complicated for us easily to work out the sixth; but, on reference to Table 2, it is seen that the fourth is never more, numerically, than about two per-cent of the second, and generally much less; while from age 82 onwards it is of opposite sign. We may reasonably assume that the proportion of the sixth to the fourth is analogous, and probably also the sign again changes; in which case, to bring in the sixth differential coefficient would reduce the estimate of error very slightly. This view is confirmed by Table 3, from which it appears that, by stopping at the fourth differential coefficient, we on the whole overestimate the error.

Graduation by l_x or d_x.

Table 4 gives particulars of the graduation by l_x or d_x. The first part shows the ungraduated value of l_x according to the $O^{M(5)}$ Table recomputed as explained, and its graduated value; and in the next two columns there is given the error, actual and estimated, in l_x itself. At the younger ages the error is not large as compared with the magnitude of the function, but the sign varies at different parts of the table. At the oldest ages the error, however, becomes considerable.

An error in l_x is not in itself of main importance, but only in so far as it affects q_x; and therefore, in the second part of the table, the error in q_x is given, which arises from graduating l_x. On comparing Tables 3 and 4, it will be seen that, as far as age 80, it does not much matter whether we graduate q_x or l_x. In either case we get very good results. But from age 80 onwards the l_x graduation is defective, and at the oldest ages the error is very large, affecting the second place of decimals in q. Also, the estimated error does not come so near the actual as when q_x is

graduated. For ages 80 to 94 inclusive, the total actual error is $+\cdot046583-\cdot005126$, or $+\cdot041457$; while the total estimated error is $+\cdot057756-\cdot005830$, or $+\cdot051926$. This is due to the omission of the sixth differential coefficient from the estimate. Table 2 shows that the second and fourth differential coefficients at the highest ages are of the same sign, and the fourth does not bear so small a proportion to the second as in the case of q_x. If the sixth has relations to the fourth like those of the fourth to the second, then the partial failure of the estimate of error is accounted for. The form of the differential coefficient is too complicated to permit of the numerical value of the sixth being easily worked out.

These investigations have not been carried beyond age 95, because the last age which appears in the published $O^{M(5)}$ Table is 102, and Woolhouse's graduation formula requires seven values beyond that last graduated. On this point Mr. Woolhouse remarks that, beyond the limiting age of the table, l_x may conveniently, as well as accurately, be put down as zero; and, acting on this view, he accordingly graduates the table by his formula to the very end. But anomalous results are thereby produced, and necessarily so. The graduation formula presupposes a function of the third degree; and a function of the third degree can never become permanently zero. A better plan seems to me to be, to carry on the graduation as far as the formula used will naturally permit, and then, for the few remaining values to employ one of Mr. Higham's short summation formulas, such as

$$35(u)=7[5][3]-10[7].$$

This formula requires only seven terms in all, and, by using it, only three terms remain ungraduated, and these can easily be dealt with by inspection. There may possibly be a slight break in the continuity of the curve at the point where the change of graduation formulas takes place, and, moreover, these short formulas do not always give smooth results, but all such irregularities can be rectified without trouble.

It has been mentioned above that Mr. Spencer was very successful in graduating q_x of the O^M Table by a summation formula, but he carried the graduation only as far as age 79. It is at ages above 80 that the greatest error produced by these formulas appears; and had Mr. Spencer gone further, the results might not have been quite so happy. Applied to q_x, however, the formula would probably prove sufficiently satisfactory to the

end, but it is not likely that that would be so if it were applied to l_x. The formula may be written

$$350(u) = [7][5]^2\{[1]+[3]+[5]-[7]\}$$
$$= 60u_0 + 57\gamma_1 + 47\gamma_2 + 33\gamma_3 + 18\gamma_4 + 6\gamma_5$$
$$-2\gamma_6 - 5\gamma_7 - 5\gamma_8 - 3\gamma_9 - \gamma_{10}$$

No less than twenty-one terms are involved, and, consequently, the errors due to the differential coefficients are comparatively large. Up to the sixth, the error is

$$-12{\cdot}6\frac{d^4u_0}{dx^4} - 34{\cdot}\dot{8}5714\dot{2}\frac{d^6u_0}{dx^6}.$$

The error due to the fourth differential coefficient is $2\frac{1}{2}$ times as great as that produced by Woolhouse's formula, while that due to the sixth is rather more than $3\frac{2}{3}$ times as great. We may, therefore, reasonably assume that neither Mr. Spencer's formula, nor any other that involves a considerable number of terms, is suitable for the graduation of l_x.

Graduation by colog p_x.

The function, log l_x, might be selected for graduation, but colog p_x is practically more convenient, because the numbers are much smaller, and it yields identical results. The relation of colog p_x to log l_x is the same as that of d_x to l_x, each being the result of taking the difference negatively, and we have seen that it is theoretically immaterial whether we apply summation formulas to a function itself or to its difference.

I am not aware that colog p_x has ever been used for graduation purposes, and yet this function is eminently suited to summation formulas. In numerical value, colog p_x is only of about half the magnitude of q_x, and its graduation error is very small. In Table 5 particulars of the graduation of this function are given. The first part of the table shows the error, actual and estimated, in colog p_x itself, and the second shows the resulting error in q_x. The error is always negative, but it is insignificant, being at its largest only about 1 in 4,000 of the value of the function. Also, the divergence between the actual error and that estimated is very slight. From ages 80 to 95 inclusive the total actual error in q_x, due to the graduation of colog p_x is $-{\cdot}001222$, and that estimated is $-{\cdot}001206$. This close agreement is due to the fact that the error arising from the

sixth differential coefficient, although it is also negative, is almost nil, seeing that at its greatest it does not amount to more than a unit in the sixth decimal place of q_x, while generally it is much less. We have $\frac{d^6 \operatorname{colog} p_x}{dx^6} = \frac{d^4 \operatorname{colog} p_x}{dx^4} \times (\lambda c)^2$, and similarly for higher orders; and $(\lambda c)^2 = \cdot 00806418$; so that the differential coefficients of $\operatorname{colog} p_x$, as we pass to the higher orders, rapidly vanish. We could not have a better function than $\operatorname{colog} p_x$ to graduate.

Graduation by m_x.

The function m_x does not naturally present itself in the construction of mortality tables from the statistics of assured lives, and therefore under ordinary circumstances it would not be thought of in connection with graduation. But in the construction of mortality tables from population statistics, m_x is generally the first function found, and sometimes we may wish to graduate it before passing to the other columns. Therefore it will be useful to enquire whether summation formulas may legitimately be used for the purpose. We have seen that the differential coefficients are very simple, being the same in form of those of $\operatorname{colog} p_x$, the only difference being that for m_x we have the Napierian logarithm of Makeham's constant, s, and for $\operatorname{colog} p_x$ the common logarithm. That being so, it was not thought to be necessary to perform an actual graduation by m_x, but the estimated error is given in Table 6. The resulting error in q_x is very small, and almost identical with that produced by graduating $\operatorname{colog} p_x$. We may therefore say that m_x is a very suitable function for graduation.

Mr. G. F. Hardy's Modified Formulas.

In his paper already referred to, xxxii, 371, Mr. Hardy introduced summation formulas of graduation belonging to a different category, and which, admittedly, are not quite true to second differences. He remarked that a formula based on a series of summations such as are represented by the symbols [4] [5] [6] will generally give a smoother curve than one based on the approximately equivalent operations $[5]^3$; and, on this principle, starting from the formula $125(u) = [5]^3\{2[3] - [5]\}$, which is true to third differences, he constructed his Friendly Societies formula, which may be written

$$120(u) = [4]\,[5]\,[6]\{2[3] - [5]\},$$

and which involves an error of one-twelfth of the second difference, as well as errors due to the fourth, sixth, &c. Regarding the error, Mr. Hardy remarked that it is of no great importance in practice; and, elsewhere, xxvii, 277, he said that it is insignificant, "not being greater in value than that due to neglect of fourth differences in all such formulas."

In passing, it may be remarked that, when we substitute $[n-1][n][n+1]$ for $[n]^3$, the error produced is one-twelfth of the second difference, or of the second differential coefficient, no matter what may be the value of n, and no matter what may be the other summations. This is convenient, because the error is thereby easily measured.

Mr. Hardy's Friendly Societies formula may be written

$$120(u)=24u_0+22\gamma_1+17\gamma_2+10\gamma_3+4\gamma_4-2\gamma_6-2\gamma_7-\gamma_8\,;$$

and the error to the sixth differential coefficient is

$$+\frac{1}{12}\cdot\frac{d^2u_0}{dx^2}-6{\cdot}50\dot{1}3\dot{8}\frac{d^4u_0}{dx^4}-13{\cdot}10\dot{1}1574\dot{0}\frac{d^6u_0}{dx^6}.$$

We have seen that Woolhouse's formula may be written

$$125(u)\doteq[5]^3\{10[1]-3[3]\}$$

and, if we modify it according to Mr. Hardy's principles, we have

$$120(u)=[4][5][6]\{10[1]-3[3]\}$$

which may be written

$$120(u)=24u_0+23\gamma_1+17\gamma_2+10\gamma_3+3\gamma_4-2\gamma_6-3\gamma_7\,;$$

and the error to the sixth differential coefficient is

$$+\frac{1}{12}\cdot\frac{d^2u_0}{dx^2}-5{\cdot}50\dot{1}3\dot{8}\frac{d^4u_0}{dx^4}-9{\cdot}85\dot{1}1574\dot{0}\frac{d^6u_0}{dx^6}.$$

The remarkable similarity between the two formulas is apparent; and, evidently, it does not much matter from the point of view of the present enquiry which of them is used. Seeing then that, thus far, Woolhouse's formula has been retained for purposes of illustration, it will be well to employ the modification of that formula, rather than another, in order to test to what extent Mr. Hardy is justified in treating as of no account the error arising from second differences.

In Tables 7 to 9 the estimated errors are given, due to graduation of q_x, l_x, and colog p_x, respectively; and they are shown in their component parts, the amounts arising from the second differential coefficient being separated from those arising from the fourth.

Looking first at the graduation of q_x, and comparing Tables 3 and 7, it will be seen that, up to age 81, the two portions of the error in Table 7 are of opposite sign, and tend to neutralize each other; but that the second difference error is usually much the larger, and, therefore, the total error of the modified formula is greater than the error of the formula in its original shape. Above age 81, the errors are all of one sign, and the modification of the formula tends to aggravate them and to render the results of the graduation, from this point of view, worse. Whether the greater smoothness in the curve, which Mr. Hardy claims, compensates for the increase in the error, is not investigated here. Even in its enlarged amount, the error, at its worst, is never of vital consequence when q_x is the function graduated.

Tables 4 and 8 afford comparisons of the errors introduced by graduating l_x. For the most part the second difference error is of the same sign as that of the fourth, and therefore the modification makes the formula decidedly worse. Neither the original formula, nor Mr. Hardy's modification of it, can be considered to apply well at the higher ages when l_x or d_x is the function graduated, and Mr. Hardy's form is the worse of the two.

The errors arising from the graduation of colog p_x by the modified formula are shown in Table 9; and it will be seen that the two errors are of opposite sign at every age. The net result is that the modified formula is a trifle more accurate than the original, but that the very small tendency to error is in the opposite direction. We may, therefore, affirm with confidence that Mr. Hardy's modification of Woolhouse, when applied to graduate colog p_x, will give results as accurate as need be desired.

Moreover, in Hardy's modification of Woolhouse, the coefficient of $\frac{d^4u}{dx^4}$ is $-5\cdot5013\dot{8}$, while in his own Friendly Societies formula it is $-6\cdot5013\dot{8}$. That is, the error arising from the fourth differential coefficient is greater by the amount of that coefficient in the Friendly Societies formula than in the modified Woolhouse; and, as it is negative, this will tend still

further to counteract the error arising from the second differential coefficient. Therefore, in the graduation of colog p_x, the Friendly Societies formula would seem to give the best results of all, in fact, results that are almost absolutely accurate. That, however, is not so when the formula is used to graduate l_x or q_x.

A few words may not be out of place here on the general question of the graduation of mortality tables. It seems to me that, excluding those tables which follow Makeham's law with sufficient accuracy, practically only two methods of graduation are available to us, namely, the graphic method, frequently and ably advocated by Dr. Sprague, and more particularly in the paper already referred to (xxvi, 77), and the method of summation formulas. Where eminent authorities have differed so irreconcilably, it is, perhaps, rash to venture an opinion; yet I feel constrained to say that, according to my own view, each of the methods, *in its own place*, is to be preferred. The one does not supplant, but rather it supplements, the other. In some cases no mathematical formula would apply. See, for instance, Dr. Sprague's remarkable graduation of marriage curves, xxii, 77. There, summation formulas would be entirely at fault, as also at the early ages of a select mortality table. On the other hand, the graphic method may be uncertain, and even misleading under certain circumstances; and there could not be a more telling illustration of this fact than one supplied by Dr. Sprague himself. In graduating the ultimate table of A. J. Finlaison's Government Annuitants, xxvi, 77, he first tried two preliminary groupings, and on them remarked, "The laws "indicated by these two groupings are essentially different, and "we have to consider which of them is to be preferred. According "to the former of them, the rate of mortality must be considered "as approximately constant from age 19 to 49. . . . According "to the latter, the rate of mortality is considerable at the "youngest ages, falls to a minimum between the ages of "30 and 35, and thereafter continually increases." He then discussed the rates resulting from the respective groupings, and said, "If we had no further information beyond what is contained "in the table, it would be difficult to say which of the two "groupings should be preferred, and the safer plan would be to "take the former and make the rate of mortality approximately "constant from 19 up to 49." Nevertheless, he eventually discarded this grouping and adopted the second, which presented a high rate of mortality at the commencement, a minimum

between ages 30 and 35, and thereafter an uninterrupted increase; and his reason for doing so was that he found in other statistics an indication of the same minimum.

Now Dr. Sprague himself showed that, if the series with which he was dealing be graduated by Woolhouse's formula, or by another of the summation formulas, the minimum between ages 30 and 35 is clearly brought out, as also a second well marked minimum between ages 45 and 50 which is not disclosed by any of Dr. Sprague's groupings. In fact, the summation formulas faithfully analyze the curve, and infallibly lay hold of its true law, even if they do not remove all irregularities. Therefore I submit that, rather than test the curve by a number of preliminary groupings, it is, as a rule, safer and better first to make a graduation by a summation formula, and then, if greater smoothness be deemed necessary, to resort to the graphic process; or else, by inspection, or by differencing, to remove any remaining irregularities, which can be easily and efficiently done. There is nothing specially sacred in the results of the first graduation, which only gives unmistakable indications of the true law; but when the table, thus graduated, is before us, we may with perfect propriety make such further adjustments as may be found to be necessary, in order to bring the final curve into as close conformity as possible with the law revealed to us, and, what is really the same thing, in order to produce a curve entirely free from irregularities.

Dr. Sprague pursues this course with the graphic method. He is not content with the first attempt, but tries again and again, correcting here, there, and everywhere, and testing frequently by taking out the expected deaths, until he attains to a curve with the required degree of perfection; and, when that is reached, he applies a final test by once more comparing the expected with the actual deaths. Equal perfection can be secured with, so far as my own personal experience goes, far less labour, and far less risk of going astray, if we start in the first instance with graduation by a summation formula.

In conclusion, it gives me very great pleasure to record my thanks to my friend Mr. S. B. Neill for the great help he rendered to me, before he proceeded to take up a high post in far Cathay, in the extensive and laborious calculations which the illustrations in this paper entailed. Without his painstaking and intelligent aid it would have been difficult to bring the work to a speedy and successful issue.

APPENDIX TABLE 1.

$O^{M(5)}$ *Mortality Table. Recomputed.*

Age	Colog p_x	l_x	d_x	Age
80	0·064731	15531·	2151·	80
1	·070571	13380·	2007·	1
2	·076963	11373·	1846·6	2
3	·083953	9526·4	1674·4	3
4	·091601	7852·0	1493·1	4
85	·099966	6358·9	1307·5	85
6	·109120	5051·4	1122·3	6
7	·119131	3929·1	942·6	7
8	·130085	2986·5	773·0	8
9	·142066	2213·5	617·6	9
90	·155175	1595·9	479·4	90
1	·169513	1116·5	360·83	1
2	·185200	755·67	262·35	2
3	·202360	493·32	183·74	3
4	·221134	309·58	123·53	4
95	·241669	186·05	79·40	95
6	·264136	106·65	48·596	6
7	·288712	58·054	28·192	7
8	·315599	29·862	15·424	8
9	·345010	14·438	7·914	9
100	·377187	6·524	3·787	100
1	·412385	2·737	1·678	1
2	·450892	1·059	·684	2

Age	p_x	q_x	m_x	Age
80	0·861527	0·138473	0·148774	80
1	·850020	·149980	·162139	1
2	·837601	·162399	·176751	2
3	·824227	·175773	·192710	3
4	·809840	·190160	·210140	4
85	·794390	·205610	·229170	85
6	·777822	·222178	·249944	6
7	·760097	·239903	·272602	7
8	·741165	·258835	·297312	8
9	·720998	·279002	·324233	9
90	·699560	·300440	·353550	90
1	·676842	·323158	·385436	1
2	·652830	·347170	·420092	2
3	·627538	·372462	·457700	3
4	·600988	·399012	·498457	4
95	·573233	·426767	·542535	95
6	·544332	·455668	·590117	6
7	·514385	·485615	·641336	7
8	·483505	·516495	·696317	8
9	·451846	·548154	·755113	9
100	·419578	·580422	·817739	100
1	·386914	·613086	·884101	1
2	·354085	·645915	·954024	2

Appendix Table 2.

$O^{M(5)}$ *Mortality Table. Recomputed.*

Differential Coefficients of Graduation Functions.

Age	p_x		l_x		Age
	$\frac{d^2 p_x}{dx^2}$	$\frac{d^4 p_x}{dx^4}$	$\frac{d^2 l_x}{dx^2}$	$\frac{d^4 l_x}{dx^4}$	
20	·0000053	·0000000	1·3341	0·03244	20
30	·0000129	·0000001	7·7383	0·06840	30
40	·0000314	·0000002	21·2153	0·12048	40
50	·0000763	·0000006	46·1407	0·09225	50
60	·0001819	·0000013	76·0623	0·47624	60
70	·0004163	·0000022	49·0598	1·86681	70
80	·0008522	·0000010	125·5215	1·60059	80
1	·0009054	·0000004	143·9822	2·42272	1
2	·0009590	·0000003	160·0181	3·23698	2
3	·0010122	·0000011	172·8661	3·96835	3
4	·0010645	·0000021	181·7502	4·54548	4
85	·0011146	·0000032	186·0868	4·89846	85
6	·0011614	·0000046	185·5480	4·96615	6
7	·0012037	·0000062	180·0982	4·70977	7
8	·0012397	·0000080	169·9438	4·11538	8
9	·0012677	·0000100	155·7020	3·21625	9
90	·0012858	·0000122	138·2656	2·08473	90
1	·0012916	·0000146	118·7666	0·83006	1
2	·0012829	·0000171	98·4147	0·41442	2
3	·0012570	·0000197	78·4744	1·51130	3
4	·0012115	·0000222	60·0257	2·34288	4
95	·0011438	·0000247	43·8864	2·83531	95

Age	Colog p_x		m_x		Age
	$\frac{d^2 \text{colog } p_x}{dx^2}$	$\frac{d^4 \text{colog } p_x}{dx^4}$	$\frac{d^2 m_x}{dx^2}$	$\frac{d^4 m_x}{ax^4}$	
20	·0000023	·0000000	·0000053	·0000000	20
30	·0000056	·0000000	·0000130	·0000001	30
40	·0000138	·0000001	·0000318	·0000003	40
50	·0000339	·0000003	·0000781	·0000006	50
60	·0000831	·0000007	·0001916	·0000015	60
70	·0002042	·0000016	·0004701	·0000038	70
80	·0005014	·0000040	·0011523	·0000093	80
1	·0005485	·0000044	·0012600	·0000102	1
2	·0006000	·0000048	·0013779	·0000111	2
3	·0006564	·0000053	·0015066	·0000121	3
4	·0007181	·0000058	·0016471	·0000133	4
85	·0007855	·0000063	·0018006	·0000145	85
6	·0008593	·0000069	·0019681	·0000159	6
7	·0009401	·0000076	·0021508	·0000173	7
8	·0010284	·0000083	·0023501	·0000190	8
9	·0011250	·0000091	·0025672	·0000207	9
90	·0012307	·0000099	·0028036	·0000226	90
1	·0013464	·0000109	·0030607	·0000247	1
2	·0014729	·0000119	·0033500	·0000270	2
3	·0016113	·0000130	·0036437	·0000294	3
4	·0017626	·0000142	·0039722	·0000320	4
95	·0019282	·0000156	·0043276	·0000349	95

N.B.—Positive quantities in Old Style Type.
Negative quantities in Modern Type.

APPENDIX TABLE 3.

Woolhouse's Formula.

Error caused by the Graduation of q_x.

Age	Original q_x	Graduated q_x	Error: Actual	Error: Estimated	Age
20	·006522	...	...	·000000	20
30	·007467	...	...	·000001	30
40	·009784	...	...	·000001	40
50	·015447	...	...	·000003	50
60	·029212	...	...	·000007	60
70	·062190	...	...	·000012	70
80	·138473	·138466	·000007	·000005	80
1	·149980	·149979	·000001	·000002	1
2	·162399	·162398	·000001	·000002	2
3	·175773	·175779	·000006	·000006	3
4	·190160	·190173	·000013	·000011	4
85	·205610	·205631	·000021	·000018	85
6	·222178	·222205	·000027	·000025	6
7	·239903	·239940	·000037	·000033	7
8	·258835	·258878	·000043	·000043	8
9	·279002	·279056	·000054	·000054	9
90	·300440	·300503	·000063	·000066	90
1	·323158	·323236	·000078	·000079	1
2	·347170	·347259	·000089	·000092	2
3	·372462	·372566	·000104	·000106	3
4	·399012	·399127	·000115	·000120	4
95	·426767	·426896	·000129	·000133	95

N.B.—Positive quantities in Old Style Type.
Negative quantities in Modern Type.

Appendix Table 4.

Woolhouse's Formula.

Error caused by the Graduation of l_x or d_x.

Age	Original l_x	Graduated l_x	Error in l_x		Age
			Actual	Estimated	
20	100770·	...	...	0·18	20
30	94050·	...	...	0·37	30
40	86493·	...	...	0·65	40
50	76717·	...	...	0·50	50
60	62265·	...	...	2·57	60
70	40636·	...	...	10·08	70
80	15531·	15539·69	8·69	8·64	80
1	13380·	13393·07	13·07	13·11	1
2	11373·	11390·03	17·03	17·48	2
3	9526·4	9546·32	19·92	21·43	3
4	7852·0	7874·37	22·37	24·55	4
85	6358·9	6382·71	23·81	26·45	85
6	5051·4	5075·34	23·94	26·82	6
7	3929·1	3951·62	22·52	25·43	7
8	2986·5	3006·13	19·63	22·22	8
9	2213·5	2228·92	15·42	17·37	9
90	1595·9	1606·18	10·28	11·26	90
1	1116·5	1121·06	4·56	4·48	1
2	755·67	754·69	0·98	2·24	2
3	493·32	487·39	5·93	8·16	3
4	309·58	299·77	9·81	12·65	4
95	186·05	173·73	12·32	15·31	95

Table 4—*continued.*

Error in q_x caused by the Graduation of l_x or d_x.

Age	Error in q_x		Age
	Actual	Estimated	
20	...	·000000	20
30	...	·000000	30
40	...	·000000	40
50	...	·000001	50
60	...	·000011	60
70	...	·000023	70
80	·000335	·000342	80
1	·000422	·000455	1
2	·000529	·000626	2
3	·000632	·000730	3
4	·000727	·000839	4
85	·000780	·000902	85
6	·000772	·000901	6
7	·000635	·000732	7
8	·000294	·000303	8
9	·000390	·000580	9
90	·001596	·002069	90
1	·003646	·004723	1
2	·007010	·008898	2
3	·012500	·015514	3
4	·021441	·025972	4

N.B.—Positive quantities in Old Style Type.
Negative quantities in Modern Type.

Appendix Table 5.
Woolhouse's Formula.
Error caused by the Graduation of colog p_x.

Age	Original colog p_x	Graduated colog p_x	Error in colog p_x Actual	Error in colog p_x Estimated	Age
20	·002842	...	...	·000000	20
30	·003255	...	...	·000000	30
40	·004270	...	...	·000001	40
50	·006761	...	...	·000001	50
60	·012876	...	...	·000004	60
70	·027885	...	...	·000009	70
80	·064731	·064708	·000023	·000022	80
1	·070571	·070548	·000023	·000024	1
2	·076963	·076936	·000027	·000026	2
3	·083953	·083924	·000029	·000029	3
4	·091601	·091569	·000032	·000031	4
85	·099966	·099932	·000034	·000034	85
6	·109120	·109081	·000039	·000037	6
7	·119131	·119090	·000041	·000041	7
8	·130085	·130039	·000046	·000045	8
9	·142066	·142017	·000049	·000049	9
90	·155175	·155120	·000055	·000054	90
1	·169513	·169454	·000059	·000059	1
2	·185200	·185135	·000065	·000064	2
3	·202360	·202289	·000071	·000070	3
4	·221134	·221055	·000079	·000077	4
95	·241669	·241584	·000085	·000084	95

Table 5—*continued.*
Error in q_x caused by the Graduation of colog p_x.

Age	Error in q_x Actual	Error in q_x Estimated	Age
20	...	·000000	20
30	...	·000001	30
40	...	·000001	40
50	...	·000003	50
60	...	·000008	60
70	...	·000019	70
80	·000044	·000044	80
1	·000045	·000047	1
2	·000052	·000050	2
3	·000055	·000055	3
4	·000059	·000057	4
85	·000063	·000063	85
6	·000069	·000066	6
7	·000072	·000072	7
8	·000078	·000077	8
9	·000081	·000081	9
90	·000089	·000087	90
1	·000092	·000092	1
2	·000098	·000096	2
3	·000103	·000101	3
4	·000110	·000107	4
95	·000112	·000111	95

N.B.—Positive quantities in Old Style Type.
Negative quantities in Modern Type.

Appendix Table 6.

Woolhouse's Formula.

Error caused by the Graduation of m_x.

Age	Original m_x	Graduated m_x Estimated	Estimated Error In m_x	Estimated Error In q_x	Age
20	·006541	·006541	·000000	·000000	20
30	·007498	·007497	·000001	·000000	30
40	·009828	·009826	·000002	·000001	40
50	·015570	·015567	·000003	·000002	50
60	·029643	·029635	·000008	·000007	60
70	·064186	·064165	·000021	·000020	70
80	·148774	·148724	·000050	·000043	80
1	·162139	·162084	·000055	·000047	1
2	·176751	·176691	·000060	·000051	2
3	·192710	·192645	·000065	·000054	3
4	·210140	·210068	·000072	·000059	4
85	·229170	·229092	·000078	·000063	85
6	·249944	·249858	·000086	·000068	6
7	·272602	·272509	·000093	·000072	7
8	·297312	·297209	·000103	·000078	8
9	·324233	·324121	·000112	·000083	9
90	·353550	·353428	·000122	·000088	90
1	·385436	·385303	·000133	·000094	1
2	·420092	·419946	·000146	·000099	2
3	·457700	·457541	·000159	·000105	3
4	·498457	·498284	·000173	·000111	4
95	·542535	·542347	·000188	·000116	95

Appendix Table 7.

Hardy's Modification of Woolhouse's Formula.

Error caused by the Graduation of q_x.

Age	Estimated Error in q_x: Due to $\frac{d^2}{dx^2}$	Due to $\frac{d^4}{dx^4}$	Total	Age
20	·000000	·000000	·000000	20
30	·000001	·000001	·000000	30
40	·000003	·000001	·000002	40
50	·000006	·000003	·000003	50
60	·000015	·000007	·000008	60
70	·000035	·000012	·000023	70
80	·000071	·000005	·000066	80
1	·000075	·000002	·000073	1
2	·000080	·000001	·000081	2
3	·000084	·000006	·000090	3
4	·000089	·000011	·000100	4
85	·000093	·000018	·000111	85
6	·000097	·000025	·000122	6
7	·000100	·000034	·000134	7
8	·000103	·000044	·000147	8
9	·000106	·000055	·000161	9
90	·000107	·000067	·000174	90
1	·000108	·000080	·000188	1
2	·000107	·000094	·000201	2
3	·000105	·000108	·000213	3
4	·000101	·000122	·000223	4
95	·000095	·000136	·000231	95

N.B.—Positive quantities in Old Style Type.
Negative quantities in Modern Type.

Appendix Table 8.

Hardy's Modification of Woolhouse's Formula.
Error caused by the Graduation of l_x or d_x.

Age	Estimated Error in l_x — Due to $\frac{d^2}{dx^2}$	Due to $\frac{d^4}{dx^4}$	Total	Age
20	0·11	0·18	0·29	20
30	0·65	0·38	1·03	30
40	1·77	0·66	2·43	40
50	3·85	0·51	4·36	50
60	6·34	2·62	3·72	60
70	4·09	10·27	6·18	70
80	10·46	8·81	19·27	80
1	12·00	13·35	25·35	1
2	13·33	17·81	31·14	2
3	14·41	21·83	36·24	3
4	15·15	25·01	40·16	4
85	15·51	26·95	42·46	85
6	15·46	27·32	42·78	6
7	15·01	25·91	40·92	7
8	14·16	22·64	36·80	8
9	12·98	17·69	30·67	9
90	11·52	11·47	22·99	90
1	9·90	4·57	14·47	1
2	8·20	2·28	5·92	2
3	6·54	8·31	1·77	3
4	5·00	12·89	7·89	4
95	3·66	15·60	11·94	95

Appendix Table 9.

Hardy's Modification of Woolhouse's Formula.
Error caused by the Graduation of colog p_x.

Age	Estimated Error in colog p_x — Due to $\frac{d^2}{dx^2}$	Due to $\frac{d^4}{dx^4}$	Total	Age
20	·000000	·000000	·000000	20
30	·000000	·000000	·000000	30
40	·000001	·000001	·000000	40
50	·000003	·000002	·000001	50
60	·000007	·000004	·000003	60
70	·000017	·000009	·000008	70
80	·000042	·000022	·000020	80
1	·000046	·000024	·000022	1
2	·000050	·000027	·000023	2
3	·000055	·000029	·000026	3
4	·000060	·000032	·000028	4
85	·000065	·000035	·000030	85
6	·000072	·000038	·000034	6
7	·000078	·000042	·000036	7
8	·000086	·000046	·000040	8
9	·000094	·000050	·000044	9
90	·000103	·000055	·000048	90
1	·000112	·000060	·000052	1
2	·000123	·000065	·000058	2
3	·000134	·000071	·000063	3
4	·000147	·000078	·000069	4
95	·000161	·000086	·000075	95

N.B.—Positive quantities in Old Style Type.
Negative quantities in Modern Type.

Abstract of the Discussion.

Mr. A. W. WATSON said that the history of the subject dealt with by the paper, as recorded in the *Journal* of the Institute, was exceedingly interesting. Perhaps the classical papers on the subject were those of Dr. Sprague and Mr. Woolhouse, in which those eminent gentlemen differed widely from each other, and, as Mr. King had said, left the question in a very inconclusive condition. Prior to that famous controversy, there were excellent articles by the late Mr. J. A. Higham, in which the field was widened very considerably, and what was known as Mr. Woolhouse's "system" was dethroned from its position of isolation, and shown to be merely one formula of many that were suitable, more or less, for the purpose of graduation. After the lapse of a few years, there was a short note from Mr. G. F. Hardy (*J.I.A.*, xxxii, 371), in which the subject was attacked with much originality. Besides giving one or two new formulas, Mr. Hardy gave, what appeared to be of more importance, some practical though tentative criteria for determining the value of any particular formula. Mr. Levine had preceded Mr. Hardy in the same volume (p. 290) with a short note elucidating the algebraic form of Mr. Higham's and Mr. Woolhouse's expressions, and also in the same volume (p. 378) was a very admirable paper by Mr. Todhunter, in which the algebraical side of the subject was thoroughly investigated. Mr. Todhunter's paper was preceded by a note which, reading it at the present time, almost implied that the article was intended as a sort of obituary notice. But the method did not seem to have expired, and in the Transactions of the Second International Congress of Actuaries (pp. 31–109) there was a remarkable paper by Dr. Karup, in which he suggested that the system of Summation formulas was destined to be of far more utility in the future than any method of analytical graduation. Probably the truth was that an analytical method of graduation, such as Makeham's, would be preferred, when the peculiar qualities of such a method as that were desired to be brought into operation for secondary purposes; but, if nothing were required but a smoothly progressing curve, then one of the methods of summation, "mechanical graduation", as Dr. Karup had happily termed it, would be adopted, since this, if the formula were carefully chosen, would give an admirably flowing curve without laborious effort in its application.

It seemed that the questions of most importance were, first of all, which function was to be graduated, and, secondly, which formula was to be employed in the graduation. With regard to the first question, he agreed entirely with Mr. King that it made no difference in practical working whether one graduated l_x or d_x, but the proof of that was not quite easily seen at the first reading, and he thought it would be of advantage if Mr. King would amplify his references to that point in the final proof of his paper. He was perfectly satisfied that the fact was as Mr. King stated, but he did not think Mr. King had quite correctly used Mr. G. F. Hardy's notation $(2[1]-[2])$; Mr. Hardy's plan was

always symmetrical, and a summation of two successive values would go between the lines, whereas twice the summation of the unit value would be on the line, and a deduction of one from the other would not in these circumstances be in accordance with Mr. Hardy's scheme. The point was a small one, and in no way impugned the validity of Mr. King's argument on the general principle of graduating l_x or d_x. Mr. King showed that, so far as Woolhouse's formula was concerned, the graduation of q_x gave better results than the graduation of l_x at the extreme end of the table, and he was inclined to believe that every summation formula would give better results, when applied to q_x, than when applied to l_x. Dr. Karup had investigated that point, and came to the conclusion that that method was the best which gave the smallest theoretical errors, and like Mr. King he found that q_x gave very much smaller errors than d_x or l_x; it therefore seemed reasonable to assume that q_x should be preferred. There was nothing, so far as he could see, in Mr. Woolhouse's writings to suggest why he was so emphatic in preferring l_x, and the only conclusion he could come to was that his statement appeared to be merely a controversial weapon, and need not be taken very seriously. It should not be overlooked that remarkably good results were given by Mr. King's graduation of colog p_x. Whether the graduation of colog p_x would give universally good results was, of course, a matter for further enquiry; but on the figures given by Mr. King, colog p_x would seem to be a perfectly suitable function for graduation by a mechanical process. Mr. King questioned whether the application of Mr. Spencer's 21-term formula would not give defective results, if applied to the column l_x. That, perhaps, was a matter for further investigation, but he was satisfied that, applied to q_x, the formula gave results that were nearly, if not indeed quite, all that could be desired.

As to the best formula to be used for graduation, he was not sure that he entirely agreed with Mr. King in his method of investigation. Mr. King had taken a curve that was admittedly of the Makeham type, and had ascertained what error was produced in that curve by a re-graduation by Mr. Woolhouse's formula. In the first place, he was not at all sure that at the end of life any of the unadjusted bodies of data that had to be dealt with in recent years conformed very closely to a Makeham curve. In one very large investigation which he had to conduct a short time ago, there was a distinct tendency in the rates of mortality at the highest ages to form a series in arithmetical progression, and, when he turned to the account of the graduation of the $O^{M(5)}$ Table, it seemed to him that there had been an appreciable transference of the deaths from ages 80–84 to ages 85–89 for the purpose of making the facts fit into a Makeham curve. Therefore he was not quite disposed to agree with Mr. King that all known statistics did tend to take the shape of a Makeham curve at the end of life. If, on the contrary, the tendency of the rates of mortality at these extreme ages was to proceed by constant first or second differences, then a summation

formula would give better results than a Makeham graduation, and discrepancies between them could not be ascribed to errors in the former. But, however that might be, the test of a good graduation formula of the summation type was not the amount of the theoretical error which it introduced, but its power of graduating. It was admitted that a certain theoretical error was introduced, but Mr. G. F. Hardy was so little impressed, apparently, with its amount that he purposely increased it by the introduction of an error of one-twelfth of the central second difference, in order to obtain a formula which had greater graduating power. Mr. Hardy demonstrated in his paper that a formula which gave a theoretical error, such as Woolhouse's, was nevertheless very efficient in reducing the probable error in the ungraduated facts, and he also gave a criterion for ascertaining the smoothness of the third differences. He would venture to suggest that both of those criteria should be applied to a summation formula, before judgment on its utility was pronounced. Although Mr. Spencer's 21-term formula was erroneous to a greater extent, as regarded the 4th and 6th differential coefficients, than Woolhouse's or Higham's, yet, judged by the standard of graduating power supplied by Mr. Hardy, it was more powerful than either of them. Mr. King doubted whether the formula gave quite such happy results at the extreme of life, after age 80, as it did in the published results. Reference to a table in which the formula was used throughout showed, however, that at the ages 80–84 the expected deaths by the 21-term formula were 1,511, and the actual were 1,524, whilst at the age group 85–89 the expected were 502, and the actual 511. That was a much closer result than would have been obtained by the use of a Makeham graduation, and brought him back to the suggestion that a series might not invariably follow a Makeham curve, and that, if it did not, the errors in the graduated results might be even less than the very small quantities which Mr. King's enquiry had disclosed.

There was, perhaps, another very easy criterion as to whether a formula of graduation was a good one, which was first given by Mr. J. A. Higham, and had been subsequently referred to by Dr. Karup, namely, the flow of the coefficients of the successive terms entering into the graduating formula. Mr. Woolhouse's curve of coefficients had been described as "high-shouldered." Mr. Higham stated that the reason why his formula gave better results was that the flow of the coefficients was very much more regular than Mr. Woolhouse's. In Mr. Spencer's formula, on p. 68 of Mr. King's paper, the coefficients were 60, 57, 47, 33, 18, 6, – 2, – 5, – 5, – 3, – 1, for u_0, and respective values of γ from 1 to 10, forming a curve of a wavy type. That seemed to satisfy the criterion of waviness upon which Mr. Higham insisted, and seemed to show that the formula, other things being equal, was likely to give satisfactory results. In conclusion, he wished to express his satisfaction, personally, with the results of Mr. King's investigation. He felt a certain sense of responsibility in having used the summation formulas in an

important investigation, but whatever feeling of hesitation he might have had as to their acceptance by the profession would be dissipated, now that Mr. King had shown, first, that the differences between the errors involved under age 80 were so small as not to be worthy of notice, and secondly, when q was the function graduated, that errors up to the extremity of life were practically evanescent.

MR. T. G. ACKLAND said the Institute was once more indebted to Mr. King for a valuable paper, based upon skilful mathematical investigations, and upon his large experience of the subject of graduation. The object of the paper was set out on p. 56—"To ascertain the nature and to measure the magnitude of the error which these summation formulas unquestionably do introduce, and so to gauge its importance", and "also to enquire as to the functions to which the formulas should be applied in order to keep the error at its minimum." Necessarily, in a scientific investigation of the error introduced by summation formulas, Mr. King had had to assume a law of mortality, because it must be evident that the ungraduated facts of a mortality table do not in themselves actually follow any ascertained law. Quite properly, Mr. King had taken the Makeham curve, and the $O^{M(5)}$ Table, although it might be remarked that that table, from special circumstances indicated by the author of the graduation, Mr. G. F. Hardy, did not run quite so closely to the ungraduated facts as it might have done, if those circumstances had not existed. But, having adopted that Makeham curve as a basis, it seemed to him that it was no longer possible to consider the paper, valuable though it was, as an investigation of the error introduced by a summation formula of graduation, as compared with the ungraduated facts. The paper was really, as he thought, an investigation of the error arising between a Makeham graduation and a Woolhouse graduation, and gave no indication as to the departure of the latter from the unadjusted data. He could not help thinking that the qualifications he had suggested must be fully in Mr. King's mind. He said, for instance, that the error which he was investigating was negligible up to about age 80, and that only from 80 to 95 did it become of sufficient importance for him to set out, in respect of different functions, the actual magnitude of the error, based on mathematical formulas. But, comparing the Makeham graduation of the $O^{M(5)}$ Table with the actual ungraduated data, it would be found that in many points the error as between the Makeham curve and the ungraduated facts exceeded the value of the error, as deduced by Mr. King's mathematical formula. That could be shewn at once by reference to the table given in the "Account of Principles and Methods" (p. 149) by Mr. G. F. Hardy, in which he showed that at ages 15-19, 40-44, 60-4, 80-4 and 85-9, the error was practically double that which might be considered accidental, or the "probable error", and therefore, that the Makeham curve materially departed from the facts at those particular groups of ages.

But he might illustrate the matter further by considering some of the functions which Mr. King had graduated, and stating the actual

magnitude of the error, as between the ungraduated facts and the Makeham curve. Taking the column of l_x, for instance, at ages 20, 40 and 60 (and making the necessary adjustment as to the different values of the radix used in the ungraduated results and in the $O^{M(5)}$ Table), it would be found that the difference in the value of l in each case exceeded 1,000, on a radix of 107,324 at age 10. Mr. King's figures given in Table 4 represented, quite accurately no doubt, the difference in l_x between the Makeham and Woolhouse graduations, and that difference was expressed (to the same radix) in decimals at ages 20 and 40, and was only equal to 2·57 at age 60. Making similar comparisons with the graduation of q_x (Table 3) and colog p_x (Table 5), similar results were indicated, but not to the same extent; that was to say, the error, which was not dealt with in the paper, between the ungraduated facts and the Makeham graduation, was very much larger than the error which Mr. King had actually investigated. Therefore he suggested that it could not be considered that the actual error was negligible up to age 80; and, as regarded ages 80 to 95, as Mr. Watson had pointed out, the $O^{M(5)}$ Table departed rather from the facts, and after age 85 it would be found that, speaking generally, the graduated mortality throughout was higher than the ungraduated facts. He was not suggesting that Mr. King could have done any more than he had done, or acted differently; but he was trying to warn the younger members of the profession that, if his own view was correct, this was not in any sense an investigation of the actual error involved by a summation formula of graduation, as compared with the actual facts.

Mr. KING said that the error dealt with in his paper was not that "involved", but that "introduced into" mortality tables.

Mr. ACKLAND said that might be a necessary qualification. With regard to the graduation of particular functions, he was afraid his remarks would also apply; he should not admit, on the reasoning given by Mr. King (although it might be perfectly correct on other grounds), that particular functions were more suitable for graduation by Woolhouse's method than other functions, becaused it seemed to him that the author had always proceeded on the lines of comparing his results with the Makeham curve, which was not the curve that truly represented the original facts. For instance, taking the deduction arrived at by Mr. King with regard to colog p_x, which was new and interesting, this might probably be soundly based, but one would not be surprised to find that such a function as colog p_x could be so graduated by Woolhouse's method as to come very close indeed to the value by Makeham's curve, because colog p_x was in the simple form $\alpha+\beta c^x$. In graduating a function of that form by Woolhouse's method, evidently the constant α would remain unaltered in the result. The constant β was a common factor of the second term, and would also emerge unaltered, so that one was only dealing with the geometrical progression c^x, afterwards multiplying by the factor β. If they were dealing with a geometrical progression in general, he had no doubt that Woolhouse's graduation

might very materially disturb the results,* but when dealing (as in the present case) with a geometrical progression, where the ratio, the value of c, did not materially differ from unity—being equal, in the $O^{M(5)}$ Table, to 1·0939564—it would be found that the application of Woolhouse's method did not materially disturb the values of the series graduated. He had tested this by a simple experiment, and found that, taking the necessary series of 15 consecutive values for Woolhouse's formula, and dealing with the function c^x, $\log_{10}c$ being $=\cdot039$, a central graduated value was arrived at of $124\cdot955c^x$ instead of $125c^x$, the difference being $\cdot045c^x$, which, divided by 125, gave an error of $\cdot00036c^x$. If that result was taken out for values of x from 80 to 95, and multiplied by the factor β, there would be obtained the precise values of the "estimated errors" indicated in column (5) of Mr. King's Table 5, in respect of the graduation of colog p_x. It of course followed that the estimated error deduced by Mr. King in respect of colog p_x was a quantity in geometrical progression in Tables accurately following Makeham's law, since $\dfrac{d^4 \operatorname{colog} p_x}{dx^4}=\beta c^x(\lambda c)^4$, with a common ratio of $c=1\cdot0939564$. He thought therefore that there was a special reason, evident on consideration of the above facts, why colog p_x was susceptible of graduation by Woolhouse's method without at all materially disturbing the values by the Makeham curve. Similar remarks would apply to the functions μ_x and m_x.

He thought, however, that the paper would be eminently suggestive, in that it might be possible to develop Mr. King's idea (on p. 60) by applying Makeham's curve in *sections* throughout the mortality table. Dealing, for instance, with ages 80 to 95, there were 16 consecutive values, and if the ungraduated values of colog p_x were taken for these ages, and summed in four sets of four, it would be possible, by well-known methods, to deduce values of a, β, and c which would reproduce accurately the sums of such values, and which would bring out a curve, with values of the constants, all materially differing from those employed in the whole table, but which would faithfully and very closely represent the unadjusted values from age 80 to age 95. Taking the appropriate differential coefficients of the values of colog p_x, so graduated, he thought that there would be a very close indication of the true error involved by a summation formula in the ungraduated facts. He had no doubt that this line of procedure would have occurred to others, and was probably in Mr. King's mind, and it might be a suggestion worth following up.

Mr. Watson had referred to Dr. Karup's method, published in the singularly interesting paper in the "Transactions of the Second International Actuarial Congress." That paper was translated into English on pp. 78-109; and he might here interpolate the wish that members could have the benefit of full translations of the whole of the Congress papers, or, at least, the more valuable of them, instead of brief and insufficient abstracts. Dr. Karup

* *See* Dr. Sprague's investigation of this point (*J.I.A.*, xxvi, 109).—[ED. *J.I.A.*]

proceeded in his paper on lines similar to those adopted by Mr. King, but he did not get out actual numerical values of the differential coefficients, but dealt only with the relations of the fourth to the sixth, and so on, as illustrating whether one formula for graduation was better or worse than another. With regard to the mathematical work in Mr. King's paper, it was of great interest to all the members to have the differential coefficients of the different functions thus carefully worked out. Personally he should like to have seen them set out in the more familiar form of the constants a, β, and c, because one was rather confused by seeing such results as, for example, $\log p_x - \log s$, which, in the other notation resolved itself simply into βc^x; and similarly in other cases. Mr. King had, in this respect, followed the plan of the *Text-Book*, and, no doubt, from his large experience with students, had found that this was the best way to set out the formulas and results, but those who were more accustomed to deal with the a and β form found it a little troublesome to familiarize themselves with the other method.

There was just one personal matter to which he would refer in conclusion. On p. 59 of his paper, Mr. King was good enough to mention his name, and, to give him credit for that which was not really due to him. In the twenty-third volume of the *Journal* (p. 352 *et seq.*) the speaker had shown that Mr. Woolhouse's summation formula could be arrived at by a columnar method, but it was not for the first time that that was demonstrated. Mr. G. F. Hardy, in the remarks he made on a paper read in April 1882 by Mr. J. A. Higham, was the first to indicate a formula, which, although a little different in form from his own, practically led to the same result, in the way of deducing Woolhouse's formula by a columnar method. His own formula and method were independently arrived at, but appeared a little later in the *Insurance Record*, and were ultimately published, at the request of the then Editor, Dr. Sprague, in the *Journal* of the Institute, where his contribution happened to appear on the same page as the report of the method deduced by Mr. Hardy, to whom was, however, due the credit of first discovery and publication. Mr. J. A. Higham followed the matter up, and deduced a third and perhaps simpler formula for columnar summation, and the whole case was fully and accurately stated by Mr. Todhunter in volume xxxii of the *Journal* (p. 384). In conclusion he thanked Mr. King for an eminently suggestive paper, and one that would, he thought, probably lead to further investigations and interesting developments.

Mr. G. J. LIDSTONE thought that one of the uppermost feelings in the minds of the members must be one of regret that Mr. G. F. Hardy was not present that evening—not only because he was a past master in the art of graduation, but also because he had such a singular power of luminous exposition, which, in view of some differing opinions, would be of the highest value that evening. The mention of Mr. G. F. Hardy's name brought him to two preliminary points. First of all, it appeared to him, with great submission,

that Mr. King made too much distinction between Mr. Hardy's plan and Mr. Higham's. So far as he understood the subject, they were practically alternative representations of the same method. He thought that all Mr. Higham's results could be obtained by Mr. Hardy's plan, and certainly Mr. King's equation 7 (page 58), which he understood was really the expression of Mr. Higham's formula, could be obtained much more simply from the first and second equations given in Mr. Hardy's short paper.

[Mr. Hardy's equation (1) contains an awkward misprint, as $\frac{24}{n^2-1}$ should read $\frac{n^2-1}{24}$. His equation (2) follows at once, and in Mr. King's notation it may be written—

$$S = P\left[1 + \frac{s_2 - t}{24}b\right]u_{\frac{n}{2}}$$

or

$$\frac{S}{P} = \left[1 + \frac{s_2 - t}{24}b\right]u_{\frac{n}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\alpha)$$

remembering that Mr. Hardy's u_0 (central term) is Mr. King's $u_{\frac{n}{2}}$. Similarly,

$$\frac{\Sigma}{\Pi} = \left[1 + \frac{\sigma_2 - \tau}{24}b\right]u_{\frac{n}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\beta)$$

Eliminate b, by multiplying (α) by $\sigma_2 - t$ and (β) by $s_2 - t$ and taking the difference of the results, and there follows at once

$$u_{\frac{n}{2}} = \frac{(\sigma_2 - \tau)\frac{S}{P} - (s_2 - t)\frac{\Sigma}{\Pi}}{(\sigma_2 - \tau) - (s_2 - t)}$$

which is Mr. King's equation (7) in a slightly more condensed form.]*

Another point was that he desired to ask Mr. King whether he could reconsider the title "Mr. G. F. Hardy's modified formulas" as applied to the one which was dealt with in the paper. Of course, Mr. King used it only as an illustration, but incidentally it might be thought that the formula was put forward by Mr. Hardy as one which he would himself use. That, he thought, was not the case. Mr. Hardy's Friendly Society formula was developed from Woolhouse's by two successive stages; first of all from Woolhouse's was obtained Higham's formula, as Mr. Hardy pointed out, by substituting the sum of three successive central differences $(b_{-1} + b_0 + b_1)$ for three times the central second difference $(3b_0)$; and thereby a great improvement in smoothness was obtained by Mr. Higham. Mr. Hardy went further, and developed and smoothed the formula by substituting three successive summations [4] [5] [6], instead of summing three times in fives [5] [5] [5]. There was therefore a double improvement, one adopted from Mr. Higham, and another introduced by Mr. Hardy himself; with the result that his Friendly Society formula was certainly very

* The paragraph given above within square brackets is inserted as an Addendum, at Mr. Lidstone's request.—[ED. *J.I.A.*]

much smoother than the one to which Mr. King had attached Mr. Hardy's name.

MR. KING said that he had distinctly disclaimed that at the foot of page 70 of his paper.

MR. LIDSTONE quite understood that Mr. King had used it only for the purpose of illustration, but his suggestion was that it might be thought, by those who did not read the paper with extreme care, that the formula was one belonging to Mr. Hardy, which it certainly was not.

On the main subject of the paper, he found himself in agreement with Mr. King, and somewhat in disagreement with Mr. Ackland. It appeared to him that Mr. Ackland had based his remarks, and in particular his numerical application, on a Makeham graduation which applied without any change of constants from age 20 to the limit of life, whereas the whole spirit of Mr. King's investigation was that, if a mortality table did not follow Makeham throughout, it would follow it with sufficient accuracy for successive sections of fifteen years, and that Mr. Ackland demonstrated himself at the end of his own speech. He (the speaker) agreed that Mr. King's investigations had shown that in applying the summation methods it would be better to use q_x, or colog p_x, rather than l_x, if it was desired to obtain extreme accuracy from the theoretical point of view. At the same time, the general impression left on his mind was that it really did not matter very much which was used, even l or d, because, as Dr. Sprague himself pointed out, the theoretical error only became at all sensible at the extreme limit of life, where the data must be necessarily rough and unreliable, and in almost any event the graduation would require a little adjustment.

If it were thought worth while to eliminate or further reduce the small theoretical error, there was another point of view from which the subject might be attacked. The theoretical errors arose because of the very rapid change of curvature in the curve. Those rapid changes were to a very large extent common to all mortality tables, and if, therefore, instead of graduating the original observed function, say colog p_x, the difference between the observed function and the value according to some standard table was first taken, the greater part of that disturbing curvature would be eliminated, and the resulting function left for graduation would have two advantages: first of all, it would be much smaller in itself than the original function, and, much more important, it would be a very much flatter curve. Therefore for a double reason it would lead to very much smaller theoretical errors when one of the summation formulas was applied, and this without bringing in more distant terms of the original series, which is the effect of a double graduation or subsequent correction, such as has sometimes been suggested. Therefore it would be possible to use a very much more powerful graduation formula, one which might introduce a sensible error into the value of colog p_x, but which would not introduce an error of any importance into the difference between the observed

value and the standard value. That was a method that might be called graduation by reference to a standard table, and he once gave a small example of it in the *Journal* (vol. xxx, p. 212), except that the difference, instead of being graduated by a summation formula, was graduated graphically. If Mr. King would set out the difference, say, between the O^M unadjusted q_x, or colog p_x, and the corresponding values by any other standard table which was really well graduated, it would be seen at once how much smaller the differences were, and how much flatter the curve.

He entirely agreed with Mr. A. W. Watson, when he said that Mr. King had set out to deal with only one side of the subject, and had dealt with it very completely; but that the other side of the question, namely, what formula would do most smoothing of the roughnesses, was of much more importance. For instance, Mr. Todhunter in his note, by a *tour de force* produced a little graduation formula which had the merit, from Mr. King's point of view, of being correct to fifth differences, but he thought Mr. Todhunter would probably be the first to say that he would not advocate its adoption, because it would not have a very great smoothing power.

In dealing with the end of the table, Mr. King had made an interesting suggestion that, at the point where the main formula failed, the curve should be continued by one of Higham's shorter formulas; but he pointed out that there might be some slight disjunction; that is, the final curve might not run smoothly into the old curve. In that connection, he should like to ask Mr. King to consider whether that smoothness of junction might be obtained automatically, for all practical purposes, by using the last *graduated* values, found by the main formula, instead of the ungraduated values, in applying the short formula. It seemed to him that would probably have the effect of making one curve run gently into the other, without the necessity for subsequent adjustment. Another way in which the ends of the table might be conveniently dealt with, was by the assumption that at the last ten years at least Makeham's law would apply. Probably everyone would agree that one should not attach too much importance to the actual facts at the end of life, and that Makeham's law would be quite adequate, and probably even superior. By that law there were three constants, namely, A, B and c, and therefore three relations or points of agreement to dispose of. If one of those relations were used, by making the graduated term, at the junction of the two curves, to agree by the summation method and by the Makeham method; and if another relation were used, by making the differential coefficient or the flow of the curve also equal at that point, there was one relation remaining to dispose of, and if that were used by making the total deaths reproduce themselves for the short term that was being dealt with, there would be obtained a graduation which would be as satisfactory as anything that could be done by the summation method, and at the same time, from the way in which it had been formed, it would necessarily fit on quite smoothly to the end of the summation graduation.

At the end of his paper (p. 72), Mr. King arrived at the conclusion that there were practically only two methods available, unless the experience closely followed Makeham's law throughout its course. He should like to plead for an open mind on the subject. In the first place there was Mr. G. F. Hardy's masterly graduation of a table which did not follow Makeham's law, by means of a supplementary continuous mathematical curve grafted on to a base line which did follow Makeham's law. It seemed to him that in that case very satisfactory results had been produced, and the method was one which must be borne in mind. Then there was another method, which Mr. Elderton introduced in his paper on "Temporary Assurances" (*J.I.A.*, xxxvii, 501). He thought he was not making any breach of confidence when he said that in Mr. Elderton's treatise on "Frequency Curves", now about to be published, there would be found an absolutely satisfactory graduation of the $O^{NM(5)}$ Table produced by a method which was not included in the two Mr. King had mentioned. In conclusion, although he had ventured to comment on one or two points in the paper, he thought everyone would be very grateful to Mr. King for one of his usually thorough investigations, and one which, to his mind, was satisfactory, notwithstanding Mr. Ackland's criticisms.

Mr. H. P. CALDERON said that it appeared to him that there was no reason why it should be assumed that the summation was equivalent to the central term, since the formula would not be absolutely correct. Mr. King having attained graduated values which were equal to $u_x - 5{\cdot}4\Delta^4 u_x$, the graduated result can be equated to that quantity. With regard to the point with which Mr. Lidstone had dealt, he had to make an alternative suggestion as to the series left at the end of the graduation formula. It would be perfectly possible to adopt, say, the first, third, and fifth terms from the end of the graduated table; and also as a second group the second, fourth, and sixth terms, and to construct two curves consisting of the alternate terms of the residual series, taking the mean of these curves to form the rest of the curve required, the fourth differences in each case being left to be found from the terms to be so adjusted.

Mr. F. B. GALER said he had been asked by Mr. Crisford to mention that, in his recent investigation into the mortality experience of the Rock Life Assurance Company, the graduation had been made in two ways—first, l_x, by Woolhouse, and, secondly, q_x, by "central ages after grouping in groups of 5 and taking mean rate and mean age of final groups." They certainly obtained a result more approximate to the actual mortality by the latter method than by the former, but whether that was an accident or not he could not say.

Mr. R. TODHUNTER said that the debate had been maintained at a very high level, and that was the best evidence of the great interest which the paper had aroused. In considering the question of graduation by summation formulas, he would suggest that a distinction should be drawn between summation formulas, considered as independent methods of graduation, and deduced

merely from an algebraical expression, and those formulas which had been devised as a convenient means of applying processes based upon some method of interpolation. Of the latter class of formulas there were two well-known examples, Woolhouse's and Karup's, and a third example in a work recently published by Herr Altenburger.* The formulas which depended upon a method of interpolation seemed to stand on a somewhat different footing from the various formulas "more or less suitable for graduation" which Mr. G. F. Hardy had shown them how to produce wholesale. Mr. King had essayed the difficult task of defending summation formulas as a whole, and had succeeded—so far as he had gone—in his object. He had shown that the graduation of the smooth series by any of the summation formulas which were commonly employed did not produce any material deformation of the curve, and also that the functions q_x and colog p_x were least unfavourably affected by the process. But he felt sure that Mr. King would be the first to admit that that did not carry them very far. It was quite easy, by some such process as that which Mr. J. A. Higham explained in one of his later papers, a process of adjusting the differences between the unadjusted and graduated results, to reproduce a series correctly to the seventh differences. No doubt a formula could be devised correct to tenth differences, if it were worth anyone's while to do it. But, as Mr. Watson and Mr. Lidstone had pointed out, it was the graduating power of the formula that had to be considered, and not its effect upon a smooth curve. That point Mr. King had avowedly not dealt with.

It seemed to be open to question whether, at the present stage of their knowledge, there was really sufficient justification on the ground of theory alone for employing a long range summation formula. The only generally accepted criterion of the graduating power of a summation formula appeared to be the "flow of the coefficients." It was true that Mr. G. F. Hardy, in his note in volume xxxii of the *Journal*, had made some investigations as to the effect of such formulas upon the probable error, but it was not clear that Mr. Hardy had committed himself to the proposition that the probable error test was strictly applicable. His conclusions were based on the assumption that the various quantities included in the range of the formula were affected by similar probable errors. As a matter of fact, that was not the distribution of the errors—and it was not even to be justified by the similar weights of the observations. If one took a long-term formula, a 21-term formula, embracing say ages 20 to 40 of the unadjusted data, the weights of the observations at the two ends of that curve were so dissimilar that the assumption that each quantity was affected by a similar probable

* $[\frac{1}{16}(10S_2 - S_4)]^{2n}$. The single operation $\frac{1}{16}(10S_2 - S_4)$ gives the value of $u_{\frac{1}{2}}$ obtained by drawing a curve of the third order through u_{-1}, u_0, u_1, and u_2. The formula will be found in a paper on Graduation contributed by Herr Altenburger in 1905 to the Proceedings of the Austro-Hungarian Insurance Offices' Association (noted under "Additions to the Library" on page 414, of vol. xxxix of the *Journal*).—[ED. *J.I.A.*]

error seemed to be inadmissible. He was rather inclined to accept the summation methods for the purpose for which Mr. King recommended them, namely, for preliminary graduation, but suggested that in that case the best thing was to employ a short range formula, including a comparatively small number of terms and calculated to deal merely with the ripples in the unadjusted data, as Mr. Hardy called them, and to leave the waves alone. The function e_x seemed to be one that might lend itself to graduation by the summation formula method. In the calculation of the curtate expectations, a good deal of graduation was implicitly done. In the case, for example, of the unadjusted O^M experience, it would be found that the expectations ran with comparative regularity, and that their differences could almost be adjusted by inspection.

THE PRESIDENT said that the paper exhibited that depth of knowledge shown in all the papers which Mr. King had contributed. He did not himself propose to offer any criticisms, because the few notes he had made had been already dealt with by those who had spoken. There was, however, one small point in which he ventured slightly to disagree with Mr. King. Mr. King said: "In fact, the summation formulas faithfully analyze the curve, and infallibly lay hold of its true law, even if they do not remove all irregularities." He was prepared to be convinced by Mr. King on that point, but his experience was that that was not rigidly correct, and that the application of summation formulas did not always bring the law of the series to light. If he was wrong, he was sure the members would be glad if Mr. King would elaborate the point a little further.

THE PRESIDENT then read the following communication which had been received from Dr. T. B. Sprague, and remarked that he was glad to see that this veteran actuary, who had now retired from work, was still active in mind, and ready to take up the cudgels on his old subject of graduation.

DR. SPRAGUE wrote as follows:

I am glad to see that Mr. King has taken up the subject of graduation, and has studied so thoroughly the conflicting views of Mr. Woolhouse and myself. As I have now for several years laid aside all actuarial work, I cannot undertake to follow Mr. King over all the new ground he has traversed in the course of his long paper, but must practically confine myself to an examination of his criticisms on my papers.

The first point that struck me on reading his paper, was that he seems to have greatly exaggerated the extent of my "partial change of views." It will be seen on reference to my papers (*J.I.A.*, xxvi, 77; xxix, 59, 236) that I objected to Mr. Woolhouse's formula of graduation, on two grounds, which it has in common with all other summation formulas; that is to say, (1) because it does not remove the irregularities of the original series of facts, but only reduces them; and (2) because it distorts the law of the facts to a greater or less extent. Mr. Woolhouse admitted the former of these objections to be well founded as a matter of fact, but put it aside because, in his view, smoothness of graduation was quite a secondary

consideration; whereas I hold it to be the most important point. There remained, therefore, nothing more to be said about this objection, and the discussion was limited to the second objection. Mr. Woolhouse denied that his method had any tendency to distort the law of the facts, as I had alleged; and it was for me to prove that such a tendency existed. For this purpose, I made a graduation, by his method, of a table that was already perfectly graduated and free from all irregularities; and in order that he might not object that his method had not had a fair trial, I was careful to follow the precise course recommended by him; and, therefore, took l_x as the function to be graduated, and not q_x, as some other writers had done. The figures that I thus obtained showed that the law of the series had been distorted, as I had predicted; but, the distortion was so small, as to be of no practical importance except at extreme old ages; and I therefore stated plainly that my second objection to his method might be disregarded in practice. This, however, left objection No. 1, which was by far the more important, untouched; so that my "change of views" was much less than would be inferred by anybody who simply read Mr. King's paper.

Such were the "circumstances" of my partial change of views; and I learn, with much surprise, that Mr. King and other persons have inferred from them, that I was of opinion that Mr. Woolhouse's formula gives better results when applied to the l_x column, than when applied to q_x. This was strongly asserted by Mr. Woolhouse, but, like many other assertions of his, was not accepted by me. I thought it better, however, not to call it in question, nor even to express my dissent from it; because that might have led to an increased bitterness of controversy, which it was clearly desirable to avoid. I have never considered the question whether it is better to graduate l_x or q_x; and my reason for taking the former was, as stated above, because Mr. Woolhouse held that his formula was not applicable to q_x; and, therefore, if I had graduated q_x, it would have been open to him to say that I had not given his method a fair trial.

Mr. King says with regard to the figures given by me (xxvi, pp. 82 and 87) that the graduations by the summation formulas used by Mr. Higham, bring out, not only the minimum mortality between ages 30 and 35, but also a second well-marked minimum between ages 45 and 50, which is not disclosed by any of my groupings. I cannot allow this to pass without notice; but all that can be said is that, in my opinion, the latter minimum is not well marked, but is one of the minor irregularities which it is the business of a good graduation to eliminate.

The only other point I have to remark upon is Mr. King's statement that it comes to exactly the same thing whether we graduate l_x or d_x. This is not generally true, for when we graduate d_x, this does not at once give us the values of l_x that are required to get a mortality table, and it depends on what value of l we take as our starting point, whether we shall get the same identical results as we get by graduating l_x. For instance, if the series of l_x proceeds

by constant fourth differences, and we graduate it by means of Mr. Woolhouse's formula, there is a constant error of $-5\cdot4\Delta^4l_x$. Next, graduating in the same way the values of d_x, we leave them unaltered, because they proceed by third differences; and if we then take one of the ungraduated values of l_x as our starting point, we shall reproduce the original values of the l_x series. But the series obtained by graduating l_x is, as we have seen, different from this, inasmuch as it contains the constant error above mentioned.

A hearty vote of thanks was accorded to Mr. King for his paper.

MR. KING, in reply, said that at such a late hour he could scarcely take up all the points raised, nor did he think it was necessary. Mr. Watson had criticized his remarks with regard to the operation of taking $2[1]-[2]$ in order to get d_x, and complained that that brought the result between the lines. That was what it did, because d_x was not central, but was between the lines, and therefore the little demonstration in the paper was perfectly accurate. It was only one case of a much more general theorem, the remembrance of which might sometimes save a good deal of trouble. It bore upon the question of interpolation. If there be two series, u_0, u_t, u_{2t}, &c., and w_0, w_t, w_{2t}, &c., and if the differences, u_0-w_0, u_t-w_t, &c., be taken, and the intervals of the resulting series be subdivided by interpolation, the result produced is the same as if each of the two original series had been first separately interpolated, and the differences between the interpolated values had then been taken. That led him to the remark of Dr. Sprague on the question of graduating l_x or d_x. All that he said in the paper was that, whether l or d was graduated, the same d was obtained. It was another question altogether how to start with the d's to reconstruct the table. Of course if the start was not with a graduated l, but if some other value were taken as the starting point for the summation, a different table was obtained, but that was apart from the question he had discussed.

With regard to other matters, he thought he should have had a good deal to say in reply to Mr. Ackland, but he had replied to himself at the end of his own speech. The paper did not discuss the question of graduation generally, nor what was the best course to pursue; nor did it discuss the divergence between the graduated series and the ungraduated rough series. He had taken up for a very good reason only one point, what was the error introduced into a perfectly smooth table by summation formulas of graduation? The question had been left unsatisfactory, because Dr. Sprague did not entirely withdraw his charge that all such formulas should be included in one condemnation. He might have referred more to Mr. Todhunter's able paper on the subject of these graduation formulas, but it did not cover the ground of his own paper, and did not help in the solution of the problem. But, in an introductory note to that paper, Mr. Todhunter referred to the objections to summation formulas, and said that therefore it was necessary to resort to the graphic process or to a

Makeham formula. His own object was to see whether that was so, and he thought he had shown that they might absolutely ignore the theoretical errors, and go forward and discuss the larger question as to which formulas might properly be used, which were the best to use, and, generally, what was the best method of graduation.

If, however, anyone so desired, it was easy to eliminate practically all the error. When a table had been graduated by one of the formulas, one had merely to difference centrally four times, and multiply the fourth central difference by the proper factor, which is 5·4 for Woolhouse's formula, and add that to the original result, and the table was obtained graduated correctly to the sixth differential coefficient. But personally he did not think it was worth while doing that, the error being so small that it might be neglected altogether. With the President's remarks he agreed to a certain point, that the graduation formula did not in every case disclose the true law of a curve, and we might take, for instance, the early years of a select table; but for all practical purposes he still thought that his statement that the law was shown was correct, and that therefore it was possible afterwards to apply a graphic method with safety. He was much interested in Mr. Watson's figures regarding the carrying out to the old ages of the graduation of certain tables by Mr. Spencer's twenty-one term formula, but they simply proved what he said, that the formula further on did not give quite such good results. The error shown by Mr. Watson came up to 1 percent, whereas the error in q_x given by Woolhouse's formula was 1 in 3,000. Even according to Mr. Watson's figures, however, the error was not of supreme importance; it was not very large, and at those old ages it was so far discounted that it might be practically neglected. He was obliged to the members and felt grateful to them for the reception they had given to his work.

[Mr. King asks us to supplement his speech in reply by the following remarks.—ED. *J.I.A.*]

It was unnecessary on the part of Mr. Ackland to make the disclaimer at the end of his speech. He has misunderstood the passage on p. 59 of the paper, because Mr. G. F. Hardy's original and elegant method of applying Woolhouse's formula is not "summation" in the sense in which the word is there used. He (Mr. King) had attributed rightly to Mr. Ackland the discovery that the formula is of the "summation" class; and Mr. Ackland had attributed rightly to Mr. Hardy the discovery that the formula can be applied by a "columnar" method; but the two things are different, and the distinction is indicated by Mr. Todhunter, *J.I.A.*, xxxii, 384, where he discusses the respective discoveries of Messrs. Ackland and Hardy.

No doubt the "summation" form could be evolved from Mr. Hardy's "columnar" form, but that was not fully shown by Mr. Hardy when he first promulgated his discovery.

ACTUARIAL NOTES.

I.

A New Demonstration of the Formula for the value of an Apportionable Annuity payable by instalments m times a year. By GEORGE J. LIDSTONE, F.I.A.

THE common formula for the value of an apportionable annuity of 1 payable by $m$thly instalments, namely—

$$\mathring{a}_x^{(m)} = \left(1-\frac{\delta}{2m}\right)\bar{a}_x + \frac{\delta}{12m^2}$$

is usually derived* by a double application of the Euler-Maclaurin (Woolhouse) summation formula. It is proposed to show how it may be obtained, by a simple and instructive process, from a consideration of the way in which $\mathring{a}^{(m)}$ must differ from $\bar{a}$. Since in each case the annuity is payable up to the date of death, the difference is evidently one of interest only.

In each interval of $\frac{1}{m}$th of a year, the payment of $\frac{1}{m}$th by continuous instalments is replaced, in the case of the $m$thly annuity, by one payment of $\frac{1}{m}$th at the end of the interval; hence,

$\bar{a}_x \times \dfrac{a^{(m)}_{\overline{\frac{1}{m}}|}}{\bar{a}_{\overline{\frac{1}{m}}|}}$ or $\bar{a}_x \times \dfrac{\delta}{j_{(m)}}$, *i.e.*, the value of a continuous annuity of $\dfrac{\delta}{j_{(m)}}$, will provide precisely the same payments as $\mathring{a}_x^{(m)}$ *up to the end of the last completed interval.*

In the interval in which death occurs, the continuous annuity of $\dfrac{\delta}{j_{(m)}}$ will provide a total payment of $t \times \dfrac{\delta}{j_{(m)}}$ (where t is the time completed from the beginning of the interval to the date of death), and this payment—being made in momently instalments—will amount at the end of the time t (*i.e.*, at the moment of death) to

$$\bar{s}_{\overline{t}|} \times \frac{\delta}{j_{(m)}} = \frac{e^{t\delta}-1}{\delta} \cdot \frac{\delta}{\delta + \frac{\delta^2}{2m} + \frac{\delta^3}{6m^2} + \ldots}$$

$$= \left(t + \frac{t^2\delta}{2} + \frac{t^3\delta^2}{6} + \ldots\right)\left(1 - \frac{\delta}{2m} + \frac{\delta^2}{12m^2} + 0\cdot\delta^3 + \ldots\right)$$

$$= t - \delta\left(\frac{t}{2m} - \frac{t^2}{2}\right) + \delta^2\left(\frac{t}{12m^2} - \frac{t^2}{4m} + \frac{t^3}{6}\right) + \ldots$$

* See *Text-Book,* Part II, Chaps. ix and xi.

The *m*thly annuity will provide after the same interval of time a payment of t, *i.e.*, an excess payment of

$$\delta\left(\frac{t}{2m}-\frac{t^2}{2}\right)-\delta^2\left(\frac{t}{12m^2}-\frac{t^2}{4m}+\frac{t^3}{6}\right)$$

as compared with the last result. Since t ranges from 0 to $\frac{1}{m}$, the average amount of this quantity (assuming equal distribution of deaths in this small interval) will be

$$\frac{\int_0^{\frac{1}{m}}\left[\delta\left(\frac{t}{2m}-\frac{t^2}{2}\right)-\delta^2\left(\frac{t}{12m^2}-\frac{t^2}{4m}+\frac{t^3}{6}\right)\right]dt}{\frac{1}{m}}$$

$$=\delta\left(\frac{1}{4m^2}-\frac{1}{6m^2}\right)-\delta^2\left(\frac{1}{24m^2}-\frac{1}{12m^2}+\frac{1}{24m^2}\right)=\frac{\delta}{12m^2};$$

and the present value of a payment of this amount receivable at the moment of death is

$$\frac{\delta}{12m^2}\bar{\mathrm{A}}_x=\frac{\delta}{12m^2}(1-\delta\bar{a}_x)$$

This, then, is the amount to be added to $\bar{a}_x\times\frac{\delta}{j_{(m)}}$ to produce $\mathring{a}_x^{(m)}$; that is

$$\mathring{a}_x^{(m)}=\bar{a}_x\times\frac{\delta}{j_{(m)}}+\frac{\delta}{12m^2}(1-\delta\bar{a}_x)$$

$$=\bar{a}_x\left(1-\frac{\delta}{2m}+\frac{\delta^2}{12m^2}+0\cdot\delta^3+\ldots\right)+\frac{\delta}{12m^2}(1-\delta\bar{a}_x)$$

$$=\bar{a}_x\left(1-\frac{\delta}{2m}\right)+\frac{\delta}{12m^2}\cdots$$

Q.E.D.

The assumption of equal distribution of deaths is equivalent to rejecting small quantities of the second and higher orders in evaluating the integrals of $\left(\frac{t}{2m}-\frac{t^2}{2}\right)$ and $\left(\frac{t}{12m^2}-\frac{t^2}{4m}+\frac{t^3}{6}\right)$; but as these integrals are themselves multiplied by δ and δ^2 respectively, *i.e.*, by small quantities of the first and second orders, the rejected terms in the product are of the *third* and higher orders. Similarly in the term $\bar{a}_x\left(1-\frac{\delta}{2m}+\frac{\delta^2}{12m^2}+0\cdot\delta^3+\ldots\right)$, the first rejected

term is of the order $\bar{a}_x \times \delta^4$, *i.e.*, also of the *third* order since $\bar{a}_x$ is, in general, of the same order as $\frac{1}{\delta}$. Thus the final result is correct as far as terms of the second order (as, of course, is well known from the Euler-Maclaurin solution), although the result has been obtained on the assumption of equal distribution of deaths; the reason is that (by a device which is of not infrequent application) the required value is thrown into such a form that the approximation is applied only to a small correctional term and not to the main terms involved in the required value.

Mr. W. Palin Elderton points out that an interesting variant of the above proof can be obtained by integrating to find the average value of the correction, *before* expanding in terms of δ. It is shown above that the excess payment, as at the moment of death, is

$$t - \bar{s}_{\overline{t}|} \times \frac{\delta}{j_{(m)}} = t - \frac{e^{t\delta}-1}{\delta} \cdot \frac{\delta}{j_{(m)}} = t - \frac{e^{t\delta}-1}{j_{(m)}}.$$

The average value is m times the integral of this quantity between the limits 0 and $\frac{1}{m}$, *i.e.*,

$$m\left[\frac{t^2}{2} - \frac{\frac{e^{t\delta}}{\delta} - t}{j_{(m)}}\right]_0^{\frac{1}{m}} = m\left[\frac{1}{2m^2} - \frac{1}{j_{(m)}}\left(\frac{e^{\frac{\delta}{m}}-1}{\delta} - \frac{1}{m}\right)\right]$$

$$= \frac{1}{2m} - \frac{1}{\delta} + \frac{1}{j_{(m)}}, \text{ since } e^{\frac{\delta}{m}} - 1 = \frac{j_{(m)}}{m}.$$

Multiplying this, as before, by $\bar{A}_x$ or $1 - \delta\bar{a}_x$ and adding the product to $\bar{a}_x \times \frac{\delta}{j_{(m)}}$, we have

$$\mathring{a}_x^{(m)} = \bar{a}_x \frac{\delta}{j_{(m)}} + \left(\frac{1}{2m} - \frac{1}{\delta} + \frac{1}{{}_{(m)}}\right)(1 - \delta\bar{a}_x)$$

$$= \bar{a}_x\left(1 - \frac{\delta}{2m}\right) + \left(\frac{1}{2m} - \frac{1}{\delta} + \frac{1}{j_{(m)}}\right).$$

Expanding $\frac{1}{j_{(m)}}$ in terms of δ, we get

$$\frac{1}{\delta} - \frac{1}{2m} + \frac{\delta}{12m^2} + 0 \cdot \delta^2 \ldots$$

so that the expression finally reduces to

$$\bar{a}_x\left(1-\frac{\delta}{2m}\right)+\frac{\delta}{12m^2},$$

as before, and it may be similarly shown that the rejected terms are of the third and higher orders.

II.

On the Determination of the Rate of Interest in Annuities-Certain.

By DR. FRANCESCO P. CANTELLI.

IN the theory of annuities the problem of finding the rate of interest, having given the present value of a specified number of payments of an annuity, is a very familiar one.

Various authors have given solutions, but their formulas have been obtained by developing series which do not, in all cases, prove sufficiently convergent to permit of a small number of terms being taken into account.

M. H. Laurent has criticized Baily's formula on the ground that it is impossible to fix the limit of the error in using the formula, and has proposed* for the determination of i, the formula

$$i=\frac{1}{a}-\frac{1}{a}\left(1+\frac{1}{a}\right)^{-t}-t\left(\frac{1}{a}\right)^2\left(1+\frac{1}{a}\right)^{-2t-1}-\ldots$$

where a is the given value and t the term.

The solution of the problem without placing any restriction on the positive values which a and t (t integer) may have, and so as to avoid development in a series, may, I think, be worthy of note, especially if the method of solution leads easily to the determination of i in cases where this value may prove to be negative.†

The method which I set forth, which depends solely on the solution of an algebraic equation having only three terms,

* Theorie des Operations Financières.

† Among the most recent investigators of the problem under notice, I may quote M. A. Achard, *Bulletin de l'Institut des Actuaires Français* (June, 1902); S. Wallis Newling, *J.I.A.*, vol. xxxvii, p. 437; J. Spencer, *J.I.A.*, vol. xxxviii, p. 280.

permits of the determination of the rate of interest even when the formula of Baily or the development in series cannot be conveniently applied.

Writing x for $1+i$ in the equation

$$a=\frac{1-(1+i)^{-t}}{i},$$

we obtain—

$$ax^{t+1}-(a+1)x^{t}+1=0 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This equation, where $a>0$, presents two variations, hence, by the theorem of Cartesio, the number of its real positive roots may be zero or two; but it is satisfied by $x=1$, and thus it will certainly hâve another positive root α.

Equation (1) may be written

$$\frac{a}{a+1}x+\frac{1}{(a+1)x^{t}}=1 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and, for the positive values of x which satisfy (1),

$$\frac{a}{a+1}x<1 \quad \text{and} \quad \frac{1}{(a+1)x^{t}}<1$$

whence

$$\sqrt[t]{\frac{1}{a+1}}<x<\frac{a+1}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Returning now to (1) and differentiating with respect to x, we have

$$x^{t-1}\{(t+1)ax-t(a+1)\}=0 \quad . \quad . \quad . \quad . \quad (4)$$

an equation which cancels itself for only one real and positive value of x other than zero, namely, for

$$x=\frac{a+1}{a+\frac{a}{t}}.$$

This value is evidently greater than 1 if $t>a$ or less if $t<a$, and therefore by the well-known theorem of Rolle, it must be comprised between the roots 1 and α of (1); thus, in the first case the positive root α will be greater, and in the second case less than unity.

Putting in (2)

$$\frac{a}{a+1}x=\sin^2\theta \text{ and } \frac{1}{(a+1)x^t}=\cos^2\theta \quad . \quad . \quad . \quad (5)$$

we obtain

$$\sin^{2t}\theta\cos^2\theta=\left(\frac{a}{a+1}\right)^t\cdot\frac{1}{a+1}=\lambda$$

hence $$t\log\sin\theta+\log\cos\theta=\tfrac{1}{2}\log\lambda \quad . \quad . \quad . \quad . \quad (6)$$

Equation (6) is satisfied by two values of the angle θ comprised between 0° and 90°, namely, by those which correspond to the values $x=1$ and $x=\alpha$. By means of this, making use of logarithmic tables of the trigonometrical functions of the sine and cosine the angle θ can be easily determined, and thence by means of one of the values in (5) the positive value of x corresponding thereto. It is evident that the exactness of the value obtained for x depends on the approximation afforded by the logarithmic tables used.

It is convenient to determine the limits between which the angle θ is to be sought. In the common case $t>a$ the value of α must be greater than unity, and this will, as already stated, fall between 1 and $\frac{a+1}{a}$.

Indicating by θ_1 and θ_2 respectively, the limits of the angle θ, and making use of the relation

$$2\log\sin\theta=\log\frac{a}{a+1}+\log x \quad . \quad . \quad . \quad . \quad (7)$$

obtained from the first expression in (5), we have

$$\log\sin\theta_1=\frac{1}{2}\log\frac{a}{a+1}, \quad \log\sin\theta_2=0,$$

these limits being excluded.

For example, let $t=30$, $a=19$.

Here α must be greater than 1, and, using Kohler's logarithmic tables to seven places of decimals, we have

$$\log\frac{a}{a+1}=\bar{1}\cdot9777236$$

$$\log\sin\theta_1=\frac{1}{2}\log\frac{a}{a+1}=\bar{1}\cdot9888618\,; \quad \log\sin\theta_2=0$$

$$\theta_1=77°\cdot5' \qquad \theta_2=90°$$

The angle θ corresponding to the value of x which is to be found is thus comprised between $77^\circ{\cdot}5'$ and 90°.

Equation (6) gives

$$30 \log \sin \theta + \log \cos \theta = \bar{1}{\cdot}0153390.$$

From a first examination we find that the angle θ is comprised between 82° and 83°, since

$$30 \log \sin 82^\circ + \log \cos 82^\circ = \bar{1}{\cdot}016$$

$$30 \log \sin 83^\circ + \log \cos 83^\circ = \bar{2}{\cdot}988$$

Further examination gives for

$$\theta = 82^\circ\ 2'\ 10'' \qquad 30 \log \sin \theta + \log \cos \theta = \bar{1}{\cdot}0153392$$

$$\theta = 82^\circ\ 2'\ 11'' \qquad 30 \log \sin \theta + \log \cos \theta = \bar{1}{\cdot}0153331$$

from which we have, lastly, for $\theta = 82^\circ\ 2'\ 10''{\cdot}033$,

$$30 \log \sin \theta + \log \cos \theta = \bar{1}{\cdot}0153390$$

with $\log \sin \theta = \bar{1}{\cdot}9957912$.

From this we obtain

$$2 \log \sin \theta = \bar{1}{\cdot}9915824$$

$$\log \frac{a}{a+1} = \bar{1}{\cdot}9777236$$

$$\text{Difference} = \log(1+i) = {\cdot}0138588$$

and $i = {\cdot}032425$, which is correct in the last place.

In conclusion, it may be mentioned that the analogous problem of finding the rate of interest, having given the amount of an annuity-certain as distinct from its present value, may be dealt with on lines similar to those laid down above.

CORRESPONDENCE.

A FRAGMENT OF HISTORY.

To the Editor of the Journal of the Institute of Actuaries.

MY DEAR SIR,—When recently destroying a mass of papers, I saved from the flames copies (made by me thirty-five years ago) of two private formal "Opinions", by Joshua Milne and Griffith Davies, upon (what they conceived to be) the proper method of distributing profits. You may probably consider that they possess an antiquarian interest sufficient to justify their insertion in the *Journal*, more especially as the products of two of the most original minds in our profession.

I furnish the Opinions in the authors' own words:

I.—*Opinion of* JOSHUA MILNE *expressed in* 1821.

1. From the total value of the property at present held by the company deduct the sum which would be required to discharge its present engagements, including the purchase of all its policies now in force, and call the remainder the *surplus fund.*

2. Call the part of this fund, which is to be divided among the assured, the *gross dividend.*

3. From the amount of the premiums received for each assurance now in force, with its improvement at interest up to this time, deduct the present value of the policies; the remainder will be the contribution from such policies towards the surplus fund as well as to all the outgoings of the company.

A being insured by any one policy, call this remainder in his case *the contribution of A.*

4. Then, as the sum of such contributions derived from all the policies in force is to the gross dividend, so is the contribution of A to his share of that dividend, or the value in present money of the augmentation of his claim which should now be made.

5. From such present value the equivalent augmentation of the claim may be arrived at.

II.—*Opinion of* GRIFFITH DAVIES *expressed in* 1821.

The *only* correct method of dividing profits is to apportion them in such a way to the different policyholders as to make the *present values* of their respective bonuses in the ratio of the profits which have accrued to the office from those persons during the time they have been respectively assured.

Suppose A, aged 20 at the time of effecting his policy, has been assured six years for £1,000 at a premium of £21. 15*s.* 10*d.*, the amount of the six premiums which he will have paid during those six years, improved at 3 per-cent compound interest during that time, will equal £145. 3*s.* 8*d.*, and the value of his policy at the

end of that time will be only £50. 14*s.* 5*d.*: hence, the profit accruing to the office from him if he be then alive must be £94. 9*s.* 3*d.*

Suppose D, aged 35, assured for the same sum and for the same number of years at a premium of £29. 18*s.* 3*d.*, the amount of his contributions will be £199. 5*s.* 6*d.* and the value of his policy £77. 16*s.* 5*d.*: hence, in case he survive, the office must realize by him £121. 9*s.* 1*d.*

I therefore infer that the present values of the bonuses of these two persons must, in equity, be to each other as 94·462 is to 121·454. But in order to have the present values of these bonuses to each other in the above ratio, we must divide the former number by ·457, the present value of £1 at the death of A, who must then be 26 years old, and the latter by ·5451, the present value of £1 at D's death, who will then be 41 years old; the quotients will be $\frac{94\cdot462}{\cdot457}=206\cdot7$, and $\frac{121\cdot454}{\cdot5451}=222\cdot8$.

Let £66. 9*s.* 5*d.* be divided—

206·7
222·8

429·5 : 66·471 :: 206·7 : 31·989 = £31. 19*s.* 9*d.* for A's bonus; and taking this from £66. 9*s.* 5*d.* we get for D's, £34. 9*s.* 8*d.*

We notice, by the way, a characteristic of these two actuaries: the methodical manner of Milne, and the partiality of Davies for expressing his views in figures.

Yours very truly,

T. E. YOUNG.

1 *October* 1906.

THE VALUATION OF LIMITED-PAYMENT POLICIES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In July 1898, Herr Altenburger, in a letter to the *Journal*, described a method of valuing special class policies by which all those on lives of the same age might be grouped together and valued as whole-life policies, a correction being afterwards made, depending on the nature of each particular contract. The method is especially applicable to Limited Payment Policies, as it avoids the separation of the sums assured and premiums into different sections, if the policies are to be valued in groups. The general formula for age attained x is $V = A_x - Pa_x + \frac{P\mathbb{N}_{x+n}}{D_x}$ where P represents the limited payment premium, and $P\mathbb{N}_{x+n}$ is inserted as a constant at the outset in the valuation registers; so that the total of the column divided by D_x gives the value of

the deferred premiums to be deducted from the value of the premiums treated as payable during the whole of life. I have recently had occasion to look into the method as applied to a valuation by the O^{M} and $O^{M(5)}$ Tables, and the results are given below, in case they may be of use to others.

It is desirable that only one set of factors should be used for multiplying the premiums and loadings, in order to avoid the labour of writing off the O^{M} and writing on the $O^{M(5)}$ constants after the policies have been five years in force; and it will be seen that this may be done with results which, for all practical purposes, are exact, as in the trial valuation appended the greatest error in any year does not amount to unity.

The trial valuation was made by the O^{M} Table at 3 per-cent interest and included only policies effected within the five years preceding the date of valuation. The premiums were assumed to be due eight months after the valuation date, so that the values of these were $P_{\frac{2}{3}|}\mathbf{a}_{x\overline{n}|}$, or, under Herr Altenburger's method, $P_{\frac{2}{3}|}\mathbf{a}_{x}-\dfrac{P\mathbb{N}_{x+n+\frac{2}{3}}}{D_x}$. The policies were tabulated according to age nearest birthday at 31 December, and the constants were obtained by multiplying the premiums and loadings by the $O^{M(5)}$ values of $\mathbb{N}_{x+n+\frac{2}{3}}$, $(x+n)$ being the age nearest birthday at the end of the calendar year of last payment.

To deal with the premiums only, the sums of these and of the corresponding constants were obtained at each age, and the sums of the constants were divided by the $O^{M(5)}$ D_x, giving the values of the deferred premiums. Each of these values was in turn divided by the corresponding sum of the premiums, which gave the average deferred annuities ${}_{n+\frac{2}{3}|}\mathbf{a}_x$, and these subtracted from ${}_{\frac{2}{3}|}\mathbf{a}_x$ gave the $O^{M(5)}$ temporary annuities, ${}_{\frac{2}{3}|}\mathbf{a}_{x\overline{n}|}$, by which the premiums might have been multiplied for an $O^{M(5)}$ valuation. The values of n were found by interpolation from a table of the $O^{M(5)}$ values of ${}_{\frac{2}{3}|}\mathbf{a}_{x\overline{n}|}$, and these same values of n were used in obtaining, from a similar table of the O^{M} values of ${}_{\frac{2}{3}|}\mathbf{a}_{x\overline{n}|}$, the annuities by the latter table of mortality by which the premiums were finally multiplied. The two interpolations necessary were taken by first differences to three decimal places. A separate calculation requires to be made for the values of n in the case of the loadings, as the incidence is generally different.

I have taken the case where the constants are calculated by the $O^{M(5)}$ Table, but the contrary will obviously hold, and the method affords a ready means of finding the increased reserve required for an O^{M} and $O^{M(5)}$ valuation, where an office makes an O^{M} valuation, and has the constants already calculated by that table.

Yours faithfully,

ALEX. FRASER.

Scottish Life Assurance Co.,
19, *St. Andrew Square, Edinburgh,*
7 *November* 1906.

Valuation of Premiums.

Age	Premiums	n	Approximate Value	Exact Value	Error
48	383·4	17·744	4519·5	4519·8	−·3
43	238·6	15·334	2658·5	2658·5	0
38	135·4	14·573	1486·0	1486·8	−·8
33	291·5	20·430	4093·2	4093·3	−·1
28	261·2	19·475	3612·7	3613·0	−·3

Valuation of Loadings.

Age	Loadings	n	Approximate Value	Exact Value	Error
48	64·9	17·916	769·4	769·4	0
43	46·5	15·318	517·7	517·7	0
38	26·3	15·328	299·2	299·5	−·3
33	66·9	20·681	946·6	946·5	+·1
28	64·3	19·617	893·6	894·1	−·5

OBITUARY NOTICE.

Dr. Ferdinand Hahn.

THE President (Mr. Frank B. Wyatt), at the commencement of the Sessional Meeting of the Institute of Actuaries, held on 17 December last, referred in the following terms to the death of Dr. Ferdinand Hahn, the President of the Fifth International Congress of Actuaries held in Berlin in September last :

The President said that before proceeding to the business of the evening he had a sad announcement to make. No doubt many of the members had read with regret in the papers of the death of Dr. Hahn, the President of the German Federation for Insurance Science, and of the last International Congress held in Berlin, which was attended by several of the members of the Institute. That Congress was the first opportunity the members had had of making the acquaintance of Dr. Hahn ; they went there as strangers, but when they left at the end of a week they felt no longer strangers. Dr. Ferdinand Hahn was an exceedingly genial man, of distinguished position and of great legal and scientific attainments, and he was greatly respected by

the Society of which he was the head. A letter was being sent to Professor Manes (General Secretary of the Society), and the permission of the members was asked that that letter should convey the sympathy of the members of the Institute with the family of Dr. Hahn in the loss they had sustained, and also sympathy with the German Society at the loss they had incurred by the death of their President.

It was unanimously agreed that a resolution of sympathy should be forwarded.

We are indebted to Dr. A. Manes, the General Secretary of the Congress, for particulars kindly supplied as to the career of the late Dr. Hahn, of which the following is a brief abstract:

Dr. Hahn devoted his youth and early manhood to the study of jurisprudence, and, turning his attention to Insurance matters, he ultimately became General Manager of two important Insurance Companies in Magdeburg, and was also on the Council of several other Insurance Companies and Banks. He was President of Associations of Rail, Transport, and Accident Insurance Companies, in the founding and development of which he took a considerable share. Dr. Hahn was member successively of the Prussian, and of the Imperial, Insurance Councils. The German Federation for Insurance Science was established in 1899, largely on his initiative, with an inclusive scope which covered, not only actuarial theory and practice, but also the legal and medical study and work closely associated therewith. Dr. Hahn likewise took a large and important part in the arrangement and details of the Fifth International Congress for Insurance Science, and of the Fourth International Congress for Insurance Medicine, held simultaneously at Berlin, in September 1906, and the unequalled success of the Congress, and his unanimous election as President, were regarded by him as the zenith of his career. His energy and versatility are sufficiently evidenced by the above enumeration, but in addition he possessed high literary capacity, the best results of which, however, are hidden in reports and memoirs, mostly of a private nature. His success in life was due to a strong personality, characterized by a keen readiness to grasp new ideas; and whilst he was essentially practical, he had yet a deep respect for theory. Dr. Ferdinand Hahn passed away on 12 December 1906, in the 62nd year of his age.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Further Notes on some Legal Aspects of Life Assurance Practice.

By ARTHUR RHYS BARRAND, F.I.A., *of the Prudential Assurance Company.*

[Read before the Institute, 28 January 1907.]

SYNOPSIS.

Introduction.—Increased interest in legal matters on the part of actuaries.—Practical nature of legal questions in Institute examinations.—Need for information not found in ordinary text-books.—Why particulars of decided cases are given.—Reporting assurance cases in *Journal.*—Recent papers dealing with legal subjects.

Nature of the Contract of Life Assurance.—Assurances where a larger amount is payable on maturity than on death.—Are these contracts of life assurance?—Essentials of contract of assurance.—Accident policies with return of premium on death or attainment of fixed age.—Test of a double contract.

Insurable Interest.—Must the interest be a pecuniary one?—Has an employer an insurable interest in the life of his weekly servant?—Policy assigned immediately after issue.—Who can plead absence of insurable interest?—Return of premiums when assurance is void.—Agent as expert in assurance law.

The Proposal.—Agent's knowledge as to misstatements in proposal.—Agent as agent of proposer and not of company.—When the agent is the agent of the company.—Conflicting decisions.

The Completion of the Contract.—Receipt of first premium.—Roberts' case.—Methods of meeting difficulty.—Can the assurance company sue for the first premium?—Withdrawal before completion.—When liability commences.

Conflict of Laws in relation to the Contract.—Where contract is made.—Law governing the contract.—Status or capacity.—Mercantile contracts.—Formalities of contract.—Interpretation of contract.—Proper law of contract.—Jurisdiction of Courts.—Bankruptcy.—Discharge of contract.

The Policy as evidence of the Contract.—Representations not contained in proposal or policy.—When effect may be given to prior representations.—Effect of statements in prospectus.—Action to rescind or rectify the contract.—When effect will be given to a preliminary contract.

The Conditions of the Policy.—Grammatical error in contract.—Suicide clause.—Waiver of misrepresentation.—Acceptance of premiums after notice of breach of condition.—Conditions of policy avoiding this difficulty.

Renewal Premiums.—Death during days of grace and before payment of premium.—Instalment premiums and true fractional premiums.—Acceptance of premium under conditions.—Extension of provisions for remission of income tax.

Assignments and Charges.—Production of deed as notice.—Notice to trustees in reversionary transactions.—Are reassignments necessary?—Statutory mortgages and reconveyances.—Liquidators.—Advances on joint account.—Discharge for policy moneys by equitable mortgagee.—Exercise of power of sale.—Foreclosure.—Policy included in bill of sale.—Assignment after death of assured.—Statutes of Limitation.—Lunacy.—Convicts.

Bankruptcy.—Depositee of policy and trustee in bankruptcy.—Deeds of arrangement.—After-acquired property.—Abandonment.—Scots and Irish bankruptcy law.—Searches in bankruptcy.—Indemnity by trustee in bankruptcy.

Settlement Policies.—Conflict of laws.—Discharge of policy issued under Married Women's Property Act, 1870.—Appointment of new trustee.—Disclaimer of trust.—Can settlement be varied or cancelled?—Restriction on alienation.—Reservation of interest to assured.—Beneficiary not legally married to assured.—Loans to trustees to maintain policy.

Claims.—Payment without grant of representation.—Revenue Act 1889.—Payment direct to beneficiaries under settlement policies.—Specific legacy of policy.—Colonial probates.—Scottish

and Irish grants.—Grant to minor.—Bonuses.—Lord Campbell's Act.

Payment into Court under the Act of 1896.—Paucity of reported cases.—When money can be paid into Court.—Harrison *v.* Alliance Assurance Company.—Liability for costs.—Doubts as to sanity of assignor a sufficient reason.

Friendly Societies and Industrial Assurance Companies.—Limits of Collecting Societies and Industrial Assurance Companies Act 1896.—Nomination not revoked by subsequent will.—Effect of nomination.—Conflict of decisions as to assignment of friendly society policies.

Stamps.—Current accounts.—Covenants in mortgage deed.—Collateral security.—Reconveyance by building, friendly, and industrial and provident societies.—Receipt endorsed on duly stamped deed.—Assignment and reassignment.—Receipt for policy moneys endorsed on policy.—Periodical payments.—Separation deed.—Double statutory declarations.—Equity of redemption.—Maximum collateral duty.—Contingent consideration.—Nominal consideration.—Foreclosure order.

Conclusion.—Reasons for certain omissions.—Life Assurance Companies Acts, 1870 to 1872.—Inadvisability of public discussion.—Other reasons for not discussing them in this paper.

INTRODUCTION.

WHEN, some ten years ago, I contributed a paper to the proceedings of this Institute with a title similar to the present one,* I felt it necessary to make some sort of apology for introducing a subject which seemed to lie somewhat outside the range of those in which an Institute of Actuaries might be considered as interested. Such an apology if appropriate then is, I feel, no longer needed in view of the increased interest in legal matters bearing on life assurance work which has been manifested in recent years by actuaries individually, and also by the Institute of Actuaries and the Faculty of Actuaries, as shown by their proceedings. It is now far more common than it was a few years ago for the actuary of a life assurance company to deal with the many legal matters which arise in connection with the ordinary work of the office, a change which I venture to think is advantageous alike to the company and its clients. Where this course is adopted there is, as a rule, considerable saving of time, since a certain amount of

Increased interest in legal matters on the part of actuaries.

* *J.I.A.*, vol. xxxiii, p. 205.

delay is inevitable when a matter has to be submitted to the company's solicitors. Moreover, the matter will generally be dealt with in a way more conducive to the interests of the company as a whole, when the individual dealing with it is in close touch with all the aspects of the business, and is able to weigh the advantages likely to be gained by a concession, against the risk run by relaxing some of the strict legal requirements. Perhaps one reason for the increasing tendency to entrust the legal questions arising in the everyday work of a life office to the actuary, arises from an increased confidence in the legal attainments of the younger members of this Institute, as shown by the nature of the examinations which they have successfully encountered. Certainly, one cannot read through the examination papers of recent years without feeling admiration, sometimes tinged with awe, for the extensive legal attainments of the candidates who have passed through the ordeal unscathed; and when one realizes that these questions are only on the elements of the law of real and personal property and law of life assurance, one wonders what, in the view of the examiners, questions going beyond the elements would be like.

Objects of Paper.

This, perhaps, is not a suitable occasion for a discussion of the legal questions set in the Institute examinations, but it may not be out of place to call attention to the very practical nature of most of them, as suggesting that the knowledge which is sufficient to answer them satisfactorily is also sufficient to deal with similar questions when they arise in ordinary office practice. It is partly on account of this practical aspect of recent examination questions that I have written this paper, for some of them relate to matters which, though familiar to those who have to deal with them in practice, may present considerable difficulty to those whose only acquaintance with legal matters is derived from the ordinary text-books. I hope to refer to some of these points myself, but, even more, I hope to elicit, in the discussion to follow, a portion of that large store of information on such matters which the senior members of this Institute must have accumulated in the course of their official experience.

The fact that I am writing mainly for the benefit of the students of the Institute will also explain why I have thought it necessary to give particulars of decided cases at some length. There has been, I think, a tendency in recent years, to require from our students some knowledge of recent cases affecting life assurance matters; and while such knowledge on their part is

greatly to be desired, the means of obtaining it have not always been easily accessible. I might, of course, have contented myself with giving the effect of a decision in a few words; but had I done so, the cases would have lost much of their educational value, and moreover, since so much depends on the actual circumstances of each case, I might mislead rather than help by giving merely the effect of a decision without the facts on which it was based; and might cause students to apply a general principle to a set of facts to which it was altogether inapplicable. If I may be permitted here to make a suggestion to the Council of the Institute, I think it would be helpful to the members if arrangements were made such as, I believe, have been made by the Faculty of Actuaries, for reports of cases more or less closely connected with life assurance matters to appear in the *Journal* from time to time. Such reports would enable students and others to keep themselves informed as to these matters, and would, moreover, remove to a great extent the necessity or excuse for such a paper as this, an argument which I feel sure will appeal strongly to many who are doubtless appalled at its length.

Scope of Paper. It will readily be understood that my remarks must necessarily be somewhat fragmentary and disconnected, for other members of this Institute have dealt in the past with certain legal aspects of life assurance far more ably than I can hope to do. I can only endeavour, therefore, to call attention to new points that have arisen in recent years, to fill up gaps that appear to have been left by my predecessors, and to point out changes that have taken place since they wrote. Such papers as have been contributed to the proceedings of this Institute by Mr. J. E. Faulks, Mr. J. R. Hart, Mr. C. D. Higham, Mr. W. Hughes, Mr. G. King, Mr. W. K. Lemon, Mr. G. J. Lidstone, Mr. B. Newbatt, and Dr. T. B. Sprague, are so well known to all, that I need only refer to their authors by name. They are, or should be, familiar to our students, and are of course absolutely necessary to them for the purposes of their examinations. I may, however, call attention to an admirable paper appearing in the *Transactions of the Faculty of Actuaries*, and therefore, perhaps, not so well known to them. I refer to the recent paper by Dr. A. E. Sprague, entitled "Titles to Life Assurance Policies", read before the Faculty of Actuaries, on 27 February 1905. I shall have occasion to refer to one or two points dealt with in that paper, but in general, where a matter of recent interest has

been discussed by Dr. Sprague, there is nothing more to be said on the subject, and I shall not, therefore, refer to it. Attention should also be called to a most useful paper by Mr. J. L. Wark, read before the Faculty of Actuaries on 4 June 1906, and entitled "Notes on Recent Legal Decisions relating to Life Assurance."

Nature of the Contract of Life Assurance.

Perhaps the first question to be considered, if anything like a logical order is to be followed in what, as I have pointed out, must necessarily be a somewhat disconnected paper, is as to the nature of the contract of life assurance. It may seem somewhat late in the day to start a discussion on such a subject as this, but as a matter of fact, as most of us are aware, this particular point has been the subject of an important decision in the recent case of the Prudential Assurance Company, Limited *v.* The Commissioners of Inland Revenue 1904 2 K.B. 658. The question arose in consequence of difficulties raised by the Inland Revenue authorities as to the stamping of certain forms of endowment assurance policies. In this particular case, the policy provided for the payment of £95 if the assured should attain the age of sixty-five, and in the alternative, for the payment of £30 if he should die before attaining that age. The principle involved was therefore the same as that in a double endowment assurance. The Commissioners took the view that the main part of the contract was the undertaking to pay £95 at age sixty-five, and that the promise to pay the lesser amount on earlier death was merely incidental thereto. They therefore held that it was not a policy of life assurance within the meaning of the definition contained in section 98 of the Stamp Act, 1891, and accordingly assessed the duty at 2*s.* 6*d.*, as being a covenant for securing the payment of money not exceeding £100, instead of 1*s.*, the duty payable as a policy of life assurance. The assurance company appealed against this decision, and Channell, J., in giving judgment in their favour, said: "In order that it may "come within the definition of a policy of life insurance it must, "first of all, be a policy of insurance. And whether it is a "policy of insurance is, I think, substantially the only question "in this case. For if it be a policy of insurance at all, I cannot "help thinking that it is free from doubt that it is a policy upon "an event relating to, or dependent upon, a life. . . . "When you insure a ship or a house, you cannot insure "that the ship shall not be lost, or the house burnt;

Is a double endowment policy a policy of life assurance?

Essentials of the contract of insurance.

"but what you do insure is that a sum of money shall be paid upon "the happening of a certain event. That, I think, is the first "requirement in a contract of insurance. . . . Then the next thing "that is necessary is that the event should be one which involves "some amount of uncertainty. . . . The remaining essential "is that which was referred to by the Attorney-General when he "said the insurance must be against something . . . . that is to "say, the uncertain event which is necessary to make the "contract amount to an insurance, must be an event which is, "*primâ facie,* adverse to the interest of the insured. . . . A contract "of insurance, then, must be a contract for the payment of a "sum of money, or for some corresponding benefit, . . . to become "due on the happening of an event, which event must have some "amount of uncertainty about it, and must be of a character "more or less adverse to the interest of the person effecting the "insurance. Then does the particular contract with which we "have here to deal come within that definition of a contract of "insurance? If it were to be split up and treated as two "separate contracts, I should incline to the view that even the "old age endowment portion of it . . . . would satisfy the "definition. In the first place, the event on which the money "is to be paid is uncertain. . . . Secondly . . . the "event, in addition to being uncertain, is, *primâ facie,* "adverse to the interests of the insured. . . . The "reaching of that age, with its attendant disadvantages is, to my "mind, an event which is sufficiently adverse to the interest of a "poor person to make it a proper subject against which to insure. ". . . . But . . . . when you take the whole contract "together, there does not seem to be any real difficulty in the "matter. Sometimes it is provided that the sum insured shall be "payable, either upon the insured reaching a certain age or upon "death, whichever first happens. It is clear that that would be "a contract of insurance. That is very like this case, the only "difference being that here a larger sum is payable in the "former event, and that is a difference which, in my opinion, is "immaterial. I have come to the conclusion that this contract, "taken as a whole, is clearly a contract of life insurance within "the meaning of the Stamp Act, and that the appeal must "therefore be allowed." I have quoted somewhat fully from the judgment, as it is of interest to note the grounds on which the decision is based. To many, at the time, it seemed that the reasoning was unnecessarily elaborate, and that a contract of the

nature in question came so manifestly within the description of a policy of life assurance as defined by the Stamp Act, as to leave no room for doubt or dispute. This view is, perhaps, due to the long period during which insurance officials have treated such contracts as a matter of course as being policies of life assurance, without any question being raised; and does not make sufficient allowance for the possibility of a different point of view being taken by one who approaches the consideration of such a contract with an open mind, untrammelled by prepossessions induced by constant contact with life assurance practice. I may say that the decision was not appealed against by the Inland Revenue Commissioners, but was apparently accepted by them as settling the matter, as regards all contracts involving the same principle as that on which the assurance in question was based. Quite recently, however, my attention has been called to the fact that practically the same point is being again raised by them, in the shape of an objection to allowing the whole of a premium paid in respect of such an assurance to count as a life assurance premium, when an exemption from income tax is claimed in respect of such payments. This objection on their part appears to be untenable as long as the foregoing decision stands; and it would therefore seem probable, either that their contention is not meant seriously, or that they intend to contest the decision from another point of view. Whatever may be the intention of the Commissioners of Inland Revenue in the matter, the position of the assurance companies seems to be a very strong one, and the result of any further action may be awaited with a considerable amount of confidence. By way of contrast to the above-mentioned case, attention may be called to that of The General Accident Assurance Corporation, Limited *v.* The Commissioners of Inland Revenue 1906 43 S.L.R. 368, where the positions were reversed, and the Inland Revenue authorities contended that a policy which, in the event, was held to be an accident policy, was, in reality, a policy of life assurance. In this case an accident policy contained a provision under which the assurance company undertook to return a portion of the premiums, not exceeding a certain fixed amount, on the assured attaining a specified age or dying earlier, provided no payment had been made under certain clauses relating to payment for accidents involving death or total disablement. The Com-

Are double endowment policies entitled to rebate of Income Tax?

Accident assurance coupled with return of premiums on death or the attainment of a given age.

missioners held that this was not only an accident policy, but also a life insurance policy, and assessed it as attracting both duties; but on appeal it was held that the policy was merely an accident one, and therefore only liable to the accident policy duty of one penny. The Lord President, in delivering judgment, said: "The first thing that I think we have to discover is, what "is the general character of this instrument before us? Now "as to that, I do not think there can be any doubt. There is no "doubt that it is an accident policy. . . . That a company or "individual could contract with another person in one deed, "by which they should effectuate both a policy against accident "and also a policy of life insurance, I do not doubt. But the "point is, has that been done here? Now I cannot think of any "better test—at the same time I think it is a true one—than to "take the second so-called contract and see if it would stand "alone. . . . Testing it by that test, I have no hesitation in "saying that it would not. . . . Accordingly I come to the "conclusion . . . . that this policy is truly an accident policy "and nothing else, and that it is appropriately stamped if it "bears a one penny stamp."

Insurable Interest.

The next question to be considered is that of insurable interest, the existence of which is a condition precedent to the formation of a valid contract of life assurance. On this point, attention may be called to a slight difference between the third and fourth editions of Bunyon's "Law of Life Assurance", especially as many of us have, no doubt, derived most of our knowledge of life assurance law from the third edition, and may not have read the present edition as carefully as we did the preceding one. In the addenda to the third edition it is stated to have been decided, in the case of Barnes *v.* London, Edinburgh and Glasgow Assurance Company, that a woman who maintained, and was morally liable for the expenses of burying her half-sister, had an insurable interest in her life. This statement, which is based on the report of the case in 8 T.L.R. 143, has given rise to the idea that a good insurable interest may exist, even in the absence of an actual pecuniary one. It was however pointed out by Lord Alverstone, in delivering judgment in another case (Harse *v.* Pearl Life Assurance Company 1903 2 K.B. 92), that the report contained in the *Times Law Reports* has led to some misunderstanding,

Must insurable interest be a pecuniary one?

the facts, as shown by the report in the *Law Reports* 1892 1 Q.B. 864, being as follows: The plaintiff had effected an assurance on the life of her stepsister, she having promised the child's mother (since deceased) that she would take care of the child and help to maintain her. No objection was taken that the plaintiff had not, in fact, spent any money on the child, or as to the amount (if any) expended by her. Lord Coleridge, sitting with A. L. Smith, J., as a Divisional Court, said: "I agree that the insurable interest must be a pecuniary "interest, and that the interest must be in existence when the "policy is effected. That is perfectly clear on the authorities. "Is there such an insurable interest here? I think there is . . . "In my judgment the plaintiff had an insurable interest in the "child's life, at least up to the amount of the payments "actually made by her on the child's account. . . . The "question of amount is not before us." A. L. Smith, J., said: "No doubt the contention of the defendants is correct, that "unless the plaintiff had a pecuniary interest in the child's life "at the time the contract of insurance was made, the policy "would be void under the provisions of the Statute. I think, "however, the plaintiff had such an interest." In the fourth edition of Bunyon's "Law of Life Assurance", page 17, attention is called to the fuller report of the case contained in the *Law Reports*, but even in that edition the statement made can hardly be accepted without qualification, for it is there stated, on the authority of the case quoted above, that "a promise to maintain "and educate a child, given by a person who was under no legal "liability to do so, conferred on that person a right to insure the "child's life", whereas all that the case in question can be considered as deciding is that there is an insurable interest to the extent of the amount actually so expended. In the light therefore of this case, and of the more recent cases of Cunliffe *v.* British Workman's Assurance Company 1902 18 T.L.R. 425, 502; and of Harse *v.* Pearl Life Assurance Company 1903 2 K.B. 92; 1904 1 K.B. 558, I think it must be said that there is no warrant at present for holding that anything short of an actual pecuniary interest in the life proposed for assurance, is sufficient to support the assurance contract, although it may perhaps be noted that there is a tendency in some of the American States to hold that what may be termed a moral consideration, is in certain cases sufficient. I must not, of course, be understood as expressing any opinion

No authority for the idea of a moral insurable interest.

as to whether it is advisable to limit insurable interest in the way it is at present limited in this country, or whether it would not be better to extend the law in the direction indicated by the American decisions. Such an expresion of opinion, though legitimate, and perhaps useful in a discussion on proposals for altering and improving the present laws regulating life assurance, would be out of place in a statement of the law as it at present exists.

Has an employer an insurable interest in the life of his weekly servant?

Another case on the subject of insurable interest, which attracted some little attention when it was decided, is that of Simcock *v.* Scottish Imperial Insurance Company 1902 (O.H.) 10 S.L.T. 286. Here an employer insured his foreman, who was a weekly servant, under two policies, to cover what he considered he would lose by the death of his servant. In an action on the second policy, the absence of insurable interest was relied upon as a defence; and it was held that if any insurable interest existed in such a case, it was at any rate limited to the value, to the master, of one week's employment of the servant; and as this was much more than covered by the amount received under the first policy, the action failed.

Case of a policy taken out ostensibly by the life assured and immediately assigned.

A case that occasionally presents some difficulty in the matter of insurable interest is where the policy is apparently taken out by the life assured and immediately assigned. There is no doubt that in some instances this is merely a subterfuge to avoid the difficulty caused by the absence of a true insurable interest; but it is usually easier to suspect the existence of such a state of affairs than to prove it. Of course, if it can be proved, the assurance is just as much void for want of insurable interest as if it had been directly effected by the person for whose benefit it is really taken out. The situation is clearly set out by Pollock, B., in his judgment in the case of McFarlane *v.* Royal London Friendly Society 1886 2 T.L.R. 755, where he says: "There is nothing "to prevent any person from insuring his own life a hundred "times, paying in each instance only one premium, provided it "is, *bonâ fide*, an insurance on his life, and, at the time, for "his benefit; and there is nothing to prevent him from dealing "with such policies by assigning them to someone else; and "nothing to prevent this being done, even though at the time "when he effected the policies he had the intention of so "dealing with them. There is no law against this, and it is not

"within the evil or mischief of the Statute. But if, *ab initio*, "the policy effected in the name of A is really and substantially "intended for the benefit of B, and B only, and that is kept "back, a different state of facts is presented, and other "considerations arise. That is within the evil and mischief "intended to be met by the Statute." Two modern cases may be cited as illustrations, both singularly enough decided at the same time, though in different places. In the first, that of Holt *v.* English and Scottish Law Life Assurance Company, tried by Mr. Justice Grantham at Leeds, and reported in *The Times* for 5 August 1899, a woman was assured, professedly by herself, for £500, the annual premium being £31. 8*s.* 4*d.* At the time in question she was fifty-eight years of age, and her husband was only earning 22*s.* per week. It was alleged that the first two premiums were paid by her and the subsequent ones by a neighbour, who borrowed £75 for the purpose from the plaintiff, to whom the policy was ultimately assigned. The question left to the jury was: "Had the "policy sued upon been really effected by others, or was "it a *bonâ fide* transaction on the part of the deceased "woman?", and they had but little difficulty in finding in favour of the assurance company. The second case, that of Downing *v.* Marine and General Mutual Life Assurance Society, tried by Mr. Justice Channell, at Birmingham, and reported in *The Times* for 7 August 1899, is perhaps better known, as it attracted some attention at the time. Here a policy for £5000 was effected, nominally by the life assured, and six weeks afterwards, in accordance with an intention expressed by the assured before it was effected, it was assigned to his son who paid the first premium. The assurance company in disputing the policy contended, *inter alia*, that the assurance was really for the son's benefit, and was void under the Statute. The question left to the jury was as to whether the father insured his own life or whether the son really effected the assurance; and after some deliberation, they found that the son insured his father's life, and judgment was therefore given in favour of the assurance company. Other cases might be quoted in illustration of this principle, but as already indicated, the difficulty in such cases is not to establish the principle, for that was clearly established at least as long ago as 1858, when the case of Shilling *v.* Accidental Death Insurance Company 1 F. & F. 116 was decided on these lines. The real difficulty is rather in

Difficulty of proof in such cases.

getting sufficient evidence to prove the case for the assurance company.

Who are entitled to plead the absence of insurable interest?

It may be noted that it has been held that while the assurance company can plead the absence of insurable interest as a defence, other parties are not entitled to raise it in support of their claims. Thus in the case of Hadden *v.* Bryden 1899 1 F. 710 there was a dispute as to the ownership of a policy between the representative of the life assured, and the representative of his father who had paid the premiums and held the policy. It was contended by the former, *inter alia*, that the father, having no insurable interest in the life of his son, could not effect an assurance on his life which would not be void under the Statute. The Lord President said, in regard to this contention, "The answer is, that it has been "decided, on grounds which are clearly valid, that the Statute "merely furnishes a defence to the insuring company against a "claim on the policy; but that if the company waive the defence, "the question, who is entitled to the proceeds of the policy, falls "to be determined as if the Statute did not exist." Attention should, however, be called to the fact that in the case of Gedge *v.* Royal Exchange Assurance Corporation 1900 2 Q.B. 214, which was a case of marine assurance, the Court apparently took a somewhat different view; and refused to enforce a policy where the plaintiff's case disclosed that the transaction on which his claim was based was illegal under the statute 19 Geo. II c. 37 s. 1 although the illegality was not pleaded or relied on by the defendants.

Are the premiums returnable when the assurance is void for want of insurable interest?

Closely connected with the subject of insurable interest is the question as to whether, where a policy is void for want of such interest, the premiums paid can be recovered back. For a considerable time it had been held, on the authority of Howard *v.* Refuge Friendly Society 1886 54 L.T. 644, that such premiums could not be recovered back; but this view received somewhat of a shock from the decision in Cunliffe *v.* British Workman's Assurance Company 1902 18 T.L.R. 425 502. In this case, the plaintiff had effected an assurance on the life of a relative, without having any insurable interest in the life. The agent who took the proposal knew that there was no such interest, but stated that the policy would be all right. The Court of Appeal ordered the return of the premiums on the ground that the knowledge of the agent as to there being no insurable interest, coupled with his statement

as to the policy being a valid one, prevented the parties from being *in pari delicto;* and that therefore the ordinary rule, that money paid under a mistake of law cannot be recovered, did not apply. This case was followed shortly afterwards by that of Harse *v.* Pearl Life Assurance Company 1903 2 K.B. 92 1904 1 K.B. 558 which resembled the preceding one in most respects, but with this important exception, that the agent in representing that the policies were good, in spite of the absence of insurable interest, acted innocently. This fact, however, did not alter the result when the case went before the Divisional Court, for Lord Alverstone, C.J., held that the agent of the assurance company was a person who ought to be presumed to have a peculiar knowledge of the particular branch of law relating to insurance; and was therefore practically precluded from setting up his innocence in the matter. The Court held, therefore, that in this case also, the parties were not *in pari delicto,* and ordered the return of the premiums. This judgment was, however, reversed by the Court of Appeal, and Romer, L.J., disposed of the somewhat dangerous contention of Lord Alverstone by saying: "Nor do I think that agents of insurance companies must be "treated as under a greater obligation to know the law, than "ordinary persons whom they approach in order to effect "assurances." He went on to state that, "the parties must be "taken to have been *in pari delicto,* and that the Company "cannot stand in a worse condition than their agent." This last remark disposes of a far-reaching contention of the counsel for the plaintiff, that, "it was not a question between the plaintiff "and the agent of the company, for the contract was made with "the company; and it is their knowledge which is material. It "was their representation, made through their agent, that there "was an insurable interest."

Must an agent be considered to be an expert in assurance law?

The Proposal.

The cases to which I wish next to draw attention relate to the proposal. A very important modern case dealing with this subject is that of Biggar *v.* Rock Life Assurance Company 1902 1 K.B. 516. Here the proposal for an accident assurance was filled up by the company's agent, who inserted many incorrect statements on material points, such statements being incorrect to his knowledge, and made without instructions or authority from the proposer. The latter

Agent's knowledge as to misstatements in the proposal.

did not know that the incorrect statements had been made, and signed the proposal without reading it. The company disputed the policy, and judgment was given in their favour, Wright, J., saying: "I agree with the view . . . that if a person in the "position of the claimant chooses to sign, without reading it, a "proposal form which someone else has filled in, and if he "acquiesces in that being sent in as signed by him without "taking the trouble to read it, he must be treated as having "adopted it. Business could not be carried on if that were not "the law. . . . I cannot imagine that the agent of the insurance "company can be treated as their agent to invent the answers "to the questions in the proposal form. For that purpose, it "seems to me, if he is allowed by the proposer to invent the "answers, and to send them in as the answers of the "proposer, that the agent is the agent, not of the "insurance company, but of the proposer." The result in this case is, no doubt, satisfactory, and the law as laid down therein would appeal to an ordinary business man as being eminently reasonable, even without the endorsement of so distinguished a Common Law judge as the late Mr. Justice Wright; but, at the same time, it is not wise for us to shut our eyes to the fact that this is only a decision of the High Court, and that there is an older case, decided by the Court of Appeal, which is much less satisfactory, and which is not easily reconciled with this case. I refer to that of Bawden *v.* London, Edinburgh and Glasgow Assurance Company 1892 2 Q.B. 534. In this case, which was also one of accident assurance, the proposer had only one eye, and this fact was known to the agent, who nevertheless made no mention of the fact on the proposal, although there was a space thereon for recording any deviation from the normal. The proposer lost his other eye by accident and claimed as for a total loss of sight. The case was decided against the assurance company by a very strong Court, all three judges holding that the knowledge of the agent was, in the circumstances, the knowledge of the company. Thus Lord Esher, M.R., said, speaking of the agent: "He saw that the man had "only one eye. . . . In that sense, the knowledge "of the agent was the knowledge of the company." Lindley, L.J., said: "In my opinion the company are bound "by Quin's (the agent's) knowledge", and Kay, L.J., said: "The knowledge of the agent was the knowledge of the "company. The knowledge was obtained by him when he was

Agent, not of the company, but of the proposer.

When the knowledge of the agent is the knowledge of the company.

"acting within the scope of his authority, and it must be "imputed to the company." It is true that there has been a tendency of late years to whittle away the effect of this case, and to very readily distinguish it from other cases which arise, and which to the ordinary mind seem indistinguishable. Thus in the recent case of Levy *v.* Scottish Employers' Insurance Company 1901 17 T.L.R. 229, decided by Wills and Phillimore, JJ., sitting as a Divisional Court, Wills, J., said: "All that was decided in the case of Bawden *v.* London, "Edinburgh, and Glasgow Assurance Company was that the "agent, under the circumstances of that case, was an agent of "the company to settle the terms of the proposal"; and Phillimore, J., said: "Bawden's case turned upon the special "terms of the contract, and the utmost the decision came to was "this, that it was the agent's duty to put into the proposal form "obvious and patent matters." Mr. J. L. Wark, in his paper already referred to, mentions in this connection that in the case of M'Millan *v.* The Accident Assurance Company, Limited 1906 13 S.L.T. 858, of which I have not seen a full report, the decision in Bawden's case was questioned on the ground that it was not clear that the agent in that case had any authority from the company to negotiate and settle the terms of the proposal, and also on the ground that the judgment overlooks the fact that the contract sued on was a contract in writing. I cannot help thinking, however, that if the difficulties raised by Bawden's case are to be got over, it must be on the common sense, business-like lines laid down in the decision of Biggar's case.

As illustrating the difficulties of the situation, attention may be called to the case of Ramsbottom *v.* British Union Assurance Society, Limited, tried by Mr. Justice Grantham at Manchester, and reported in *The Times* for 15 February 1906. This was a case of fire insurance, and it appeared from the evidence that the defendant's agent called on the plaintiff and invited him to insure. The plaintiff consented, and the agent filled up the proposal form from information received from the plaintiff. The proposal contained the questions: "(3) Have you ever had a fire, "either in these premises or elsewhere? If so, state particulars. ". . . . (6) Have you ever been declined by any office?" The plaintiff answered both these questions in the negative. The proposal also contained a declaration . . . "I hereby declare that "the whole of the answers to the above questions . . . are true, "and I agree that this declaration shall be the basis of the

"contract between me and the Society." The plaintiff did not conceal from the agent the fact that there had been two fires elsewhere, for which he had claimed compensation, and it was also discovered later that another assurance company had declined to continue the plaintiff's policy. The assurance company disputed the policy on the ground that the plaintiff's answers were untrue and avoided the policy. Grantham, J., in summing up, said that he preferred to follow a decision of the Court of Appeal (Bawden's case) rather than the decision of a judge of concurrent jurisdiction with himself (Biggar's case), particularly when he was not quite certain what that learned judge had meant. In the result a verdict was returned for the plaintiff, and judgment was entered accordingly. In view of the conflict of legal opinions and decisions on this point, perhaps all that can safely be said is that, while there seems to be a tendency to distinguish Bawden's case from others resembling it, and to confine its authority to cases standing exactly on all fours with it, nevertheless, while hoping for the best, we must be prepared for the worst, as represented by that case.

The Completion of the Contract.

Turning now from questions relating to the proposal to those concerning the completion of the contract and the issue of the policy, the first case that claims attention is the well-known one of Roberts *v.* The Security Company 1897 1 Q.B. 111. The material facts of the case are as follows: The plaintiff made a proposal for insurance against burglary on 14 December 1895. A policy in conformity with the proposal was prepared and was signed by two directors and the secretary of the company, and sealed with its common seal. The proposal contained a provision that no insurance would be considered in force until the premium had been paid. The policy contained a like provision, but it went on to recite that the proposer had paid the first premium to the company. The policy remained in the hands of the company, and before any premium had been paid thereon a burglary took place. The company disclaimed liability under the policy, but it was held that the policy constituted a completed contract of insurance, that by the recital as to the payment of the premium the defendants had waived the condition for prepayment, and therefore that the risk had attached. The case attracted so much attention when it was decided, and has since given rise to so much criticism, that it

Receipt of first premium acknowledged in policy.

may be worth while to quote from the judgments delivered by the members of the Court of Appeal, as showing the grounds on which their decision was based. Lord Esher, M.R., said: "The "question raised is, whether an insurance was effected by the "sealing and signing of the policy, or the execution of the policy "was only intended to be conditional. I do not see any evidence "of a conditional delivery, or that this document was intended "not to be a policy unless certain conditions were fulfilled . . . "There is no suggestion that it was delivered to any one as an "escrow. If it was in the hands of the Company itself, it could "not be delivered as an escrow. . . . It is a contract to insure the "plaintiff . . . and it recites that the assured has paid the "premium for that insurance. It was said that the recital was "incorrect, and that the premium so stated to have been paid "never was, in fact, paid. I do not think that the "defendants are for the present purpose, at liberty to "show that, in contradiction of the terms of their own "deed. They have treated the premium as paid; and if it has "not been paid, I think they have thereby waived the previous "payment as a condition of the existence of an insurance." Lopes, L.J., said: "One of the recitals in the policy contains a "positive statement that the premium has been paid; but the "company, nevertheless, seek to contradict that statement in "their own deed. I think they are estopped, for this purpose, "by their own statement, or, if not properly speaking estopped, "there is evidence of a waiver by them of the term as to previous "payment of the premium." Rigby, L.J., said: "I think that, "even if it were shown—which does not appear to be the case— "that the deed was delivered as an escrow, on condition that it "should not be handed over to the person who was to benefit "thereby until he had paid the premium, the deed would be as "good a deed, whenever the condition was performed, as if it had "been originally delivered unconditionally. It would be open to "the assured to tender the premium and demand the deed; "and it would make no difference that a loss had then occurred. "It seems to me that it would be his right to have the policy "delivered to him, notwithstanding that a loss had happened."

Company estopped from denying the actual receipt of premium.

Methods of meeting the difficulty.

The decision aroused a considerable amount of interest, as the practice of many life assurance companies was similar to that of the Security Company in the matter of the first premium; and various devices have been suggested for getting round the difficulty raised by the case, the latest of

which, I think, is that due to Mr. Wark, who suggests that the proposal should contain a condition that there shall be no liability until the policy is actually delivered to the assured. Such a method as this, however, is likely to give rise to considerable dissatisfaction where, as frequently happens, the first premium is collected immediately on acceptance, but the policy is not actually completed and ready for delivery until a week or two later. As a matter of fact, I believe most of the companies concerned have got over the difficulty by adopting the plan that was already employed by several companies, that of not preparing the policy until the first premium had actually been received. I notice Mr. Wark suggests that the decision in this case would probably be treated as a very special one by the Scottish Courts, and in so far as that opinion depends on differences between Scottish and English law, I am, of course, not in a position to dispute his view. It may, however, be worth while to point out that the question was decided by a strong Court of Appeal, that the members of that Court were unanimous in their opinion, and that, apparently, they arrived at their decision without doubt or hesitation. It is somewhat improbable that the House of Lords would reverse such a decision of the Court of Appeal on a point of law; and therefore, in so far as the law of Scotland agrees with the law of England on the questions involved, it is probable that the ultimate result of a similar case would be the same, whether it arose in Scotland or England, at least if it were carried to the ultimate Court of Appeal for both countries, in the shape of the House of Lords.

Can the assurance company sue for the first premium if the assurance is not taken up?

An interesting case to contrast with the foregoing is that of The General Accident Assurance Corporation *v.* Cronk 1901 17 T.L.R. 233. Here the defendant completed a proposal for an assurance against accidents caused by his employees. The proposal form provided that if the risk was accepted, the defendant would pay the premium when called upon to do so. The risk was accepted by the assurance company, and a policy was issued containing some terms which did not appear in the proposal form. After it had been completed, the defendant informed the company that he did not desire to proceed with the insurance. The company sued him for the first premium, and he was held liable for the amount. Wills, J., in delivering judgment for the plaintiffs, pointed out that "the defendant, in his "proposal, undertook, if the risk was accepted by the plaintiffs, "to pay the premium. That meant that as soon as the risk was

"accepted he became liable to pay the premium; and it did not "mean, as it was contended on his behalf, that before he could "be asked for the premium, he must approve of the policy "tendered to him. . . . If the wrong form of policy was "tendered to him, he no doubt had the right to insist on receiving "the correct one."

It may, of course, be contended that this decision turns on the special provision as to payment of premium contained in the proposal, and that it is therefore inapplicable to ordinary cases, even if an assurance company should deem it advisable to take the very unusual course of suing for the first premium, when the proposer does not wish to take up the assurance. In particular, it may be urged that it would not apply where, as is frequently the case, there is a distinct provision that the risk is not to attach until the first premium is paid. This contention is, however, considerably weakened by the decision in Roberts *v.* Security Company, and it was pointed out by the counsel for the plaintiffs in Cronk's case, when contending that they were entitled to the full premium and not merely to nominal damages, as awarded in the Court below, that, on the authority of Roberts' case, the contract of insurance was complete when the policy was executed. It will be remembered that the policy in the last-named case contained the usual provision as to the commencement of the risk; and it seems probable that such a provision would be construed as being for the assurance company's protection, and not as a means whereby the proposer could evade his contract. If this view be correct, then, in the event of the proposer failing to take up his policy, the Company could, if it thought fit, in spite of the provision as to attaching of risk referred to, recover, at least, the amount of damage it had suffered by the proposer's failure to complete his contract, though in the absence of a special provision as to the payment of the premium, as in Cronk's case, it is doubtful whether the whole of the first premium could be recovered.

A proposer can withdraw before the contract is complete, and can recover back a premium paid.

As an example, on the other hand, of an incomplete contract, and one therefore not capable of being enforced, reference may be made to the case of Wolfe *v.* Equitable Life Assurance Society of the United States, which is reported in *The Times* for 26 January 1906. In this case the plaintiff held an endowment policy in the defendant society, and certain options, including one to take a cash surrender-value of £166. 14*s.* 10*d.*, became exercisable on

14 October 1905. The defendant's representative induced the plaintiff to propose for a fresh assurance, and on 18 October the latter signed a proposal form, was examined by the society's doctor, and allowed the defendants to take the first year's premium, amounting to £121. 8*s.* 4*d.*, from the money due to him. The document signed contained the words: "I hereby "agree that this application, and the policy hereby applied for, "taken together, shall constitute the entire contract between the "parties hereto." On the terms of the arrangement, the defendants were bound, within a period that could not exceed ten days, either to execute the assurance and cover the assured from the date of the receipt, or to return him his money. On the following day, 19 October, the plaintiff informed the society that he had decided not to insure his life, and applied for the return of the premium. The defendants declined to do this, on the ground that the matter had been completed; and they forwarded him a temporary policy. It is difficult to see on what grounds their contention could be supported, since at the time of the plaintiff's withdrawal, the society retained the right of either accepting or rejecting the proposal; and in the course of the judgment in favour of the plaintiff, ordering the return of the premium, it was pointed out that "The money was paid simply "on a proposal unaccepted, and the defendants did not bind "themselves to anything. The proposal was recalled before "anything was done on it, and therefore the plaintiff was entitled "to judgment." The main point which distinguishes this case from the preceding one is of course that in this instance, the withdrawal took place before the definite acceptance of the proposal, and it is therefore analogous to the withdrawal of an application for shares before allotment; whereas in Cronk's case, the attempted withdrawal took place after acceptance, and it is therefore similar to the case where the withdrawal of the application arrives after the allotment letter has been posted.

Effect of provision as to risk not attaching until the premium is paid.

With regard to the very common provision, contained in the policy in Roberts' case, to the effect that the risk shall not attach until the premium is paid, it may be noted that it was held, *inter alia*, in the case of Sickness and Accident Assurance Association *v.* General Accident Assurance Corporation 1892 19 R. 977, that such a stipulation relieved the insurer of liability in respect of a claim arising after the date at which the policy was (*ante*) dated but before the premium had been paid.

Conflict of Laws in relation to the Contract.

Having arrived at a complete contract of life assurance, as set forth in the proposal and policy, the question may now be considered as to where the contract is made, and the closely allied one, as to what law is to govern it, when the head office of the company is situated in one country and the proposer is domiciled in another. These questions are, of course, of more interest to companies doing foreign business than to companies confining their operations to the United Kingdom. They are however of considerable importance to most companies, for it must be borne in mind that England and Scotland, constituting as they do, different jurisdictional areas, are foreign to each other for most legal purposes.

Where is the contract made and what law is to govern it?

With regard to the first question, as to where the contract is made, there is not much direct authority on the point, but the correct view appears to be that the contract is made where the acceptance takes place, that is, where the actual act of acceptance is performed, not where such acceptance is communicated to the other party. Thus in the case of Cowan *v.* O'Connor 1888 20 Q.B.D. 640, where, in order that an action might be maintained in the City of London Court, it had to be shown that the contract was made in the City of London, it was held that this was shown by the fact that the telegram accepting the offer was dispatched from the City of London, although the offer was made outside the City. In all cases, therefore, of assurances granted by English companies to persons domiciled outside England, where the ordinary practice is adopted of the proposal being accepted at the head office, the contract will be regarded as having been made in England. The fact that the proposal was taken in a foreign country by an agent there, and that the policy, when issued, was sent by the company to such agent, and by him delivered to the assured, will not be sufficient to alter the fact that it is a contract made in England and to be treated, in certain circumstances, as an English contract. This is clearly seen from the case of Parken *v.* Royal Exchange Assurance Corporation 1846 8 D. 365. Here the proposal was made in Edinburgh by a man domiciled in Scotland, to the Edinburgh agents of the company, who had no power to accept proposals definitely. The policy was issued by the head office in London in English form,

The contract is made where acceptance takes place.

Proposal taken in foreign country by agent there.

and sent to the Edinburgh agents to be handed to the assured, who paid the premium to them. It was admitted, in the course of the proceedings, that neither party was bound, that is, the contract was not finally completed, until the policy was delivered and the premium paid. Nevertheless, it was laid down by Lord Moncrieff, in the course of his judgment, that "the mere "circumstance of this contract having been transmitted through "or by the hands of an agent in Edinburgh, cannot have the "effect of changing that which was, in its nature, essentially an "English contract, made under the law of England, into a "Scotch contract, to be ruled, in its nature, construction and "effect, by the law of Scotland." One important result flowing from the determination of the place where the contract is made, is referred to by the same judge in the course of his judgment, where he states, quoting approvingly from a leading Scottish text-book writer, that "it is further "very clearly laid down that if no place of payment be expressed, "it shall be presumed to be in the place where the contract is "made." Many English companies stipulate in their policies that the amount due shall be payable at the head office in England, but according to the above view, it may be still so payable, even without such a stipulation.

Place where the contract is made determines place of discharge.

Completion of the contract by a foreign agent.

It should perhaps be pointed out that if the contract were actually completed by a foreign agent, the case would be different. Thus, in the case of Mills *v.* Albion Fire and Life Insurance Company 1828 3 W. & S. 218, where the agreement for the assurance was made in Scotland with an agent there, and the subsequent policy, though made out, was never delivered to the assured, it was held that the real contract sued on was a Scottish contract, although had the actual policy been delivered and sued upon, the question as to whether it was a Scottish or an English contract might admit of doubt and question. Thus Lord Lyndhurst, L.C., in the course of his judgment said: "If I send an agent to reside in Scotland, and "he, in my name, enters into a contract in Scotland, the contract "is to be considered as mine where it is actually made. It is "not an English contract because I actually reside in England. "If my agent executes it in Scotland, it is the same as if I were "myself on the spot, and executed it in Scotland. Therefore the "original contract must be considered as a contract entered into "in Scotland, and it was that original contract, and not the "policy, which was the ground of the action."

What law is to govern the contract?

Closely connected with the foregoing subject is the question as to what law is to govern the contract, when the policy is granted by a company domiciled in one country to a person domiciled in another. This will depend, to some extent, on the particular aspect of the contract which is under consideration. Thus, questions relating to status or capacity to contract, such as those arising in the case of infants and married women, are ordinarily governed by the law of the place where the person in question is domiciled. There is, however, an important exception to this rule, from our point of view, in the case of mercantile contracts, which would appear to be governed by the law of the country where the contract is made. Thus, Dicey, in his "Conflict of Laws," page 547, points out that "there are strong grounds for holding that capacity to enter "into an ordinary mercantile contract, *e.g.*, for a loan, or for the "purchase or sale of goods, is governed, not by the *lex domicilii* "of the contracting party, but by the law of the place where the "contract is made *(lex loci contractus)*. Story certainly holds "this opinion. In one reported case, Male *v.* Roberts 1800 "3 Esp. 163, where the point is distinctly raised, though not "precisely decided, Lord Eldon held, in regard to a contract made "by an English infant in Scotland, that the effect of infancy as a "defence to an action on the contract, depended upon the law of "Scotland." The importance of the point here referred to will be seen when I come to discuss the case of The Scottish Life Assurance Company *v.* John Donald (Limited) 1901 (O.H.) 9 S.L.T. 200, which will be conveniently dealt with under the head of Settlement Policies, rather than here.

Law as to the formalities of the contract.

The validity of a contract, as far as its form is concerned, is governed by the law of the country where it is made, that is, by the *lex loci contractus;* and with certain exceptions which do not affect the questions discussed in this paper, it may be said that no contract is valid which is not made in accordance with that law. The formalities required for a valid contract by the law of the country where it is made must therefore be complied with, even though such formalities are unnecessary under the law of the country where it is sought to enforce the contract.

Formalities required by Scots law.

Thus, by Scots law, it is necessary that a deed, other than a holograph one, should be attested by two witnesses, and whether holograph or not, should be signed on every page by the person executing it, if it extends

over more than one page. If such person cannot write, it should be signed by a notary public on his behalf, and not with a mark, as in England. Since I first wrote these remarks, I have seen it contended by a Scottish solicitor that even the ordinary receipt for the policy moneys comes within the regulations referred to, and is invalid if they are not complied with in respect of it. I think, however, that this view is, to say the least, open to very grave doubt, and it seems more probable that such a document would come within the exemptions from these formalities, which have been recognized in the case of receipts for rent and of bills of exchange and other documents or writings *in re mercatoria*. The formalities in question are, of course, unnecessary by English law, but an English assurance company when called upon to act on a deed executed in Scotland must see that the requirements of Scots law in this respect have been observed.

Interpretation of the contract.

The interpretation of the contract, and the rights and obligations of the contracting parties under it, are determined in accordance with what is termed, "the proper law of the contract." This is defined by Dicey (page 540) as being, "the law by which the parties to a contract "intended, or may fairly be presumed to have intended, the "contract to be governed, or in other words, the law or laws to "which the parties intended, or may fairly be presumed to have "intended to submit themselves." So also, in the leading case of Hamlyn & Co. *v.* Talisker Distillery Company 1894 A.C. 202, it was laid down that, "in a contract between parties of different "nationalities, where a question of conflict of laws is raised "on the contract, it is a question of intention, to be gathered "from the whole facts, by which system of laws the parties "intended their relationship to be regulated." As to how this law is to be determined, certain rules are given by Dicey, which are of assistance, but the contract itself must be considered as a whole in order that the intention of the parties may be ascertained. These rules are:—(1.) The expressed intention of the parties will always determine the proper law of the contract.—(2.) *Primâ facie*, the proper law of the contract is presumed to be the law of the country where the contract is made.—(3.) When the contract is made in one country, and is to be performed wholly or partly in another, then the proper law of the contract may be presumed to be the law of the country where the performance is to take place, *i.e.* the *lex loci solutionis*. Thus, in the case of Parken

Rules for determining the proper law of the contract.

v. Royal Exchange Corporation, referred to above, it was held that the contract was governed by the law of England in matters relating to its nature and the liabilities under it. The Lord Justice Clerk said: "If certain words in the policy had a fixed "settled meaning in favour of the assured, by the law of England, "and a different meaning by the law of Scotland, would the "Company not be bound to the extent of the law of England, "and the interpretation of the terms regulated by the law of "England? That law clearly must have regulated the interpre- "tion of the contract." Lord Moncrieff said that "If the *locus* "*contractus* and also the *locus solutionis* are ascertained to be in "one country, the law of that country must rule all questions "touching the effect of the contract; and if the *locus solutionis* "were determined to be different from the *locus contractus*, the "law of the place in which performance is stipulated to be made, "must govern." It must be carefully noted, however, that the determination of the law governing the interpretation of the contract does not settle the question as to what Court has jurisdiction in the matter, but only as to what law that Court will apply. Thus, in the case just referred to, although the Scottish Court of Session held that the contract was governed by the law of England, they also decided that they had jurisdiction to deal with the case, and did so accordingly, applying, however, English law instead of Scots law.

What Court has jurisdiction?

With regard to bankruptcy, it would appear that proceedings of this nature in a foreign jurisdictional area will have the same effect on moveables in England, including such choses in action as policies of assurance, as if they were actually situated in the country where the proceedings are taken. Hence the law of the bankrupt's domicil apparently applies; but only to transfer such rights as he possesses, these rights being defined, in the case of a contract, by the proper law of the contract as shown above.

Conflict of laws with regard to bankruptcy.

Finally it may be noted that, apparently, the validity of the discharge of a contract depends on the proper law of the contract; and that a discharge, not in accordance with that law is not valid.

Validity of discharge.

The Policy as Evidence of the Contract.

It occasionally happens that disputes arise as to whether the whole contract of assurance is contained in particular documents such as the policy, or policy and proposal together; or whether

some parol conditions, written or oral, are to be read into it. Such disputes usually arise out of some alleged promise of a representative of the assurance company; and it is of some importance to ascertain the exact legal effect of such promises, assuming them to have actually been made. An important case of this description came before the Courts last year, in the shape of Horncastle *v.* Equitable Life Assurance Society of the United States 1906 22 T.L.R. 534 735. In this case, the plaintiff took out a semi-tontine policy with the defendants. Prior to signing the proposal, the plaintiff requested the defendants' London superintendent to state in writing what the benefits receivable under the policy would be. The latter drew up a memorandum which stated, *inter alia,* "Results paid in 15 years. Cash value, £7,390." The proposal form was then completed and at once submitted to the Board by the superintendent. He returned in a few minutes and handed the memorandum back to the plaintiff, saying: "That will be all right, and this amount is guaranteed to you." The policy was duly issued, containing the usual conditions of one of this class, and also this declaration: "(11). The contract "between the parties hereto is completely set forth in this policy "and the application therefor, taken together, and none of its "terms can be modified, nor any forfeiture under it waived, "except by an agreement in writing, signed by one of the "following officers, namely: . . ." The superintendent did not come within the list of officers set out. On the face of the policy, at the foot, were the following words: "No person, "except one of the executive officers designed on the back of "this policy, is authorized to make, alter, or discharge contracts, "or waive forfeitures"; and attention was then called to provision (11) set out above. When the policy matured on 2 January 1906, the defendants alleged that the amount due was £6,106. 15*s.*, which amount the plaintiff refused to accept, as he considered that the memorandum was a guarantee of a minimum sum, without which he would not have effected the assurance. The case was heard, in the first instance, by Mr. Justice Walton, who, in deciding in favour of the assurance society, said that "in his opinion the alleged warranty was not a separate" (meaning collateral) "agreement as to any matter on which the "contract was silent. The agreement, in effect, was that the "plaintiff was to get something better than that provided for in "the policy. The agreement was inconsistent with the terms of

Effect of representations not contained in proposal or policy.

"the policy. The parties intended the policy to be the complete "and final statement of the transaction." The case was taken to the Court of Appeal, and all the judges there expressed their entire agreement with the judgment of Walton, J., the appeal being dismissed without calling on defendants' counsel. Collins, M.R., said that "the suggested collateral agreement "provided for the payment of a different sum, calculable in some "other way. The two were inconsistent with each other, and "the prior parol agreement was not enforceable." Moulton, L.J., said that "Clause (11) made it a condition of the policy that "neither party would allege that the whole of the terms of the "contract were not contained in the policy—in other words, it "was a condition of the policy that nothing which was done "during the negotiations should be considered as binding "between the parties." Farwell, L.J., said "the document "was not consistent with the policy. . . . It was difficult to see "how the defendants could exclude any agreement outside the "policy more clearly than they had done by Clause (11). That "clause was intended to guard against such an agreement as "that now set up."

It must not, however, be hastily assumed, on the authority of this case, that no effect will be given to representations made by an official of the company prior to the issue of the policy.

When effect may be given to prior representations.

Horncastle's case turned, to some extent, on the peculiar wording of the policy, and, but for this, it might possibly have been decided otherwise, in accordance with the principles laid down in the well-known case of De Lassalle *v.* Guildford 1901 2 K.B. 215. In this case the defendant agreed to grant a lease of certain premises to the plaintiff. The terms were arranged, but the plaintiff refused to hand over the executed counterpart until he received an assurance that the drains were in order. The defendant verbally represented that they were in good order, and the counterpart was thereupon handed to him. The lease contained no reference to the drains. The drains were not in good order, and on an action for breach of warranty, it was held that the representation made by the defendant as to the drains being in good order was a warranty which was collateral to the lease, and for breach of which an action would lie. In the course of the judgment in the Court of Appeal, in favour of the plaintiff, delivered by A. L. Smith, M.R., on behalf of himself, and Collins and Romer, L.JJ., he said: "It appears to me in this case clear that the lease did not cover

"the whole ground, and that it did not contain the whole of the "contract between the parties. . . . The warranty in no way "contradicts the lease, and without the warranty the lease would "never have been executed." Horncastle's case was distinguished from the foregoing on the grounds that in the former: (1) The representation set up was not a collateral agreement; (2) The policy was intended to cover the whole contract; (3) The agreement and the policy were inconsistent. It does not follow that these conditions will be fulfilled in the case of a policy issued by a British company, and it is not usual in this country to take such elaborate precautions to guard against representations that may be made by minor officials as were adopted in this instance, perhaps because such precautions are not often needed. It will therefore be necessary to use considerable caution in applying Horncastle's case to apparently similar circumstances arising in connection with a British company.

Effect to be given to statements in prospectus.

Another case in which the Court refused to go outside the policy in interpreting the contract, was the recent one of Bailey *v.* British Equitable Assurance Company, Limited 1906, A.C. 35. Here the deed of settlement provided that the profits should be divided as directed by the bye-laws, and made provision for the alteration of the deed or bye-laws by means of bye-laws. By a bye-law made in 1854, it was provided that the profits of the "Mutual Life Assurance Department" should, after the deduction of expenses, be divided among the policyholders in that department. In 1886 the plaintiff took out a policy on his own life in the "Mutual Department." He made his application in reliance on the statements contained in a printed prospectus issued and circulated by the company, which stated, *inter alia,* that "the entire profits made by the company "in the Mutual Department, after deducting the expenses, are "divided among the policyholders, without any deduction for "a reserve fund." The policy was issued subject to the conditions endorsed on it, which conditions contained a reference to the deed of settlement and the bye-laws. The company subsequently proposed to alter their practice of distributing profits, by devoting part of the profits to a reserve fund; and proceeded to take the necessary steps to effect this. The plaintiff sought to restrain them from making the change, but although the Chancery Division of the High Court, and the Court of Appeal, decided in his favour, this decision was reversed by the House of Lords, and judgment was given in favour of the

assurance company. In delivering judgment, Lord Macnaughten said: "The simple question is, what was the contract between "the parties. . . . I am at a loss to understand how the "Court of Appeal came to the conclusion that the statements in "this prospectus constituted a collateral contract, or are to be "treated as incorporated in the contract of insurance." Lord Lindley said: "It is contended that the applications for "these policies were based on the faith of prospectuses containing "statements and holding out inducements, which preclude the "company from making alterations in the mode of applying "their profits, without the consent of the policyholders. . . . "The prospectus, not being referred to in the policies, cannot, in "my opinion, be legitimately referred to, in order to construe "the contracts into which the policyholders have been induced "to enter. These contracts are to be found in the policies "themselves. . . . I am quite unable to adopt the view taken by "the Courts below as to the inability of the Company to alter "their bye-laws as they have done. . . . I can find no contract "to that effect." It must, however, be carefully noted that the decision, although it was the unanimous decision of the House of Lords, does not give any countenance to the idea that the terms of the prospectus are immaterial in actions of another class affecting the policy, but only decides that they are not to be used to assist in construing the contract. As Lord Lindley pointed out, if the policyholders had been seeking to rescind or rectify their contracts on the ground of fraud or mistake, or were suing for damages caused by fraudulent misrepresentation, it would be legitimate to refer to the statements in the prospectuses, on the faith of which they became policyholders.

Action for rescinding or rectifying the contract.

By way of contrast, reference may be made to the somewhat ancient case of Mills *v.* Albion Fire & Life Insurance Company 1828 3 W. and S. 218, which has already been referred to in connection with another matter. This was a case of marine assurance, and the agent at Glasgow entered into an agreement for an assurance, and sent particulars to the company in London. The company issued a policy containing a clause excepting marine risks, they being precluded from taking such risks by the statute 6 Geo. I., c. 18. This policy was never shown to the assured, and on the ship being burned a claim was made. It was held, *inter alia*, that as the policy was never shown to the assured, the company were bound by the original agreement,

which was general in its terms, and therefore included marine risks.

THE CONDITIONS OF THE POLICY.

Error in wording of contract.

An interesting question arose in Scotland recently as to the effect to be given to the provisions of a policy when, through a grammatical error, they were inconsistent with one another. The case in question was that of Glen's Trustees *v.* The Lancashire & Yorkshire Accident Insurance Company Limited 1906 43 S.L.R. 684. Here the policy contained a provision that the right to recover under it should be forfeited on the expiration of a certain time from the date of the accident, unless within that time a settlement had been arranged, or the matter had been referred to arbitration, or, in the absence of notice from the company requiring the matter to be so referred, legal proceedings had *not* been taken by the assured. The assured died from accident on 24 April 1897, but no proceedings were taken against the company until 10 July 1905. The plaintiffs then claimed that under the last part of the condition quoted, they were not too late, and were entitled to recover. The case was decided in favour of the assurance company, and this was confirmed on appeal. The Lord President, in delivering judgment, said: "The whole question is as to the meaning of "that (the above) article. It is clear that as it stands, it is "meaningless; but reading it as a whole, as I think we are "entitled to do, it is also clear that the confusion is due to "a grammatical error. In my opinion the Court is entitled to "correct such an error. The word 'not' was evidently inserted "in the third (last) stipulation of the article by failing to notice "that the conjunction preceding was 'unless' and not 'if.' I "think we should read the stipulation as if the word 'not' was "deleted."

Suicide clause.

In connection with the question of the conditions of the policy, attention should, perhaps, be called to the case of Ellinger & Co. *v.* Mutual Life Insurance Company of New York 1904 1 K.B. 832 1905 1 K.B. 31, which turned on the construction to be placed on a somewhat unusual form of suicide clause. In this case a certain Max Firnberg effected an assurance on his own life for £4,000, limited to five years, and payable to the plaintiffs, who were creditors of his to that amount. The proposal was signed on 23 May 1902, and contained certain statements which were warranted by Firnberg.

Among these warranties was the following: "I also warrant and "agree that I will not commit suicide, whether sane or insane, "during the period of one year from the date of the said "contract." The proposal was made part of the contract by the usual reference clause in the policy. In February 1903, Firnberg committed suicide while temporarily insane. On the assurance company disputing the claim, it was contended on behalf of the plaintiffs that the effect of the stipulation as to suicide was that if Firnberg did commit suicide within the first year, the company was nevertheless bound to pay the policy money to the plaintiffs, and then had a remedy over in the shape of a right of resort against Firnberg's estate for a breach of a collateral promise. When the case came before the King's Bench Division of the High Court, Bigham, J., in deciding in favour of the assurance company, said: "The policy "was made by the defendants with Firnberg and with no one "else. It is in no sense a contract with Ellinger & Co. . . . "What then, did the parties, Firnberg and the defendant "company, intend when they inserted the warranty that Firnberg "should not commit suicide? I think it is clear that they "intended that in the event of suicide, the company should not "be liable on the policy. It does not matter whether the clause "is called a warranty or not. . . . Apt words have been "used to express this intention, and it is, I think, impossible to "read the clause as merely giving to the company a cross claim "against the estate of the deceased for damages." The plaintiffs appealed, but the appeal was dismissed without the counsel for the defendants being called upon. In dismissing the appeal, Collins, M.R., after expressing complete agreement with the passage from the judgment of Mr. Justice Bigham quoted above, and which, he said, seemed to sum up the whole situation in a manner upon which he could not improve, said: "The "question is, whether the warranty against suicide in "this case is a condition. . . . I think that the "stipulation in question really means that the assurers are not "to be at the particular risk of the applicant's committing "suicide within the year; and it appears to me to comply with "all the requisites of a condition. . . . I do not think that "the fact of this being called a creditor's policy, makes any "difference." It must, however, be borne in mind that the decision turned, not only on the particular form of the suicide clause, but also on the form of the policy, which was effected by the

Warranties and conditions.

proposer on his own life, although payable to someone else. This will be seen from the earlier part of Mr. Justice Bigham's judgment quoted above. If therefore the policy had been in the more usual form in such circumstances, namely, a policy issued to the creditor on the life of the debtor, the decision might, and probably would have been different; since the contractual relation would then have existed between the creditor and the assurance company, and the stipulation as to suicide, if entered into only by the life assured, would have been merely collateral and have given rise only to an action against the estate of the latter, for damages for breach of contract.

Declaration as to validity of policy.

Another case relating to the policy and its conditions is that of Honour *v.* Equitable Life Assurance Society of the United States 1900 1 Ch. 852, which deals with the remedies open to the assured and assurer respectively when a dispute has arisen as to the validity of a policy during its currency. Here the plaintiff was the assignee of a policy granted by the defendants. The latter alleged that the policy was void under the statute 14 Geo. III., c. 48, and further, that it was obtained by fraudulent misrepresentation and intentional concealment, and they refused to accept payment of the premiums. The plaintiff asked for a declaration that the policy was a valid and subsisting one, and that he was entitled to the benefit of it; and further, for an injunction restraining the defendants from repudiating the policy. The defendants counterclaimed for a declaration that the policy was void and ought to be delivered up to the society to be cancelled. It was held that this was not a case in which the Court ought to determine rights before the time had arrived when the right was enforceable, and that, subject to the plaintiff's not being prejudiced by the non-acceptance of the premiums by the defendants, the plaintiff's application must be dismissed. In deciding thus, Buckley, J., said: "When the policy " drops, he (the plaintiff) must not be prejudiced by the fact that " he has not paid the premiums at the due dates. If that is provided " for, he has no right now to ask for a declaration in his favour " that the policy is valid, before any claim against the society has " arisen." The defendant's counsel then withdrew all objection to the form of the action, and asked the Court to hear it on its merits. On hearing witnesses, Buckley, J., then held that the policy was obtained by fraud and fraudulent misrepresentations, and declared the policy void, and ordered it to be delivered up to be cancelled.

Waiver of misrepresentation or breach of condition.

It sometimes happens that there has been a breach of some condition by the assured, or a representation made when the policy was effected proves to be incorrect, and the matter has come to the company's knowledge. The question may then arise as to whether the company has, by its conduct, agreed to waive such breach or misrepresentation, and to affirm the contract. A case on this point which has attracted a considerable amount of attention is that of Hemmings *v.* Sceptre Life Association 1905 1 Ch. 365. In this case the proposer effected an endowment assurance to mature at age 60, the policy being dated 11 May 1888. She stated that she was born on 6 March 1847, and that her age next birthday was therefore 41 years in December 1887, when the proposal was completed. The proposal contained the usual declaration as to the truth of the statements and as to the non-concealment of any material fact, and an agreement as to the avoidance of the policy if any untrue statement had been made. The policy was subsequently assigned to the plaintiff to secure a loan, payment of premiums and interest on the loan being guaranteed by certain relatives of the assured. In 1897 it was discovered by the guarantors that the assured was born on 6 March 1844, and that her age at the time of entry was therefore 44 years next birthday and not 41 as stated, the misstatement having been made through a mistake. The defendant company were immediately informed of the mistake, but, nevertheless, accepted two further premiums, namely, on 1 March 1898 and 1 March 1899. In August 1899 the defendants informed the plaintiff of the mistake, and proposed that he should pay the annual difference of premium, with compound interest, and also, in future, pay the correct premium for the true age. The plaintiff, who then heard of the mistake for the first time, declined to do this, and tendered the original annual premium for each successive year down to and including March 1904, when the assured attained the age of 60 years. The defendants, however, declined to accept them, and alleged that the policy was void and the premiums forfeited. The plaintiff claimed that the defendants, by accepting the premiums in 1898 and 1899 had elected to affirm the policy, and it was held :—(1) That on the authority of Fowkes *v.* Manchester and London Life Assurance and Loan Association 1863 3 B. and S. 917, the defendants could not, in view of the terms of the policy and proposal, avoid the policy and forfeit the premiums on the ground of a misrepresentation which was admitted not to be

wilful; and (2) that by accepting the premiums in 1898 and 1899 the defendants must be taken to have affirmed the policy as it stood, and were not entitled, as suggested by their counsel, to treat the policy as if it matured according to the date of birth originally given—that is, on the basis of the assured being actually 41 years of age next birthday at the date of the proposal. On this latter point Kekewich, J., in pointing out that this should have been considered by the company in 1897, when the mistake was discovered, said: "It seems to me that they might then, if "they had thought fit, with propriety have said, 'We will return "'the premiums hitherto received to the proper person to receive "'them, and we will receive no more unless you wish to make a "'new contract.' On the other hand, it was open to them to "treat the policy as still subsisting, and to accept the premiums "on the old footing. It was a pure matter of business for the "directors to say which of these two courses they would adopt. "They elected to adopt the latter. . . . But they say, 'We are "'perfectly willing to pay, but we will pay when the lady "'attains 60 years according to the policy'; and they suggest "that if they are made to pay when the lady actually attains 60 "years it is making a new contract. It seems to me that it is "the defendants who are asking to make a new contract. . . . "The defendants must be taken to have elected to keep the "policy in force as it stood, and to pay when the lady attained "the age of 60 years." It is pointed out by Mr. Wark that the result would have been different if, as is often the case, there had been a provision in the policy that if the age should prove to have been incorrectly stated, the assurance should be valid only to a certain specified extent. This remark, and some other of the cases referred to in the course of this paper, suggest that the officials and legal advisers of life assurance companies would do well to keep a watchful eye upon decisions bearing directly or indirectly on life assurance matters, with a view to modifying their practice and documents where such decisions show a change to be necessary or expedient.

Renewal Premiums.

Death during days of grace and before payment of premium.

In connection with the subject of payment of renewal premiums, two recent cases of interest may be noticed. The first of these is the well-known one of Stuart *v.* Freeman 1903 1 K.B. 47. In this case, the premium was an annual one payable by quarterly instalments, with the

usual period of thirty days of grace. The plaintiff was the assignee of the policy, and on 18 August 1901 a quarter's instalment of the premium became due. The thirty days of grace expired on 17 September 1901, without the premium having been paid, but on the following day, 18 September, it was paid to a clerk of the company by the plaintiff, and a receipt was given for it. The assured died on that same day, a few hours before the premium was paid and accepted, but the fact of his death was not known to either party to that transaction. There was evidence that the plaintiff, who was an insurance broker, was entitled to make the payment on the next day; and the jury found that this course had been assented to by the company's clerk, and that he had authority to make the arrangement. The company disputed the claim on the ground that at the time of the death of the assured the policy had become void by reason of the premium being in arrear, and that the policy could not be revived by payment after the death of the person insured. It was held: (1) That there was sufficient evidence as to the extension of the days of grace by one day, and that therefore the assured must be taken to have died, and the premium to have been paid, within the days of grace; (2) That the payment of the premium having been made in accordance with the terms of the policy, it had not lapsed, and the company was therefore liable. It is evident from remarks made by Romer and Mathew, LL.J., that they were prepared to hold, if necessary, that the policy was also good on the ground that the assured had died within the days of grace. Thus Romer, L.J., said: "Presumably, therefore, the policy is to be valid and continuing, "even though the person whose life is assured died within the "thirty days and the premium has not been paid before his "death." It should be noted, however, that the judgment turned to a great extent on the fact of the premium in question being an instalment premium, and on that ground it was distinguished from Pritchard *v.* Merchants' Life Assurance Society 1858 3 C.B. N.S. 622, where an annual premium was involved. It cannot, therefore, perhaps, be cited as deciding definitely a similar question where annual or true half-yearly or quarterly premiums are involved, although some remarks by Mathew, L.J., seem to indicate that he would have taken the same view, even if premiums of these descriptions had been in question. The matter is, of course, of merely academic interest to most companies as it is, I believe, the

Instalment premiums and true fractional premiums.

almost universal rule to admit the claim as a matter of course if the death occurs during the days of grace and before the payment of the premium due, the latter being deducted from the amount payable if not previously paid.

Acceptance of premium under conditions.

The other case referring to renewal premiums, to which I wish to direct attention, is that of Handler *v.* Mutual Reserve Fund Life Association 1904 90 L.T. 192. This was a case of a policy subject to the usual conditions as to renewal. The policy lapsed owing to a mortuary call not having been paid within the days of grace; and when it was subsequently paid, a receipt was issued, stated to be subject to the conditions endorsed on it. These conditions included one to the effect that the member then was, and had been during the preceding twelve months, in continuous good health and free from all diseases. As a matter of fact, when the mortuary call was paid, the member was known to be suffering from an incurable form of heart disease; but the plaintiff gave evidence that he had not read the policy or the calls or the receipt. At the trial of the case at Liverpool Assizes, judgment was given for the plaintiff, but this was reversed by the Court of Appeal. The view taken by the latter Court is clearly expressed by Romer, L. J., who said: "When the payment was made, or purported to be "made, of the mortuary call, after the thirty days had elapsed, "there was nothing whatever which could possibly have induced "the plaintiff to suppose, or justified him in supposing, that the "premium would be unconditionally received. . . . When he "sent this money after the time within which it ought to have "been paid, he received a communication with reference to that "payment. He had no reason to suppose that it was a mere "formal acknowledgment of the money, without more. Under "the circumstances, it is clear that he was bound to look to see "on what terms the money payment had been received; and if "he had looked, he would have found it." This view with regard to life assurance companies is, perhaps, somewhat more favourable to them than the series of decisions relating to conditions printed on railway tickets has been to railway companies, but not unreasonably so, for, as may be gathered from Lord Justice Romer's remarks, there is much more, in the case of a lapsed policy, to put the person paying the premium upon enquiry as to the conditions on which it is received, than there is in the case of an ordinary contract with a railway company.

Extension of rebate of income tax in respect of life assurance premiums.

In connection with the subject of the payment of premiums, the alterations made by the Finance Act 1904, and the Revenue Act 1906, as to the remission of income tax in respect of the amount paid for life assurance premiums to an extent not exceeding one-sixth of the income, may be noticed. As is well known, it had been held, in the case of Colquhoun *v.* Heddon 1890 25 Q.B.D. 129, that the provisions of section 54 of the Income Tax Act 1853 as to such remission did not extend, at any rate, to assurances effected with companies other than those of the United Kingdom. By the Finance Act 1904 section 9 it is provided that "Section 54 "of the Income Tax Act 1853 shall apply in relation to life "insurances or contracts for deferred annuities, effected in or "with any insurance company legally established in any British "possession, as it applies in relation to life insurances or contracts "in or with the insurance companies mentioned in that section." This extension to Colonial and Indian companies was followed by a further extension to foreign companies. By the Revenue Act 1906 section 11 it was enacted that "The provisions of "section 9 of the Finance Act 1904 shall apply, in relation to "life insurances or contracts for deferred annuities, effected in or "with any insurance company lawfully carrying on business in "Great Britain or Ireland."

Assignments and Charges.

Is production of the deed, notice under the Act of 1867 ?

The first questions to be considered under this head are those relating to notice. With regard to notice under the Policies of Assurance Act, 1867, the only matter on which I propose to touch is as to whether the production of the deed of assignment is sufficient notice within the meaning of section 3 of that Act. Personally, I have no doubt whatever that it is, and I should not have thought it worth while to raise the point were it not that quite recently, in a case that came under my notice, the opposite view was taken by a solicitor of considerable experience in life assurance matters. In these circumstances it seemed to me that it might be advisable to discuss the matter briefly, as a similar view may be held in other quarters. The Act says that the statutory notice shall be "a written notice of the date and purport of such assignment", and it is difficult to see on what grounds the production of the deed, necessarily in writing and containing these particulars, can be said to be an insufficient compliance with the Act. There

has been, I believe, no direct decision on the point, but some light is thrown upon the matter by a consideration of the Transmission of Moveable Property (Scotland) Act, 1862. That Act provides for the assignment of certain forms of moveable property, including "policies of assurance of any assurance "company or association in Scotland, whether held by parties "resident in Scotland or elsewhere" and enacts, in section 2, that "an assignation shall be validly intimated (1) By a notary "public delivering a copy thereof, certified as correct, to the "person or persons to whom intimation may in any case be "requisite, or (2) By the holder of such assignation or any "person authorized by him, transmitting a copy thereof, certified "as correct, by post to such person." In the article on Life Insurance, in the Encyclopædia of Scots law, vol. 8, page 103, the author, Mr. William Harvey, in quoting the above Act by way of comment on the Act of 1867, and expressing the opinion that notice under the former Act would be a sufficient compliance with the latter Act, says that: "The company would thus "receive intimation in writing of the date and purport of "assignation, which is all that is required by the Act of 1867." If this view, with which I entirely agree, be correct, then, since the Act of 1867 applies equally to Scotland and England, an intimation in accordance with the Scottish Act of 1862 would be sufficient in England; and if the production of a certified copy be sufficient, *a fortiori*, the production of the deed itself is sufficient. The question does not often arise in the ordinary course in connection with the intimation of assignments, as in England at any rate, written notice of the deed is usually forwarded in the first instance, though not infrequently in Scottish cases, a copy of the deed is forwarded as notice. Moreover it is, I believe, the practice, where the deed itself or a copy is forwarded by way of notice, to ask for the ordinary office form of notice to be completed. It sometimes happens, however, that the first intimation as to the existence of an assignment is the production of the deed with the application for a payment under the policy; and the office then has to consider whether such production is sufficient notice to protect it under the Act of 1867, or whether it is necessary to insist on a formal written notice, in addition. I do not think, however, that there need be, or is, much hesitation in waiving this latter formality.

While dealing with the subject of notice, I may call attention to two modern cases which, though in no way relating to

notice under the Act of 1867, are concerned with the general doctrine of notice, and are of interest to assurance companies in connection with their investments in reversionary securities.

Notice to trustees in reversionary transactions.

The first of these is that of *In re* Wasdale, Brittin *v.* Partridge 1899 1 Ch. 163. The particulars of this case are as follows: In 1871 a reversioner sold his reversionary interest to one Partridge, who at once gave notice to all the existing trustees. In 1881, the survivor of these trustees died, and his widow and executrix acted as trustee. In 1896, the plaintiffs were appointed trustees. In 1897, the reversioner mortgaged his reversion to one Bell, who gave notice to the plaintiffs, the then trustees. When the interest fell into possession, it was claimed by both Partridge and Bell. Stirling, J., in delivering judgment in favour of Partridge, said: "In my "opinion it would be imposing an unreasonable burden on "assignees, to hold that they are under an obligation to give "notice to every new trustee, or are to be treated as guilty of "neglect to take reasonable precautions, if they omit to do so."

Notice to a single trustee.

The other case on the subject of notice is that of *In re* Phillips' Trusts 1903 1 Ch. 183. Here a reversionary interest was settled under a marriage settlement in 1882. The solicitor who drew up the settlement was one of the three trustees of the will under which the reversionary interest arose, and he therefore had notice of the settlement of it; but it did not appear that he ever communicated his knowledge to his co-trustees, and no formal notice was ever given to the trustees. In 1887, two of the trustees, including the above-mentioned solicitor, having died, two persons were appointed in their place. In 1894, the reversioner sold her reversionary interest to the Law Life Assurance Society, who inquired of the trustees as to whether any notice had been received by them, but failed to elicit any information as to the settlement of 1882, as none of the then trustees had received any notice of it. On completion of the purchase, notice was given by the assurance society to each of the trustees. When the reversion fell in, the trustees paid the money into Court, and it was held that the assurance society was entitled to the amount. In delivering judgment, Kekewich, J., quoting with approval the words of Byrne, J., in the case of Freeman *v.* Laing 1899 2 Ch. 355, said: "It has also been held that an assignee who has given notice to "one only of several trustees, is not entitled to priority over a

" subsequent assignee who takes his assignment after the death " of the trustee to whom notice has been given."

Passing now to the consideration of various points connected with the subject of assignments and charges, the first question I propose to consider is as to the necessity for a reassignment, where a policy has been formally assigned by way of mortgage and the debt has been paid off. Considerable difference of opinion appears to exist on this point. The general rule, at any rate of English law, is that where property has been assigned by deed and it is desired to cancel the operation of that deed, a reassignment should be executed in the same form; and provided this rule is applied with discretion to particular cases, I think it is a safe rule to follow. Dr. A. E. Sprague, in his recent paper, takes the view, apparently, that a formal reassignment of a mortgaged policy is unnecessary in any case, but many English actuaries, while holding the view that such a reassignment can safely be dispensed with in certain cases, would not be prepared to agree with so wide a generalization as this. The question of the possession of the legal interest in addition to the beneficial interest in such property as a policy of assurance is, of course, of far less importance than that of the possession of the legal estate in realty. Nevertheless, there are transactions in regard to policies in which it is generally considered advisable for the assurance company to require that the person with whom they are dealing shall be in a position to show a good title both at law and in equity to the policy; and after a formal mortgage this can only be done by means of a proper reassignment. The question is usually considered of greater importance in transactions, such as loans and payments of the cash value of the reversionary bonus, which leave the policy still in force, than in what may be termed final transactions, such as payment of surrender-values and claims; and many who would insist on a formal reassignment in the former class of cases, would readily dispense with it in the latter class, on evidence that the debt had been repaid.

Is a formal reassignment necessary?

For the benefit of those who are anxious in every case to get a good legal as well as equitable title, I may call attention to a point that has come before me on more than one occasion. There are, in the third schedule of the Conveyancing Act, 1881, certain short forms of deeds given, described as Deeds of Statutory Mortgage, Statutory Transfer

Statutory mortgages, transfers and reassignments.

and Statutory Reconveyance respectively. If all these forms are used in a transaction, the effect is to pass the legal interest in each case, and so, ultimately, to revest it in the mortgagor. It sometimes happens, however, that the ordinary form of mortgage is used, followed by the statutory forms of transfer and reconveyance, or only by the latter. In such a case, the legal interest does not pass under the transfer or reconveyance, and as far as such legal interest is concerned, the position is no better than that of a mortgage followed by a somewhat formal receipt for the mortgage money.

Deeds executed by liquidators.

Another point that may conveniently be referred to here, is that of deeds executed by the liquidators of a company in liquidation. In these days of bank amalgamations, it is not a very unusual experience to find when a policy has been mortgaged to a bank, that before a reassignment has been executed the bank has gone into voluntary liquidation, with a view to the transfer of its business to another bank. In such circumstances, I have sometimes had produced a reassignment executed only by the liquidators in their own names; and it is therefore of importance to note that, unlike the case of a trustee in bankruptcy, the property of the company in liquidation does not vest in the liquidators; and the reassignment should therefore also be executed by the company under its common seal, which is affixed by the liquidators acting as agents for the company. It may also be pointed out that where, as is commonly the case, there are two or more liquidators, in the absence of an express provision enabling one to act alone all must join in any transaction, except perhaps, one of a purely ministerial nature; and a reassignment completed by one of the liquidators alone is not therefore valid, and this is so even though the other liquidator be dead.

Advance on joint account.

Where a mortgage has been made to two or more persons without specifying that the money has been advanced either on joint account or in shares, and one of the mortgagees has died, it is usual, in reliance on section 61 of the Conveyancing Act 1881, either to pay to, or to accept a reassignment from, the survivors, on production of evidence of death of the deceased mortgagee. It seems quite clear from the words of the section, coupled with the notes on it in Wolstenholme's Conveyancing Acts, 1905, 9th edition, page 126, that such a course is a safe one. I may say, however, that a short time ago, I heard a very eminent conveyancer suggest doubts

as to the safety of this course; and he expressed the opinion that notwithstanding section 61, the legal personal representatives of the deceased mortgagee should, in such circumstances, be parties to the discharge or reassignment as the case may be.

Constructive notice of trust.

When a mortgage is made to persons who are, as a matter of fact, trustees, it is usual to keep the fact of the trust off the title by using the ordinary joint account clause. When new trustees are appointed, it is necessary to transfer the property to them, and it is customary to stamp such a transfer with 10*s*. It is however pointed out by Wolstenholme, on page 127, that it is advisable to have such a transfer stamped with an ad valorem stamp, as if the debt exceeds £2,000, the deed cannot be shown to be properly stamped with a 10*s*. stamp without disclosing the trusts; and that even if the 10*s*. stamp were adjudicated, it is questionable whether there would not be constructive notice of the trust. I may point out that the Law Society's Conveyancing Bill now before Parliament proposes, *inter alia*, to remedy this difficulty.

Discharge for policy moneys by equitable mortgagee.

In my previous paper I discussed the question as to whether a good discharge could be given for the policy moneys by a mortgagee whose security was either an equitable charge or an assignment by way of mortgage under hand only; and I pointed out that some offices, at least, taking the view that they are sufficiently safe in paying under such securities, do not require the concurrence of the legal personal representatives of the mortgagor in the receipt. This question has recently been discussed by Dr. A. E. Sprague, on page 350 of his paper, already referred to, where he takes a different view from that stated above. As a matter of fact, there is probably not very much difference in practice between the holders of the two views, for it is, I believe the practice, even of those who think that no appreciable risk is run in dispensing with the concurrence of the mortgagor's legal personal representatives, to ask for it, and only to dispense with it where an insistence would lead to unreasonable difficulty or delay. Probably, in like circumstances, those who take the opposite view would be prepared to waive what they may consider to be the strict legal requirements of the case. I called attention in my paper to a view, held by some competent authorities, that section 22 of the Conveyancing Act, 1881, which confers on mortgagors power to give a discharge for money arising under the mortgage, was not expressly limited to mortgages by deed, as

were some of the neighbouring sections. While I still think that the views referred to are entitled to very great respect, it is only right that I should call attention to a remark in Wolstenholme's Conveyancing Acts, page 81, where, in a note on section 22, it is stated: "It is conceived that this section applies only to a "mortgagee whose mortgage is by deed, after 1881. The "supposed mortgagee is one who has a power of sale conferred "by this Act." Such an expression of opinion as this will, of course, carry great weight, and may perhaps be considered as turning the scale in favour of the view that an equitable mortgagee has not, at any rate under section 22 of the Conveyancing Act, 1881, power to give a complete discharge at law and in equity for money arising under his security.

Assignment by way of mortgage under hand only.

The difficulties discussed in the last paragraph refer only to the case where the security is in the form of an equitable charge. With regard to the mortgagee whose security is an assignment by way of mortgage under hand only, the case is different. Even if it be held, for the reasons referred to above, that such a security, not being by deed, does not come within section 22 of the Conveyancing Act, 1881, nevertheless, in accordance with the decision in Tancred *v.* Delagoa Bay Railway Company 1889 23 Q.B.D. 239, it does come within section 25, subsection 6, of the Judicature Act, 1873. That section says that "Any absolute assignment by writing "under the hand of the assignor (not purporting to be by way of "charge only) of any debt or other legal chose in action, of which "express notice in writing shall have been given to the debtor, ". . . shall be, and shall be deemed to have been, effectual in "law . . . to pass and transfer the legal right to such debt or "chose in action . . . and the power to give a good discharge "for the same without the concurrence of the assignor." For many years after the Act was passed this section was considered as only applying to what are commonly known as absolute assignments; but in the case quoted above it was decided that an assignment by way of mortgage with the ordinary proviso for redemption is an absolute assignment within the meaning of section 22, subsection 6, of the Act. This Act, however, confers no power of sale, and such an assignment as that referred to here, will not therefore do so, apart from any express powers that may be contained in the instrument.

The next group of questions with which I propose to deal is that relating to a mortgagee's power of sale. A considerable

number of the difficulties in connection with assignments arise under this heading; and it is not therefore surprising to find several matters relating to this subject which call for discussion.

Does power of sale include power of surrender?

The first matter to be considered in this connection is the somewhat ancient one as to whether power of sale includes power to surrender a policy of assurance comprised in the security. I was under the impression that this question had long since been settled affirmatively, and I should not now refer to it were it not that quite recently I found the opposite view apparently held by a solicitor actively engaged in life assurance practice. There seems, however, to be no doubt that power of sale does include power to surrender, and in Prideaux's "Precedents in Conveyancing", 1904, 19th edition, vol. i, page 623, in a precedent for a mortgage of a life policy, although the usual clause expressly extending the power of sale to a surrender of the policy is included, it is stated in a note that "This declaration is inserted *ex abundanti cautelâ*, it being free "from all reasonable doubt that a surrender to the office is a "mode of sale authorized by the statutory power."

Power of sale under a charge by deed.

Reference has already been made to the powers of an equitable mortgagee to give a discharge for the policy moneys. A somewhat similar question arises where the mortgage is only a charge and not an assignment, but is under seal. Such a form of security is used by some banks, and may or may not contain an express power of sale. In either case it is quite clear that it comes within the definition of a mortgage as given in section 2, subsection 6, of the Conveyancing Act, 1881, and, being by deed, confers the statutory power of sale. Nevertheless, since the legal interest has not been transferred, the mortgagee cannot pass such interest on surrendering the policy; but it is, I believe, usual in such cases to disregard this fact and treat the mortgagee, for purposes of surrender, as if he held an assignment by way of mortgage under seal.

Evidence as to power of sale being properly exercisable.

On an application being made by a mortgagee for the surrender-value of a policy, it is, I believe, common for assurance companies to require evidence as to the power of sale being properly exercisable, such evidence varying from a statutory declaration to a simple letter, according to circumstances. In asking for this evidence, it is not unusual to be met with an objection on the ground that the company is fully protected by the provisions of section 21, subsection 2, of the Conveyancing Act, 1881, and is not therefore entitled to ask

for such evidence. A recent case dealing with this point is that of The Life Interest and Reversionary Securities Corporation *v.* The Hand-in-Hand Insurance Society 1898 2 Ch. 230. Here the action was brought by the plaintiffs, as vendors, for specific performance of an agreement for the sale by them, as mortgagees of a reversionary interest. The defendants objected that it had not been shown that power of sale was properly exercisable. The plaintiffs then urged that under section 21, subsection 2, of the Conveyancing Act, 1881, a *bonâ fide* purchaser for value without notice was fully protected, and was therefore not required or entitled to raise the question as to such power being properly exercisable. It was, however, pointed out by Stirling, J., that the clause in question gives protection only to a purchaser who has obtained a conveyance, and he went on to say: "It appears to me that this provision is not "applicable until after a conveyance is made, or, in other words, "that the three sections of the Act which I have read (section "19, subsection 1 (i), section 20, section 21, subsection 2) "confer protection only on a purchaser who has obtained a "conveyance without knowledge of any irregularity, and conse-"quently that it is left open to the purchaser, as between "himself and the vendor, to raise any question which may "properly be raised between a vendor and a purchaser. The "provisions of section 21 are, I think, for the protection of the "purchaser and not for the benefit of the vendor. In my opinion, "therefore, a mortgagee vendor, selling under the power conferred "by this Act will be bound, notwithstanding clause 2 of section "21, upon the request of the purchaser, to show that his power "of sale was exercisable." Experience shows that when objections such as those referred to are made, a reference to this case is always sufficient to overcome them; but it may be pointed out that the Law Society's Conveyancing Bill already referred to, proposes to do away with the effect of this decision or, as they prefer to express it, "provides for the difficulty disclosed in the case."

Can power of sale be exercised by the assignee of the mortgage?

It is pointed out in Coote's "Law of Mortgage", 1904, 7th edition, page 906, that "It is important that an "express power of sale in a mortgage deed should be "so framed as to be reserved in terms to the assigns of the "person or persons in whom the power is vested, so as to enable "an assignee of the mortgage effectually to exercise the power. "For otherwise he is unable to exercise the power, and the

" circumstance that the assigns are empowered to give a receipt " to a purchaser has been held, according to the strict construc- " tion that has been applied to these powers, not to extend the " power of sale to them." The need for this caution can be seen by a reference to the case of *In re* Rumney and Smith 1897 2 Ch. 351. Here a member of a building society mortgaged property to the trustees of the society by a deed conferring on those trustees express powers of sale of an ample nature, and it was declared that the receipt of the trustees should be an effectual discharge to the purchaser for the purchase money. On a transfer by the trustees and a proposed sale by the transferee, it was held, *inter alia,* that the power of sale was exercisable by the trustees exclusively and not by their assigns. Stirling, J., in delivering judgment, said : " I am asked to hold that the power " of sale contained in the mortgage deed . . . is exercisable, in " the absence of any contrary intention, by any person who, in " equity, can give a receipt for the mortgage money. I am far " from saying that that would not be a reasonable state of the " law, but the question is whether it is the present state of the " law. . . . I think that the view of the Court down to the " present time has been that trusts for sale and powers of sale " can only be exercised by the person or persons authorized so to " do by the instrument which creates them." It was accordingly held that the proposed vendor had no power to sell, and this was confirmed on appeal without calling on counsel for the purchaser to argue the case. It must, however, be noted that this decision only affects express powers of sale, and does not apply to the statutory power, for as Stirling, J., pointed out, " The Legislature, " in the Conveyancing Act, 1881, section 21, subsection 4, has " provided that the power of sale conferred by that Act may be " exercised by any person for the time being entitled to receive " and give a discharge for the mortgage money. But that " provision is limited to the power of sale conferred by the Act, " and does not extend to the express power of sale conferred by " the mortgage deed with which I have to deal."

Power of sale exercised by attorney.

Another important case dealing with the exercise of power of sale that may be compared with this, is that of *Re* Dowson and Jenkins' Contract 1904 2 Ch. 219, which, among other things, emphasizes the fact that the terms of powers of attorney have to be scrutinized with very great care and exactness. In this case a mortgagee gave a power of attorney conferring on his attorney power, *inter alia,* " to sell " any real or personal property now or hereafter belonging to

" me." On the attorney selling under the mortgagee's power of sale, the purchaser objected, on the ground that the power of attorney was insufficient to authorize the sale of property held by the principal as mortgagee; and this view was upheld by the Court and confirmed on appeal. Vaughan Williams, L.J., in the Court of Appeal, said: " The words relied on are ' also to sell " ' any real or personal property now or hereafter belonging to " ' me.' . . . These words are not wide enough . . . But the " deed also contains a power to receive all sums of money . . . " now or hereafter owing . . . by virtue of any security, and to " give . . . receipts, releases and other discharges for the same " respectively . . . Under the Conveyancing Act, 1881, " section 21, subsection 4, ' The power of sale conferred by this " ' Act may be exercised by any person for the time being " ' entitled to receive and give a discharge for the mortgage " ' money.' But we are of opinion that these words do not " extend to everybody who has power to give a receipt and " discharge. The subsection does not include a person who has " that power merely as an agent; but is intended to apply only " to those persons who are either mortgagees, or have vested in " them the property of the mortgagees, such as the executors." Of course the case must not be taken as deciding that the attorney of a mortgagee cannot exercise power of sale, but only that the words used, which are very common in powers of attorney, are insufficient for the purpose. It is quite possible to draw up a power of attorney enabling this to be done; and as a matter of fact, Vaughan Williams, L.J., said, in the course of his judgment: " I may point out that really there was no difficulty " in providing a power of attorney which should, in plain terms, " have given this power to the attorney."

Power of sale exercisable on conditions.

By way of contrast to the above, the case of Berry *v.* Halifax Commercial Banking Company Limited 1901 1 Ch. 188, may be quoted as showing that in interpreting the conditions on which power of sale is exercisable, the Court will sometimes take a broader view than when construing powers of attorney. In this case a life policy was mortgaged by the assured, by a deed dated 20 July 1889, to the defendants, to secure his banking account. The mortgage provided that the statutory power of sale should be exercisable if default were made in payment of the balance due for the space of one month after the account had been closed. On 9 November 1899, the assured informed the bank that he had

called his creditors together, and that a trustee had been appointed; and that he hoped everyone would get twenty shillings in the pound. On 17 November 1899, he assigned all his property to a trustee for the benefit of his creditors. There was no formal closing of the account, but on 18 December 1899, under the powers conferred by the mortgage of 20 July 1889, the bank sold the policy. The assured subsequently died, and his legal personal representative brought an action against the bank, on the ground that power of sale had not arisen when the policy was sold. In giving judgment for the bank, Kekewich, J., said: "I now come to what I think did amount to a closing of "the account under the mortgage, namely, the letter of "9 November. . . . If that letter means anything, it surely "means that there is an end of the transactions between the "parties . . . It was a peculiar relation constituted by that "deed, and that, obviously, had come to an end . . . It seems to me "that on 9 November there was an end of the whole transaction; "and the power of sale arose or was exercisable within a month "of that date." This case is of some practical importance, as more than one bank has a somewhat similar form of mortgage to that involved here; and it is by no means unusual, when enquiring as to the grounds on which it is claimed that power of sale is exercisable, to be met with the reply that the mortgagor has become bankrupt, or has executed an assignment for the benefit of his creditors, or is otherwise in financial difficulties. Such an answer, of course, does not help matters if the statutory power of sale is relied upon; but in the light of this case it may possibly be of service when a special power of sale of this nature is in question.

Foreclosure.

In connection with the exercise by a mortgagee of his power of sale, reference may be made to another remedy of a mortgagee, that of foreclosure. This remedy is not very often employed where a policy of life assurance is concerned, the usual course, in case of default, being to exercise the power of sale; indeed it is stated in Strahan & Kenrick's "Digest of Equity", 1905, page 278, that "There is no right of foreclosure "in the case of a mortgage of a chose in action", in which category a policy of assurance is included. This view is not, however, apparently held by other authorities on the subject of mortgage. Thus, in Coote's "Law of Mortgage", 1904, 7th edition, page 1018, it is stated that, "As a general rule, the "Court will give the benefit of foreclosure in every case where

"money is lent on a security of the nature of a mortgage"; and in Fisher's "Law of Mortgage", 1897, 5th edition, page 471, it is laid down that, "Foreclosure is a remedy of the mortgagee "which is not confined to mortgages of land. It is equally "applicable to mortgages of chattels and choses in action." These opinions seem conclusive on the question as to whether the remedy of foreclosure is available in the case of a mortgage of a policy of assurance; and, as a matter of fact, I have had before me, on more than one occasion, a foreclosure order in respect of such a mortgage, though I believe in each case the mortgage comprised other property in addition to the policy. If, however, further evidence on this point be desired, the case of Beaton *v.* Boulton 1891 W.N. 30 may be considered, I think, as putting an end to all doubt. Here the first mortgagee of a policy of life assurance obtained the usual foreclosure order *nisi*. After the time finally fixed for payment of principal, interest and costs had expired, but before the order was made absolute, the assured died, and the policy moneys became payable; and this fact was known to the Court when the order was made absolute. The parties entitled to redeem then applied for the foreclosure to be opened, as the amount of the policy moneys exceeded that due to the first mortgagee; and Stirling, J., held that the foreclosure must be opened, and ordered a subsequent account and a further period of one month for redemption.

Evidence in case of foreclosure.

When an application is made for payment in respect of a life policy under an order for foreclosure, the evidence produced will be the order for foreclosure absolute, and this has the effect of an absolute assignment of the property to the mortgagee. I have, however, seen an application for payment under an order for foreclosure *nisi*, on the ground that when the date fixed in the order for payment of principal, interest and costs had passed without such payment having been made, the order had the force of an absolute order. Such a title cannot, of course, be accepted, more especially in view of the fact that it is by no means rare for the time of payment fixed in the order to be extended.

Opening the foreclosure.

In view of Beaton's case, quoted above, it would perhaps be advisable, if the policy should have fallen in before the order was made absolute, to make enquiries as to whether the foreclosure had been opened, or whether any proceedings to that end were being taken. It is stated in Fisher on Mortgages, page 929, that: "Although the

"Court will not open the foreclosure against purchasers who "have bought a considerable time after the date of the order "absolute, and without notice of facts which would lead the "Court to interfere, it will not hesitate to do so if the purchase "be made shortly after the order, and with notice of that which "would affect the mortgagee's right to an absolute title under "the order." The same view is taken in Coote on Mortgages, page 1072, where it is said that: "The Court will be loth to "disturb a person who purchases many years after the date of "the order, with no notice of any circumstances that might lead "to opening the foreclosure. But the case is otherwise with a "purchaser who buys the estate shortly after the date of the "order, especially if he has notice of any such circumstances; "in such a case the purchaser must be taken to know that the "foreclosure is liable to be opened." In most of the cases that are likely to occur in connection with a life office, however, the mortgagee could, in any event, give a good discharge as mortgagee, even without the foreclosure order; and the question of the possible opening of the foreclosure is not therefore likely to arise.

Policy included in bill of sale.

An interesting question in connection with assignments which I have had before me on more than one occasion, is as to the position of the parties to a bill of sale when a life assurance policy has been included in the schedule. The matter has quite recently been before the Court in the case of Swanley Coal Company *v.* Denton 1906 2 K.B. 873. Here a bill of sale contained in its schedule, *inter alia,* an assignment of a lease, and it was contended that the inclusion of this assignment rendered the bill void. The County Court judge, before whom the case came in the first instance, held that the title deeds were merely personal chattels, and that their inclusion in the schedule did not avoid the bill of sale. This decision was reversed by the Divisional Court, but restored by the Court of Appeal by a majority of two to one. In delivering judgment in the last-named Court, Vaughan Williams, L.J., said: "If the "security included a chattel interest in land, it was not arguable "but that the bill of sale was void; but if, on the other hand, it "included only personal chattels, it was a good bill of sale." Cozens-Hardy, L.J., in dissenting, did so on the ground that: "The bill of sale was manifestly intended to give a charge on "the land represented by the deeds, and if this were so it was "admitted by all that the bill of sale was void." The result of

this case is, that where a policy is included in the schedule to a bill of sale, it does not render the bill void, but at the same time it does not create any charge on the contract of assurance represented by the policy, though it may create a charge on the actual piece of paper on which the policy is written and printed. The bill of sale could not even be used as evidence to prove that the policy was deposited with the intention of creating a charge on it; for to the extent to which the bill showed this, it would itself be void altogether as creating a charge on something other than a personal chattel.

Assignment of a policy when, unknown to the parties, the assured is already dead.

A case relating to the assignment of a life policy which created some interest a short time ago, was that of Scott *v.* Coulson 1903 2 Ch. 249. Here the plaintiff, in 1902, sold a policy of life assurance to the defendant, neither party knowing, when the agreement for sale was entered into, that the assured was really dead, he having died in 1899. It appeared in the course of the case that between the date of the contract for sale and the actual assignment, the defendant received information leading him to believe that the assured was dead at the date of the contract; but the fact of the death did not become actually known to him until after the date of the assignment. It was held by Kekewich, J., that the vendors were entitled to have the transaction set aside, although it had been completed by assignment; and this decision was affirmed by the Court of Appeal. In delivering judgment in the latter Court, Vaughan Williams, L.J., said: "The material date all "through is the date of the contract. If at that date a good "contract was entered into, I cannot conceive that it could be "rescinded. But it turns out that it was a contract entered into "under a common mistake, existing at the date of it; and "therefore it follows that an assignment executed in pursuance "of such a contract cannot be supported." Romer, L.J., in arriving at the same result, based his conclusion on a somewhat different ground. He said: "The contract entered into between "the parties . . . rested upon the basis of the assured "being still alive. It turns out that before the matter was "concluded by assignment to the defendants, the fact upon "which the contract was based was not the fact. The defendant "Coulson must be taken to have known that the basis upon "which the contract had been entered into, and the common "belief upon which both parties had acted, did not exist. That "was a circumstance which went to the root of the matter, and

"rendered it improper to insist upon the completion of the "contract."

Effect of Statutes of Limitation on policies.

Two cases relating to assignments of life assurance policies, which can hardly be classified under any of the preceding heads, and in each of which a somewhat curious point arose, may be noticed here. The first of these is the case of Charter *v.* Watson 1899 1 Ch. 175, and deals with the question of the effect of the Statutes of Limitation on a mortgaged policy. Here, realty and a life policy were included in the same mortgage, and the mortgagor's right to redeem the realty had become barred, by reason of the mortgagee having been in possession of it for more than twelve years without any acknowledgment of the mortgagor's title. The plaintiffs, who represented the mortgagor, claimed, nevertheless, that they were entitled to redeem the policy; and the defendants pleaded by way of defence, *inter alia,* the Statutes of Limitation. In delivering judgment for the defendants, Kekewich, J., after pointing out that there was, apparently, no existing authority bearing directly on the point, said: "I am asked to apply by analogy a statute which, in "distinct terms, only touches land, to an entirely different "property, that is to say to a policy of insurance, which is "personal estate. . . . I do not see my way to act upon "any such principle. . . . I propose to deal with the case "on a somewhat different footing, as raising a novel question, "and strictly on principle. There is one indivisible security for "one indivisible amount. The right of redemption is not as to "one part of the property, but as to the whole, and the result is "that the right is gone as to the whole; it is gone as much as "regards the policy of insurance as it is as regards the land; not "because the right to redeem the policy is barred by statute, or "by applying the analogy of the statute, but because, it "being impossible at law for the mortgagor now to require a "reconveyance of the land, it is equally impossible for him to "require a reassignment of the policy, the land and the policy "being on one footing, and being, together, the security for "the debt. . . . It comes back to this; because the land "and the policy make one security, and the plaintiffs cannot "have back the land alone, they cannot have back the policy."

Custody of deed when realty and a policy are included in the same deed.

The other case is that of *In re* Williams and Duchess of Newcastle's Contract 1897 2 Ch. 144, and also arises through land and policies being included in the same deed. In this case, a mortgage

comprised land and some policies on the life of the mortgagor. The mortgagee sold the land under his power of sale, and the purchaser claimed to be entitled to the possession of the mortgage deed, with the other documents of title, although the mortgagee retained the policies which were held under the same deed. In giving judgment for the purchaser, North, J., said: "No doubt the fact that the policies were included in the deed "would have been a reasonable ground for the vendor's inserting "a special condition that the vendor should retain the mortgage "deed while the policies subsisted. . . . In my opinion, "rule 5 of section 2 (of Vendor and Purchaser Act, 1874), when "it speaks of the vendor retaining part of an estate, clearly "refers to part of an estate in land. . . . In the present "case the vendor does not retain any part of such an estate, and "therefore he is not entitled to retain the mortgage deed. The "deed must be delivered to the purchaser, and she must give to "the vendor the acknowledgment prescribed by section 9 of the "Conveyancing Act, 1881."

In concluding the subject of assignments and charges, attention may be called to certain forms of involuntary assignment, or assignment by operation of law, which arise from some condition of disability. Of course, by far the most important of such assignments are those arising in connection with bankruptcy and death, but on account of the importance of these, I propose to deal with them in separate sections. There are, however, two forms which are occasionally met with, and which, by reason of their less frequent occurrence, are not so well known as those mentioned.

Lunacy.

The first of these arises in the case of lunacy. On a policyholder becoming insane, it is often desired to deal with the policy by way of surrender or loan, or by surrendering the bonuses for cash, in order to pay the premiums. The application in such a case is usually made by some relative of the lunatic; but in the absence of power conferred by the Court, it usually has to be refused, and a suggestion made that some person should be appointed to deal with the lunatic's property. If there be a committee, he is the proper person to act, but in the case of small estates there will rarely be a committee, as such an appointment is only made in the case of a lunatic so found by inquisition. Without an inquisition, however, the Court has power, under section 116, of the Lunacy Act, 1890, to appoint a person with the powers of a committee, such appointment being

usually made under subsection (*c*), which confers this power in respect of "Every person lawfully detained as a lunatic, though "not so found by inquisition." It is to be noted that the appointment, whether of a committee or of a person having the powers of a committee, will not, in itself, enable such person to deal with any portion of the lunatic's property, and that if it be desired to do so, an order to that effect must be obtained. Under section 117 (1) the Court may order that any property of the lunatic, whether present or future, be sold, charged, mortgaged, dealt with or disposed of, in such way as the Court may consider advisable. In conferring these powers, the Court will, as in most cases of the appointment of a receiver, require, as a rule, the person appointed to furnish security. Where this is required, it is necessary, before acting on the order of the Court, to see, in addition, the certificate of completion of security by the person so appointed. The powers of the person appointed to deal with the lunatic's property cease, of course, on the lunatic's death; and any question as to the property of a deceased lunatic will be dealt with by his legal personal representative in the ordinary way.

Convicts.

The other form of disability leading to an involuntary assignment, to which I propose to refer, is that which arises on the conviction of a person of certain offences. The provisions relating thereto are contained, chiefly, in the Forfeiture Act, 1870. When judgment of death or of penal servitude has been pronounced or recorded against any person by any court of competent jurisdiction, upon any charge of treason or felony, such person is described in the Act as a convict, and while a convict is subject to the operation of the Act, no action can be brought against him, and he is incapable of alienating or charging any property, or of making any contract. If it is desired to deal with a convict's property, an administrator can be appointed by the Home Secretary, and on such an appointment being made, all the real and personal property of the convict, including choses in action, to which he was entitled at the time of his conviction, or to which he becomes entitled while he is a convict, will vest in the administrator for all the estate and interest of the convict therein. The administrator has absolute power to let, mortgage, sell, convey and transfer all or any part of the property as he thinks fit. If no administrator has been appointed, some of his powers may be exercised by an interim curator, who can be appointed by any justices of the peace in petty sessions

assembled, or, where there are no petty sessions, by any justice of the peace having jurisdiction in the place where the convict, before his conviction, last usually resided. Such interim curator has power to sue for the possession and recovery of any part of the convict's property in respect of which he has been appointed, and to receive and give discharges for rents, dividends, interest, income and debts due to the convict. An interim curator has, apparently, no power to deal with the convict's real estate, and can only sell the personal estate on being authorized to do so by a court of competent jurisdiction; and his authority comes to an end on the appointment of an administrator. Any property acquired by a convict while lawfully at large under any licence, is vested in him and not in his administrator or interim curator. The Act has no operation when a convict dies, or is made bankrupt, or has undergone the punishment to which he is liable, or has been pardoned. It is also provided by section 48 of the Trustee Act, 1893, that as regards any property vested in a convict as trustee or mortgagee, such property shall not vest in his administrator, but shall remain in him as if he had not been a convict; but this is not to affect the title to such property so far as the convict is beneficially entitled. An illustration of the wide powers conferred on the administrator of a convict, and the way in which those powers should be exercised, can be seen in the recent case of Carr *v.* Anderson 1903 1 Ch. 90 2 Ch. 279.

If application is made by such an administrator to deal with a policy belonging to the convict, his title will consist of the certificate of his appointment signed by the Home Secretary. Such an administrator can give a discharge for any amount payable under the policy, and can either surrender or mortgage it. An interim curator will produce his appointment, made by justices of the peace in petty sessions or otherwise, and he can give a discharge for the policy moneys under that appointment. He can, however, only surrender the policy if expressly authorized to do so, and, apparently, he cannot, in any circumstances, mortgage it.

Bankruptcy.

Competition between depositee of policy and the trustee in bankruptcy.

The first case to be noticed under this head is one that does not directly affect life assurance companies, since it is concerned with a dispute as to the title to a life policy between a depositee and the assured's trustee in bankruptcy. It is, however, of some interest, and may therefore be quoted. The case is that of *In re* Wallis *ex parte*

Jenks 1902 1 K.B. 719, and the particulars are as follows: In March 1901 Wallis deposited a policy on his own life, with his wife as security for advances made by her to him. No notice of this equitable charge was given to the assurance society. On 11 October 1901 a receiving order was made against Wallis, and the same day adjudication followed. On 16 October the Official Receiver gave notice of the receiving order to the assurance society. Subsequently a trustee was appointed, and he now claimed, as against the wife, to be entitled to the policy as part of the property of the bankrupt, free from encumbrances. In deciding in favour of the bankrupt's wife, Wright, J., said: "It "is singular that there is no direct authority on this point. It "is plain that before the bankruptcy there was a good equitable "deposit of this policy for value by the bankrupt with his wife. "No doubt the general rule is that, as between several assignees "or encumbrancers of a chose in action, the assignee or encum-"brancer who first gives notice obtains priority. But the "trustee in bankruptcy is not an encumbrancer for value. "Under the bankruptcy laws he is a statutory assignee, and "this policy vested in him subject to all equities at the date of "the commencement of the bankruptcy. Therefore the trustee "could not, by giving notice to the assurance office, deprive the "bankrupt's wife of her rights as an equitable mortgagee of the "property. He can only have the policy on payment to the wife "of what is properly due to her under her security."

Deeds of assignment for benefit of creditors.

Two important cases dealing with deeds of assignment for benefit of creditors next claim attention. The first of these, that of Davis *v.* Petrie 1905 2 K B. 528, deals with the position of a debtor to the estate who makes a payment to the trustee within three months of the date of the deed. I pointed out in my previous paper that anyone making such a payment does so with notice of an available act of bankruptcy, and is therefore not protected in the event of a petition being presented within three months of the date of the deed. This view has, however, frequently been disputed by trustees under such deeds who were anxious to wind up the estate, and who contended that if such a payment were made it would be the trustee and not the person making the payment who would be liable in the event of subsequent bankruptcy. The question has now been set at rest by the case referred to above, in which the facts were as follows: One Watson executed a deed of assignment for the benefit of his creditors on 5 June 1903 in

favour of a trustee named Afford. The defendant, who owed Watson £21, paid this amount to the trustee at the latter's request. A petition in bankruptcy was subsequently presented against Watson, based on the deed of 5 June 1903, and on 20 August 1903 a receiving order was made against him, and he was thereupon adjudicated a bankrupt, and the plaintiff was appointed trustee of his estate. The £21 not having been paid over by Afford to the plaintiff, the latter sued the defendant for the amount. The County Court judge decided in favour of the defendant, and said that the nullity of the deed of assignment, consequent on the bankruptcy within three months, must be impliedly qualified so as to exempt from its operation acts rightly done under it, but, on appeal, this decision was reversed by the Divisional Court. Lord Alverstone, C.J., in delivering judgment in favour of the trustee in bankruptcy said: "A payment to a "bankrupt is not protected if the person making the payment "has notice of an act of bankruptcy, and I am of opinion that "a payment, with notice, to the assignee of the bankrupt stands "in no better position. . . . Here the defendant, with full "knowledge of the act of bankruptcy, thought fit to pay the "debt under the deed. That is not a protected transaction, and "the plaintiff is entitled to recover judgment." The case went to the Court of Appeal (1906 2 K.B. 786) upon a matter of no particular interest from the present point of view, but the judgment of the Divisional Court was not disturbed.

Unstamped and unregistered deed of assignment for benefit of creditors.

It may also be pointed out that if such a deed of assignment for benefit of creditors is unstamped, and is not registered within the period of seven days prescribed by section 5 of the Deeds of Arrangement Act, 1887, and is therefore void, as sometimes happens through the parties changing their minds and deciding not to act upon the deed, nevertheless, the execution of such a deed is apparently still an available act of bankruptcy (*re* Hollinshead *ex parte* Heapy 1889 6 Mor. 66), and payment cannot safely be made until the expiration of three months from the date of its execution. I ought perhaps to say that with regard to the deed being unstamped, some doubt as to the correctness of this view at the present time is suggested by Ringwood in his "Principles of Bankruptcy", 1905, 9th edition, p. 28. His objection seems, however, to be based on somewhat insufficient grounds, as can be seen by a reference to Alpe's "Law of Stamp Duties", 1905, 10th edition, p. 35; and the view, as stated above, is supported

by all the other leading authorities on bankruptcy, including Baldwin, Chalmers and Hough, Robson, Wace and Williams.

Release of securities to a trustee under a deed of arrangement.

The second case is that of Ponsford, Baker & Co. *v.* The Union of London & Smith's Bank Limited 1906 2 Ch. 444, and deals with the question as to whether a secured creditor can safely accept payment of his debt and give up his security, after the execution of such a deed as that referred to. It has been, I believe, the practice of some assurance companies, even where they would decline to make any payment under one of their policies until the statutory period of three months had expired, to accept repayment of a policy loan and return the policy, without waiting for the three months to elapse. The case now under consideration shows that this course cannot safely be adopted. The facts are as follows: The plaintiffs deposited securities with the defendants to secure a loan. On 27 April 1906, the plaintiffs were declared defaulters on the Stock Exchange, and it became the duty of the official assignee of the Stock Exchange to collect the assets of the firm. On 15 May the plaintiffs and the official assignee tendered to the bank the amount due, with interest, and called upon the bank to deliver up the securities, which were worth more than the loan. The bank refused to do so, on the ground that as the official assignee was assignee of all the property of the firm, an act of bankruptcy had been committed, of which the bank had notice; and they could not therefore safely hand over the property until the statutory period of three months had expired without bankruptcy proceedings having been commenced. When the case came before Mr. Justice Buckley, he ordered that on payment to the bank of the amount owing to them, they should give up the securities to the plaintiffs; but on appeal, this decision was reversed. In delivering the judgment of the Court of Appeal, Moulton, L.J., said: "Nothing is more firmly "established in bankruptcy law than that a man who has "committed an act of bankruptcy is not entitled to deal with his "estate . . . The difficulties and complications which arise from "his commission of an act of bankruptcy do not weaken the "obligation of the defendant in such an action, they only affect "the question who is to benefit by the performance of those "obligations . . . We are of opinion, therefore, that a secured "creditor is not entitled to receive payment of the debt from his "debtor after notice of an act of bankruptcy . . . If the (official) "assignee is prepared to pay the money due on the securities,

"and to undertake to hold them until bankruptcy proceedings "are taken, or the period of three months has expired, we are "of opinion that we ought to make an order empowering him "so to do. But if he is not, we do not think we ought to give "any immediate judgment in this action, but ought to direct it "to stand over until it can be seen who is the person entitled to "redeem . . . The defendants are entitled to their costs . . . "as well as to interest on the amount until actual repayment; "and they are entitled, until repayment, to exercise any rights "they may possess enabling them to realize the securities."

Policy on life of undischarged bankrupt.

A case that not infrequently occurs in office practice is where a policy is issued on the life of an undischarged bankrupt, and the office is either acquainted with the fact at the time of the issue, or subsequently receives information to that effect. Some offices, I believe, refuse to recognize any right in the assured to deal with the policy in such circumstances, unless the trustee in bankruptcy is a party to the transaction or executes an assignment of his interest in favour of the assured. It would seem, however, that such a case clearly comes within the decision in the leading case of Cohen *v.* Mitchell 1890 25 Q.B.D. 262, as after-acquired property: and that the office could safely deal with the assured and ignore the trustee in bankruptcy until the latter has actually intervened. In the case referred to, a cause of action was assigned, the assignee knowing that the assignor was an undischarged bankrupt. After the assignment, the trustee in bankruptcy intervened and claimed the proceeds. Lord Esher, M.R., in delivering judgment in favour of the assignee, said: "I am therefore prepared to lay down a proposition which has "been agreed upon by us all . . . It is this—until the trustee "intervenes, all transactions by a bankrupt after his bankruptcy, "with any person dealing with him *boná fide* and for value, in "respect of his after-acquired property, whether with or without "knowledge of the bankruptcy, are valid against the trustee." Fry, L.J., said: "It appears to me to follow that . . . dispositions "of personal or other property made by the bankrupt to a person "who receives them in good faith and for value, must be valid." It was subsequently held, in the case of *In re* The New Land Development Association and Gray (1892 2 Ch. 138), that this decision as to after-acquired property does not apply to land, and there seemed a tendency in the case of *Re* Rogers *ex parte* Woodthorpe 1891 8 Mor. 236 to still further limit it; but

nothing decided or said in these or any subsequent cases has suggested that such property as a policy of life assurance is not fully within the doctrine of Cohen *v.* Mitchell. Reference on this point may also be made to the case of *Re* A. Bennett, *ex parte* The Official Receiver, reported in *The Times*, 28 November 1906, which confirms the view stated above.

Policy on bankrupt's life with no value at time of bankruptcy.

A case of greater difficulty arises where, at the time of the bankruptcy, a policy is in force on the bankrupt's life; but owing to its short duration, it has no surrender-value, and is returned to the bankrupt by the trustee, as being of no value. The proper course in such a case is for the bankrupt to get the trustee to assign the policy to him for a nominal consideration, but this is not usually done, and when, some years afterwards, the assured desires to deal with the policy, difficulties may arise. Strictly speaking, there appears to be no alternative in such a case but for the trustee to be approached, with a view to the assignment of the policy to the assured; but this course usually leads to difficulties, partly because the original trustee has probably long since been discharged, and the bankruptcy proceedings forgotten, and partly because the policy, which at the time of the bankruptcy was valueless, has now perhaps become of considerable value. Another difficulty in such cases which I have come across once or twice lately, is that the amount offered by the bankrupt for the policy is not sufficient to pay an appreciable dividend to the creditors; and the trustee declines to re-open the bankruptcy for the purpose of receiving an amount which he could not distribute.

One method of dealing with the question which has been proposed, would treat the policy, even in these circumstances, as after-acquired property. Reasoning by analogy from the case of The Lord Advocate *v.* Fleming 1897 A.C. 145, it has been suggested that such a policy as that under consideration might be treated as after-acquired property, since its value may be said to result from the premiums paid since the bankruptcy. In the case referred to, a father had paid premiums for some years on policies on his own life, and then assigned them under a voluntary assignment to his daughter, who thereafter paid the premiums. On the death of the assured, the Inland Revenue Authorities demanded duty, *inter alia*, under the Customs and Inland Revenue Act, 1889, section 11, which extends the charge for duty to "money received under a policy of assurance effected by any "person dying on or after the 1st day of June, 1889, on his own

"life, where the policy is wholly kept up by him for the benefit "of a donee, whether nominee or assignee, or a part of such money "in proportion to the premiums paid by him, where the policy is "partially kept up by him for such benefit." The Inland Revenue authorities contended that the policy had been partially kept up by the assured within the meaning of the Act, by reason of the assured having paid the premiums prior to the assignment. The House of Lords, however, declined to take this view, and Lord Halsbury, L.C., in delivering judgment for the assignee, said: "This policy has been in existence a considerable "number of years. The person entitled ultimately to this money, "herself, in one sense, created the property—that is, she continued "the contract under which, if she continued to pay the premiums, "certain moneys would be payable on the death." These words of Lord Halsbury, taken by themselves, would seem to give some countenance to the idea that a policy without surrender-value at the date of the bankruptcy, might be regarded as after-acquired property; but it must be remembered that the case he was dealing with differed in many important respects from the case now under consideration. Moreover, he was considering the terms of a taxing statute, which is subject to somewhat special rules of interpretation, and these would not be entirely applicable where the Bankruptcy Act is concerned. I am afraid, therefore, that this mode of overcoming the difficulty can hardly be considered a satisfactory one.

Can the bankrupt acquire a title by abandonment?

Another method which has, I think, a much sounder basis, is to treat the assured as having a title to the policy by abandonment. There are certain words of Lord Watson, in the course of his judgment in the case of Whyte *v.* Northern Heritable Securities Investment Company 1891 18 R. (H.L.) 37, which tend to support this view. In that case, both the trustee and bankrupt had been discharged, the latter without composition, the effect of this being that there was no re-investiture of the bankrupt in his estate. Certain creditors applied for the appointment of a new trustee, on the ground that there were funds belonging to the estate which had not been recovered. It was objected by the bankrupt that all claim to these funds had been abandoned by the trustee. Judgment was given in favour of the creditors, as it was held that there had been no abandonment in this case; but Lord Watson said: "The creditors may deal with him (the "bankrupt) as to the footing upon which they give back part of

" the property. They may abandon it. It is not necessary, in " my view, that in every case there should be a retrocession. " The acts of the trustee and creditors in relation to it may be " such as to indicate that the bankrupt is, according to their " desire, to be deemed to be in future the master or owner of " the property, and that they have abandoned or rejected it." The case before the House of Lords was, of course, one involving Scots bankruptcy law, but the words I have quoted seem quite as applicable to English bankruptcy law; and I think, therefore, that in the case I am considering, a fairly good title by abandonment could, in many eases, be shown. I ought perhaps, however, as a warning against a too hasty acceptance of this view, to call attention to the words on page 124 of Pollock and Wright's "Possession in the Common Law", where it is stated that: "It " is even doubted whether it is possible for a possessor to divest " himself of his possession of a thing by wilful abandonment."

Dealing with bankrupt's property in Scotland after discharge of trustee.

The case of Whyte *v.* Northern Heritable Securities Company referred to above, is also of interest as clearing up a difficulty that has sometimes presented itself in Scottish bankruptcy cases, as to who can deal with the bankrupt's property after the discharge of the trustee. In England, in such a case, the property vests in the Official Receiver, but there appears to be no such provision in Scots bankruptcy law. The Court, however, decided in the above case that it had power, in such circumstances, to appoint a fresh trustee to deal with any property remaining to be disposed of, and proceeded to make the appointment.

Scots and Irish bankruptcy law.

A matter which can hardly be overlooked in dealing with the question of bankruptcy, is as to the position of English assurance companies with regard to Scots and Irish bankruptcy law. It is well settled that under the English bankruptcy law, the trustee must give notice of his interest in a policy of assurance like any other assignee; but this is not the law in Scotland or Ireland. In Scotland, under the Bankruptcy (Scotland) Act, 1856, section 102, the confirmation of the trustee in a sequestration, operates as an intimated assignation of all debts due to the bankrupt; and, therefore, as Mr. William Harvey points out in his article on Life Assurance, in Green's "Encyclopædia of Scots Law", already referred to, an assurance company cannot safely pay to an assignee of the bankrupt in accordance with a notice subsequent to the trustee's confirmation, and, presumably, *a fortiori*, cannot pay to the

bankrupt himself. He adds that therefore, in Scotland, the company, before paying under an intimated assignation, must ascertain whether the assignor is bankrupt.

Need of searches for bankruptcy in Scotland.

A striking commentary on this warning as to the need for caution in dealing with policies in Scotland, in view of the provisions of Scots bankruptcy law, is to be found in the case of Scottish Union and National Insurance Company *v.* Fairley 1900 (O.H.) 8 S.L.T. 154. In this case a sequestrated bankrupt concealed from his trustee a policy of assurance on his life. His wife kept up the policy, and on her husband's death obtained from the trustee an assignation of it, on payment of the surrender-value at the date of the sequestration. Prior to his death, and after his sequestration, the assured had obtained a loan on the policy from the assurance company who, of course, had no notice of the sequestration. It was held that the widow's title to the policy was good, not only against the executors of her husband, but also as against the assurance company who had lent on the security of the policy. Lord Low, in delivering judgment, said: "By the sequestration, the policy "was vested in the trustee to the same effect as if it had been "assigned to him, and the assignation duly intimated. Fairley, "therefore, had no more right to borrow money upon the "security of the policy than a third party, who happened to "obtain possession of the policy, would have had. The "insurance company say that they were not aware that Fairley "was bankrupt, and that they advanced the money in good "faith. I do not doubt that that is true, but I do not think "that it makes any difference; because sequestration is a public "matter, ignorance of which cannot be pleaded." Were it not for the hazardous nature of the risk involved in a criticism by a mere Southron, of a decision under Scots law, a doubt might be timidly expressed as to whether this case was correctly decided, even as between a Scottish company and a person domiciled in Scotland. If the Bankruptcy (Scotland) Act, 1856, went further than it does in making the confirmation of a trustee an intimation of the transfer of the bankrupt's property, it is hard to understand how it could override the distinct provision of the Policies of Assurance Act, 1867, which applies to Scotland, and which with regard to a special class of property, namely policies of life assurance, declares that the date on which such notice, *i.e.*, a written notice of the date and purport of the assignment, is received, shall regulate the priority of all claims under any

assignment. As I have indicated, however, I am not rash enough to venture on a serious criticism of this decision, and since, apparently, no appeal was made against it, presumably the assurance company was satisfied as to its correctness. In these circumstances, all that can be said is that, in this respect, at any rate, matters appear to be managed better and more equitably in England.

Will the decision in Fairley's case apply to a policy issued by an English company?

Assuming the decision in Fairley's case to be a correct exposition of the law in such circumstances, when all the parties are domiciled in Scotland, the question arises as to whether it will apply where an English company has issued a policy, of which the proper law of the contract is that of England, to a person domiciled in Scotland. Before discussing this point, attention may be called to the case of Scottish Provident Institution *v.* Cohen & Co. 1888 16 R. 112, which at first sight seems to suggest that, in the circumstances referred to, the matter would be governed by English law. In this case, a domiciled Scotsman obtained in England a loan from a firm domiciled in England; and deposited with them, by way of security, a policy of assurance on his own life, effected with a Scottish company. After the assured's death the mortgagees gave notice to the assurance company of their interest in the policy. Subsequently, sequestration of the assured's estate was awarded, and the trustee in such sequestration claimed to be entitled in preference to the mortgagees. Judgment was, however, given in favour of the latter. The Lord Ordinary, in interlocutary proceedings, said: "It appears to me to be reasonably clear that "the validity of the assigment must be determined by the law of "the country within which the assigment was made. Here the "policy is issued by a company having its domicil in Scotland, "and by the company's obligation a right of credit is created, "capable of assignment. . . . That right of credit follows "the domicil of the creditor wherever he goes; and is capable of "being assigned or dealt with by him, in any manner which the "law recognizes. The assignment of the right of credit in the "policy is a new contract, distinct as regards its nature, mode "of constitution and the law that regulates it, from the contract "constituted by the policy itself. And the validity of the "assignment will, in general, be determined by the *lex loci* "*contractus*—that is, according to the law of the country in "which the transference is made, or the security given. In this

" case, the policy was made over to a creditor in England, and it " appears to me that the contract falls to be determined by the " law of England." The Lord President, in expressing his agreement with the views of the Lord Ordinary, said: "The " transaction took place in England, and the constitution of the " creditor's right must be determined according to the law of the " country where the transaction took place, that is, the law of " England." As I have suggested above, this case at first sight might be supposed to give some support to the idea that Fairley's case would have been decided differently if the assurance company that granted the loan had been an English one. I think, however, that a more careful consideration of Cohen's case will lead to the conclusion that, assuming Fairley's case to have been correctly decided, the decision would be the same in the case of an English company. In the first place, it will be noted that in Cohen's case, the contract contained in the policy is distinguished from the contract made in connection with the loan, and it was with regard to the latter contract that the conflict of laws occurred. The fact therefore of the policy being issued by an English company appears to be immaterial. In the second place the point decided according to English law in Cohen's case was not the respective rights of trustee and mortgagee, but only as to whether the mortgagee's charge was a valid one. This is seen from the case of Robertson and Baxter *v.* Inglis 1897 24 R. 758, in which Cohen's case was commented upon. Here Lord Pearson, speaking of Cohen's case, said: "The competition there was " between a (Scottish) trustee in bankruptcy and an English " depositary of a life policy. There was an admission as to the " law of England to the effect that the deposit of the policy " operated as an equitable mortgage, and that if followed by " notice to the company, it conferred a preferable right. But the " English law was invoked only as to the first of these points, and " the fact that the parties admitted that intimation was required, " and was sufficient by English law, renders it all the more " significant that the question as to intimation was examined " and adjudged on, according to the rules of Scots law." It would seem therefore that even in Cohen's case, the actual question of the competition between a Scottish trustee in bankruptcy and an English mortgagee, in the case of the property of a bankrupt domiciled in Scotland, was decided according to Scots law; and since this would be the position of an English assurance company granting a loan on one of its own policies to a

policyholder domiciled in Scotland, there is no reason to suppose that the decision in Fairley's case would not be applied, certainly in Scottish Courts, and I think also in English Courts, in spite of the marked tendency of Courts to apply the *lex fori* whenever it is reasonably practicable to do so. On this point, attention may be called to the words of Lord Watson, in the case of Robertson and Baxter *v.* Inglis, already referred to, when it came before the House of Lords (1898 25 R. (H.L.) 70). He said: "It would, in my opinion, be contrary to the elementary principles "of international law and, as far as I know, without authority, "to hold that the right of a Scottish creditor when so perfected "can be defeated by a transaction between his debtor and the "citizen of a foreign country, which would be, according to the "law of that country, but is not, according to the law of Scotland, "sufficient to create a real right in the goods."

Practice as to searches in bankruptcy. The difficulty arising from the peculiarity of Scots bankruptcy law discussed above, and which is shared by the Irish bankruptcy law, that the fact of the bankruptcy has the effect of notice to the assurance company, leads to a consideration of the question as to whether searches for bankruptcy proceedings should not be made before making any payment in Scottish and Irish cases. If the decision in Fairley's case accurately represents the law of Scotland, there can be no doubt that all Scottish companies, and probably all English companies, making any payment to a person domiciled in Scotland, or, perhaps, even to any person capable of being made a bankrupt in Scotland, should, before making such payment, have a search made for bankruptcy. Apparently, also, the same rule will hold good for Ireland, and indeed, some little time ago, the Life Offices Association took the opinion of Sir Edward Carson, K.C. (then Mr. Edward Carson, Q.C.), on the question as regards Ireland, and were advised by him that such a search was necessary for the protection of an assurance company making a payment in such circumstances. It is not, however, the practice, as far as I am aware, among English offices, to make such searches, but it would be of interest to know whether any of them consider it actually necessary to take this precaution before making any payment in Scotland or Ireland.

Indemnity by trustee in bankruptcy. One minor point in relation to bankruptcy may be noticed before leaving the subject. When payment under a policy, whether of claim or surrender-value, is claimed by the trustee in bankruptcy, and for some reason,

such as the loss of the policy, the assurance company requires an indemnity, a difficulty is often presented by the fact that the trustee objects to give a personal indemnity or to obtain one from others, and his indemnity simply in his character of trustee, even if he were willing to give it, is, in the nature of things, of little or no value. In such circumstances, the proper course is for the trustee to arrange for a suitable indemnity to be given by a guarantee society, the necessary premium being paid out of the estate. Judging from a case that came under my notice a short time ago, he is entitled to do this, and there should therefore be no real difficulty, in such circumstances, in furnishing a satisfactory indemnity.

Settlement Policies.

Conflict of laws.

The first point that seems to call for discussion under this head is the question as to what law is to govern such policies when issued in one jurisdictional area to persons domiciled in another, or, to take the concrete case which usually arises under this head, as to whether a settlement policy issued by an English assurance company to a person domiciled in Scotland, is governed by the English Acts of 1870 and 1882 or by the Scottish Act of 1880. As already pointed out, where a conflict of laws occurs, questions relating to the interpretation of a contract and to rights and obligations arising under it, fall to be determined by what is called "the proper law of the contract," which simply means the law indicated directly or indirectly as being that which the parties intend shall be applied. If, therefore, the policy directly states under which Act it is issued, or even if it indicates which system of law is to apply to the contract, that will determine which Act is to apply. This is done in many cases, but some English companies in issuing settlement policies simply state that they are for the benefit of wife, or wife and children, as the case may be, without any reference to the Act they are issued under, or the law which is to govern the contract. In such a case as this there can be, I think, no doubt that a policy issued by an English company is governed by the English Acts of 1870 and 1882, whether the assured be domiciled in England or Scotland, for, as already noticed, such a policy in the absence of anything to indicate the contrary is an English contract, to be interpreted in accordance with English law. The point has not hitherto proved of very great importance, on account of the slight difference in effect between the English and Scottish Acts, but one

or two recent decisions under the latter Act seem to suggest that it may perhaps be of somewhat greater importance in the future.

Discharge required under the Act of 1870.

Leaving for the present any further question as to conflict of laws in connection with these policies, attention may be called to the nature of the discharge that should be obtained on payment of claim where the policy is issued under the Married Women's Property Act, 1870. I pointed out in my previous paper that at that time, some doubt existed as to how far the provisions of the Act of 1882 on this point had replaced the corresponding provisions of the Act of 1870 in respect of such policies. The question has now been set at rest by the decision *In re* Turnbull, Turnbull *v.* Turnbull 1897 2 Ch. 415. In this case the policy was issued under the Act of 1870 for the benefit of wife and children, and on the death of the assured his executors applied to the Court to determine, *inter alia,* whether the policy was vested in them in trust for the beneficiaries under the Act of 1882, or whether a trustee should be appointed to receive the policy moneys under the Act of 1870. The Court held that a trustee must be appointed under the Act of 1870, and, in delivering judgment to this effect, Stirling, J., said: "The effect of the last-mentioned section (section 10 of "the Act of 1870) appears to be this, that the policy is not, "under any circumstances, to be under the control of the "husband, or to form part of his estate. According to the "language of the section, that extends not merely to the policy "moneys but also to the policy itself, and it is plain that the "husband has no power to appoint a trustee of the policy, nor "has he power to assign it, even to a trustee when appointed, "because the policy is entirely out of his control." It was pointed out in a later case, that of *In re* Kuyper's Policy Trusts 1899 1 Ch. 38, that the petition for the appointment of a trustee under the Act of 1870 should be entitled "In the matter of the "Act of 1870", and not "In the matter of the Act of 1882 and "of the Trustee Act, 1893." Where however the appointment is made by a County Court, it is by no means uncommon to find the petition entitled in the latter manner, and, apart from the question discussed in my earlier paper, as to the power of the local County Court to make such an appointment under the Act of 1870, it might be contended that an appointment thus entitled was not a proper one, since only a trustee appointed under the Act of 1870 can give a discharge. It is customary, however, to

disregard such irregularities as these, and to treat the appointment as a valid one.

Can a single trustee be appointed?

It is pointed out in Wolstenholme's Conveyancing Acts, 1905, 9th edition, page 298, that in the case of the Act of 1870, a single trustee may be appointed, even though there is an infant, but on this point attention may be called to the words of North, J., in the apparently unreported case of *In re* Smith's Policy Trusts, decided on 2 April 1898. In that case the policy was issued under the Act of 1870 for the benefit of wife and children. Only a daughter survived, and the assured appointed the policy moneys to her, in accordance with the terms of a power of appointment contained in the policy. In appointing a single trustee, North, J., said: "I do not require "more than one trustee in this case, as, when appointed, he has "merely got to give a receipt for the policy moneys, and "forthwith hand them over to the petitioner." It would appear probable from these words, that where there are any other duties to be performed than those referred to, a single trustee will not be appointed in ordinary circumstances.

Appointment of a trustee in place of a deceased trustee.

In connection with the appointment of trustees, a point may be noticed which came before me a short time ago. The assured, under a policy issued in accordance with the Act of 1882 for the benefit of wife and children, had appointed a trustee as provided by the Act. This trustee, however, died in the lifetime of the assured, and the question arose as to whether a fresh appointment should be made by the assured under the Act of 1882, or by the legal personal representatives of the deceased trustee, under section 10 of the Trustee Act, 1893. The matter is not altogether free from doubt; but in view of the words of section 10 of the Act of 1893, which confers on "the person or persons nominated for the "purpose of appointing new trustees by the instrument, if any, "creating the trust," the power, in the first place, of appointing the new trustees, I think it must be concluded that the fresh appointment should be made by the assured. The Act of 1882 says that "the assured may appoint a trustee or trustees of the "moneys payable under the policy, and from time to time "appoint a new trustee or new trustees thereof, and may make "provision for the appointment of a new trustee or new trustees "thereof." This seems to indicate that the assured is a person nominated for the purpose of appointing new trustees, within the meaning of the Act of 1893, and is therefore the proper person

to make the appointment in the circumstances discussed. If, however, the trustee should survive the assured, and then die, there seems no reason to doubt that the legal personal representatives of such trustee could appoint fresh trustees under section 10 of the Trustee Act, 1893; indeed, they could probably act as trustees themselves, for, as pointed out in Underhill's "Law of Trusts", 1904, 6th edition, page 306, "Upon "the death of a last surviving trustee, since 31 December 1881, "the trust property . . . . devolves on his legal personal "representative. . . . . The person on whom the trust property "devolves can exercise the powers and duties incident to the office "of trustee; unless it is to be collected from the settlement that "the office was intended to be a personal one." On this latter point, the remark of the Lord President in the case of Schumann *v.* Scottish Widows' Fund Society 1886 13 R. 678, may be quoted. Referring to the question of a trustee under the Scottish Act of 1880, he said: "The personality of the trustee has nothing "to with the object of the clause, it is merely a convenient "arrangement."

Can an appointment of a trustee be cancelled?

It sometimes happens that after the assured has appointed a trustee, for some reason or other he desires to cancel the appointment. Unless, however, the trustee agrees to this, there does not appear to be any power under the Act enabling the assured to remove a trustee once properly appointed, except on the statutory grounds given in section 10 of the Trustee Act, 1893. If, therefore, the only reason for removal is a difference of opinion between the assured and the trustee, as is sometimes the case, nothing, apparently, can be done in the matter.

Does renunciation of probate operate as a disclaimer of the trust in the case of executor-trustees?

Another point in connection with trustees that occasionally arises may be noted here, although it is concerned with settlements generally, and not with those special ones arising under the Married Women's Property Acts. A testator frequently appoints his executors to be also trustees of the settlement contained in his will; and a question sometimes arises as to whether the renunciation of probate by one or more of the executors, operates also as a disclaimer of the trust, in the absence of any express disclaimer. Apparently it does not necessarily have that effect, although if an executor who has renounced probate does not act in any way as trustee, such a disclaimer will be presumed after the lapse of some years. If, therefore, the estate has been administered, and

the executor-trustees are acting in their capacity of trustees, it may be necessary to take into account an executor who has renounced probate, either because it is necessary to have the concurrence of all the trustees, or because all the executors who proved, or to whom power was reserved of making a grant of probate, have died, and the executor who renounced probate is therefore the last surviving trustee. In dealing with executor-trustees in their capacity of trustees, it is, of course, always necessary to take the concurrence of those who have not actually joined in proving the will, but to whom power is reserved of making a grant. In their case there is no question of disclaimer of the trusts, and they are quite as much executors and trustees as those who have actually proved the will.

Can the settlement in the policy be varied or cancelled?

Occasionally, the assured, instead of merely wishing to change a trustee under a settlement policy, wishes to alter the character of the policy altogether and change it into an ordinary "own life" one; and either consults the assurance company as to how this can be done, or endeavours to effect the desired result himself by striking out the clause stating that the policy is for the benefit of wife, or wife and children, as the case may be. Unfortunately, however just or expedient it may be that such a change should be made in the policy, as for example, by reason of the misconduct of the wife, it is not possible for the assured, or the assurance company, or both together, to make the desired change. The case of Hay's Trustees *v.* Hay 1904 6 F. 978, may be referred to on this point. Here the assured, in 1879, effected an assurance on his own life for the benefit of his wife, with an English company having a branch office in Scotland. This, of course, was before the Scottish Act of 1880, and may therefore have been either under the general law, or under the English Act of 1870, according to the terms of the policy, but there is nothing in the report to show this. The policy contained the following memorandum: "The amount assured by this policy is to be "in favour of Mrs. Georgina Birrell or Hay, wife of the assured, "Mr. John Hay, should she survive him; but in the event of "her pre-deceasing him, then the policy is to revert to his "executors, administrators or assigns." Subsequently the assured deleted this memorandum, and on being advised that this deletion was not sufficient to cancel Mrs. Hay's interest in the policy, he wrote at the foot of the policy, "I hereby revoke "and cancel the above bequest in my wife's favour. I have

"provided for her in my will." This memorandum, which was signed by the assured, was also subsequently deleted, but by whom was not known. The Court held, *inter alia,* that the deletions and alterations did not affect the wife's interest in the policy, and Lord Moncrieff, in the course of his judgment, said: "The policy was taken out originally as a provision for the "wife, and it has all the qualities of an irrevocable provision. . . . "I do not think that either the insurance company or the wife "was affected or could be affected by what the husband chose to do."

Restriction on alienation of the wife's interest under a settlement policy.

By way of contrast with the husband's unsuccessful efforts to change his wife's interest under such a policy, two cases may be noticed in which restrictions on the wife's power of dealing with her own interest are concerned. The first of these, which refers to a policy taken out by a wife on her husband's life for her own benefit, and which was not therefore a settlement policy, is that of *In re* Lavender's Policy 1898 1 Ir. R. 175. In this case, a ten-year endowment policy, taken out by a wife on the life of her husband, was described on the margin as "wife's policy endowment", and contained the words "This policy is not assignable." The policy was subsequently deposited by the husband as security for a debt, with his wife's written consent. When the policy matured there was a dispute as to the rights of the parties, and the assurance company paid the amount into Court under the provisions of the Life Assurance Companies (Payment into Court) Act, 1896. The wife claimed the full amount payable, and it was held in the Court of Appeal, reversing the decision of the lower Court, that the words, "This policy is not assignable", amounted to a restraint on anticipation, and that the deposit of the policy, though with the wife's consent, was therefore ineffectual. FitzGibbon, L.J., in delivering judgment, said: "We "shall declare that on the true construction of the policy, "Margaret Lavender, the wife of the insured, was restrained "from anticipation during the currency of the policy; and that "the memorandum of charge relied on by the respondents was, "and is, inoperative to charge the moneys payable under the "policy. The policy, and all moneys payable thereunder, must "be declared to be the separate property of Margaret Lavender. ". . . The provision operated to restrain the wife from anticipating "payment during ten years from the making of the policy, but "when that period was over the policy was her separate property "and at her disposal."

The second case is that of the Scottish Life Assurance Company *v.* John Donald (Limited) 1901 (O.H.) 9 S.L.T. 200. Here the policy was taken out under the Scottish Act of 1880, for the benefit of assured's wife. The policy was subsequently assigned, with the wife's consent, to secure a debt of the assured. On the death of the latter, the wife claimed to be entitled to the policy free from encumbrance, on the ground that she was incapable, during the existence of the marriage, of alienating her interest in the way alleged; and this contention was upheld by the Court, it being apparently held that the wife's interest, at least to a certain extent, is subject to a restraint upon alienation as in Lavender's case, even without words stating that the policy is not assignable. It would appear, however, that the case is to be considered as dealing with a somewhat special set of facts, and, as indicated in the Digest of Scots Law, must be taken only as deciding that in certain circumstances it will be held that a wife cannot alienate her interest in such a policy. As a matter of fact, I had occasion to discuss this case, soon after it was decided, with the actuary and manager of a large Scottish life office, and he informed me that after careful consideration his company had decided not to alter their practice in consequence of this decision, and that they still granted loans on such policies to the husband and wife jointly. On the other hand, I was informed by the actuary of another Scottish company that as a result of this case, his company had called in all their loans on these policies, and were declining to grant such loans in future.

Does the decision in Donald's case apply to a policy issued by an English company to a domiciled Scotsman?

The case has, of course, no bearing on such policies issued by an English company to persons domiciled in England, for it is quite clear that in the absence of words to the contrary in the contract, the wife's interest is freely assignable under either of the English Acts; though under the Act of 1870 the necessary formalities prescribed by Malin's Act, 1857, as modified by the Conveyancing Act, 1882, would have to be observed. The case of policies issued by an English company to persons domiciled in Scotland requires, however, a little more consideration. If there is anything in the contract to indicate that it is to be governed by Scots law, the case quoted above will apply equally as if the policy had been issued by a Scottish company. If, however, there is no such indication, the case is not so simple. Mr. Wark appears to consider that the case turned on the nature of the trust created

by the statute. If this view be correct, then if there is nothing in the policy to indicate what law is to apply, the English Acts of 1870 and 1882, and not the Scottish Act of 1880 will govern the policy, and the interest taken by the wife will be that given by the English Acts, and will apparently, therefore, be freely alienable. I should be inclined myself to consider that the question involved was rather one of status or capacity. If this be the correct view, then, in general, such questions fall to be determined by the law of the domicil. As already pointed out, however, there is an important exception to this rule in the case of mercantile contracts, which are usually governed by the law of the place where the contract is made. If, therefore, the contract for a loan, which comes within this description, is to be considered as made at the head office of the company in England, as would usually be the case, it seems likely that the wife's capacity to alienate would be determined by English law, in which case she could undoubtedly alienate her interest. On this point, I may call attention to the words of the Lord Ordinary in the case of Scottish Provident Institution *v.* Cohen & Co., already quoted under the head of bankruptcy. Speaking of a policy issued by a Scottish office to a domiciled Scotsman, he said: "In this case "the policy was made over to a creditor in England, and it "appears to me that the contract falls to be determined by the "law of England." I do not know whether it is upon such reasoning as this that English assurance companies have based their practice, but, as a matter of fact, I believe most English companies have not changed their practice of granting such loans in consequence of the decision here discussed.

Policy for benefit of wife if she should survive her husband.

In my previous paper, I pointed out the need, in the case of a policy issued for the benefit of the wife, for providing that she should only be entitled to the benefit of the policy if she should survive her husband. A difference of opinion seemed to exist at the time as to the need for such a provision, some contending that the words of the Act stating that the policy, so long as any object of the trust remained unperformed, should not form part of the estate of the assured, were sufficient to effect this end. The question may now be considered as set at rest, at any rate for the present, by the recent Irish decision in Prescott *v.* Prescott 1906 1 Ir. R. 155. Here a woman effected a policy on her own life, under the Act of 1882, for the benefit of her husband, without any reservation as to his surviving her. The husband predeceased his wife, and the

policy was claimed by the assured, and also by the husband's executors; and it was held that the latter were entitled. The Master of the Rolls, in delivering judgment, said: "The plain "meaning of the policy is that the contract was for the benefit of "William Prescott (the husband) . . . It (the trust constituted "by the policy) takes effect on the making of the policy; it "vests, not the money, but the interest in the policy, at once. "If the section had been intended to be confined to objects "therein named, provided they were alive when the policy "moneys became payable, some words, such as 'if surviving', "or the like, would have been inserted. . . . Therefore the "policy belongs to William Prescott's estate, and not to Mrs. Prescott."

Can endowment assurances properly be issued under the Married Women's Property Acts?

Incidentally, Lavender's case and the first one quoted in my paper, that of The Prudential Assurance Company *v.* The Commissioners of Inland Revenue, throw some light on the question which has frequently been discussed, as to whether an endowment assurance policy comes within the assurance sections of the Married Women's Property Acts. These sections expressly refer to policies of assurance effected on lives, that is, to policies of life assurance, and in the last named case, it was clearly laid down that endowment assurance policies come within this description. Moreover, in Lavender's case, where a form of endowment assurance policy was involved, the Court seems to have taken it for granted that such a policy was within section 11 of the Act of 1882. It may therefore, perhaps, be considered as now settled that such policies are properly issued under the Acts.

Rights of a second wife and children of a second marriage.

Another question relating to the interests taken under a settlement policy is that concerning the rights of a second wife and the children of a second marriage. Three cases dealing with the matter have recently been decided, each of which settles a point of some importance. The first of these came under the Act of 1882, and was the case of *In re* Browne's Policy 1903 1 Ch. 188. Here the policy was issued under the Act of 1882 for the benefit of wife and children, with a reservation in favour of the assured if he should survive them. The wife, who was alive when the policy was issued, predeceased her husband, leaving seven children. The assured married again, and on his death, left the second wife and one child of the second

marriage surviving, in addition to the family by the first wife. On the question being raised as to whether the second wife and her child were entitled to share in the benefits of the policy, it was held that they were. Kekewich, J., after remarking that it is settled that all parties interested under a policy in this form take as joint tenants, said: "It has been recognized by legal "authority that a married man, speaking of his wife, intends his "wife at that time, and does not contemplate one whom he may "marry after her death . . . But in construing an instrument "intended to make provision for a wife after the husband's "death, this seems to lose weight, and is countervailed by the "consideration that he, in all probability, intended to provide for "her who survived him and for that reason stood in need of the "provision. A similar line of reasoning points to the conclusion "that he intended to benefit all the children . . . The claim of "the children of the second wife to share in the policy moneys "is, I think, unanswerable; and if they are let in, I fail to see "any good reason for excluding their mother."

Shortly after this case was decided, the case of *In re* Griffiths' Policy 1903 1 Ch. 739 came before the Court on a somewhat similar point. Here the policy was taken out by the assured on his own life under the Act of 1870, and was expressed to be "for the benefit of his wife, or, if she be dead, between his "children in equal proportions." At the date of the policy, the assured had a wife and four children living, and subsequently four more children were born. Afterwards the wife died, and later the assured married again and had issue one child of that marriage. On the death of the assured there were surviving the eight children of the first marriage, the second wife and the child of the second marriage. On a question as to the distribution of the policy moneys, this case was distinguished from Browne's case, and judgment was given in favour of all the children, but excluding the second wife. Joyce, J., in speaking of the presumption of law in favour of an existing wife referred to in Browne's case, said: "In this particular case, I think the "presumption is stronger, because the words are 'if she be dead', "and those words seem to point to the wife who was living when "the policy was effected . . . Accordingly, I hold that 'wife' "here means the wife at the time the policy was effected, and "consequently that the widow can take nothing . . . I hold "that the subsequently born children of the first marriage are "entitled to take equally with those born before the policy was

"effected, and I can see no reason why the child of the second "marriage should not also be included. The fund will "accordingly go to the nine children as tenants in common." In considering this case, it must be carefully noted that, as was pointed out by Swinfen Eady, J., in a subsequent case, it "was "a decision on the construction of the particular policy, and not "on the Statute."

The third case to be noticed, dealing with this point arose under the Act of 1870, and is that of *In re* Parker's Policies 1906 1 Ch. 526. Here the assured effected policies on his own life under the Act of 1870, and it was expressed that his widow, or widow and children, or some or one of them, should be entitled to the policy moneys in such proportions as the assured should by deed or will appoint. The first wife, who was living when the policies were taken out, predeceased the assured, leaving three children. The assured married again, and subsequently, by deed, appointed the policy moneys in favour of the second wife, if she should survive him. On the death of assured leaving his second wife surviving, a dispute arose as to whether she was within the terms of the policy. Swinfen Eady, J., in delivering judgment in her favour, said: "In '*In re* Browne's Policy', Kekewich, J., decided that a "second wife was within the provisions of the Married Women's "Property Act, 1882, and although there are slight differences "between section 10 of the Act of 1870 and section 11 of the "Act of 1882, on examining the two sections carefully I am "unable to draw any distinction between them on this point. "If a second wife is within the latter Act she is also within the "earlier. . . . In my judgment 'widow' means the person who, "at the death of the husband, shall become the widow." It may be of interest to note in connection with this case, that in dealing with the suggestion that the policy, by reason of its terms, was not within the Statute, Swinfen Eady, J., said that the result would have been the same if it were held that the policy was not within the Statute, for, in that case, it was simply an agreement between the assured and the assurance society that the institution, on his death, would pay the money to the person appointed.

Effect of a policy which by reason of its terms is not within the Acts.

A somewhat curious case, relating to a settlement policy not under the Married Women's Property Acts, is that of *In re* a Policy of the Scottish Equitable Life Assurance Society 1902 1 Ch. 282. In this case the assurance was effected in 1850 by

Settlement policy where the beneficiary is not the legal wife.

a Mr. Sanderson on his own life, and was expressed to be "for behoof of Miss Harriott Styles." The assured's wife died in that same year, and he then went through a form of marriage with Miss Styles, who was his deceased wife's sister. Miss Styles died in 1870, and the assured in 1900. Doubts having arisen as to the persons entitled to the policy moneys, the assurance company paid the amount into Court under the Life Assurance Companies (Payment into Court) Act, 1896. The assured had always retained the policy in his own possession and paid the premiums, and apparently no evidence was produced to show that the assurance was effected for the benefit of Miss Styles, beyond the policy itself. Joyce, J., in deciding in favour of the representatives of assured, as against the representatives of Miss Styles, quoted approvingly the words of Lord Romilly: "If a purchase "be made by one in the name of another the presumption is that "the latter is a trustee for the person who pays the money, "unless the parties stand in the relation of parent and child", and said: "It really comes to this, a purchase by one in the "name of another, with no other circumstance at all proved. "Therefore, in my opinion . . . in equity the money belongs to "the legal personal representatives of Mr. Sanderson, who took "out the policy." It is with considerable hesitation that one ventures to differ from the decision of so distinguished a judge as Mr. Justice Joyce, but with great respect it may perhaps be urged that insufficient weight was given to the meaning of the words, "for behoof of" in the policy. It must be borne in mind that the presumption of law in favour of a trust where one person purchases property in the name of another is, after all, only a presumption and not a fixed rule, and this presumption may be rebutted by evidence that the purchaser actually intended to benefit the other person. In Dr. Murray's "New English Dictionary" the word "behoof" is defined as meaning "use, benefit, advantage." If this be the meaning of the word, there certainly seems to have been sufficient evidence in the policy itself of an intention to benefit the person named therein, in which case the presumption of a trust is rebutted. I may add that I have heard doubts cast on the correctness of this decision by one who should speak with some authority on questions of equity, and the opinion was expressed that had the case been taken to the Court of Appeal, the decision would very likely have been reversed.

A somewhat similar case came under my notice a short time ago. An application had been made to the Court for the appointment of a trustee under a settlement policy, and in the course of the proceedings it appeared that the wife named in the policy was really the sister of the assured's previous wife. The Court, however, declined to take any notice of the fact at the time, and left the distribution of the fund in the hands of the trustee, and stated that if the latter had any doubt as to how to proceed, he could apply to the Court for further guidance. I have no reason to believe, however, that any further application was made to the Court, but, on the contrary, I believe the deceased wife's sister was treated, for the purposes of the policy, as a lawful wife.

Remedy of creditors in regard to a settlement policy.

The Scottish case of Stewart *v.* Stewart's Trustees 1901 (O.H.) 8 S.L.T. 436, may be referred to here as showing the nature of the remedy available to creditors, when a settlement policy is alleged to have been taken out in fraud of them. Here the policy was taken out by the assured for the benefit of his wife under the Married Women's Policies of Assurance (Scotland) Act, 1880. An allegation similar to that referred to was made by the trustee of the sequestrated estate of the assured, and he proposed to retain the policy. It was held, however, that he must deliver it up to the assured as trustee for the wife, his proper remedy being to recover from the trustee of the policy, out of the proceeds, any premiums which could be shown to have been paid in fraud of the creditors.

Policies as separate property under the Act of 1870.

In concluding this section of my paper, there are two questions to which attention may be called, although they are only connected with it somewhat loosely. The first is concerned with a difficulty which occasionally arises in connection with policies issued on the lives of married women after the Act of 1870 and before the Act of 1882, and which are not expressed to be for their separate use. More than one case of hardship has come under my notice, where the policy has been effected by the woman herself and the premiums paid by her out of her own earnings, and yet she has been told that she could not deal with it in any way without the concurrence of her husband, who, perhaps, has deserted her. It seems possible, however, that a policy under the conditions referred to might be considered as her separate property under section 1 of the Act of 1870, which says that "The wages and

"earnings of any married woman acquired or gained by her "after the passing of this Act . . . and all investments of such "wages . . . shall be deemed and be taken to be property held "and settled to her separate use, independent of any husband to "whom she may be married." It certainly does not seem to be giving an unduly wide interpretation to the word "investment" to make it include a policy of assurance effected and maintained in the way mentioned.

Advances to trustees to keep a policy in force.

The other question relates to advances made to trustees to keep in force policies forming part of the trust estate, and which, but for such advances, would lapse. Of course, if there is express power under the settlement to mortgage the trust estate, no difficulty arises, but even in the absence of such a power, an advance can be made to the trustees for the purpose of keeping the policy in force, provided there is no other fund available for the purpose. A form in which this question has more than once come under my notice, is where the assured has taken out a policy under the Act of 1882 for the benefit of his wife if she should survive him, and has subsequently become bankrupt. In a policy so worded, a contingent interest is reserved to the husband, and this, of course, on his bankruptcy passes to the trustee in his bankruptcy; but this fact does not necessarily affect the position of the assured as statutory trustee of the trust constituted by the policy. If, therefore, the assured, as such trustee, is unable to keep the policy in force except by mortgaging it, there seems to be no valid reason why the assurance company should not advance the amount of the premium as a loan. There is no reason to doubt that an effectual charge on the policy can be given in these circumstances by the assured in his capacity of trustee, without the concurrence of his wife or of the trustee in his bankruptcy, in whom, together, the entire beneficial interest in the policy is vested.

Rate of interest chargeable on such a loan.

In this connection it may be noted that some years ago, the Life Offices Association took the opinion of Lord Justice Cozens-Hardy (then Mr. H. H. Cozens-Hardy, Q.C.) as to such advances as those discussed generally in the preceding paragraph; and he advised that if such a transaction came before the Court probably not more than 4 per-cent interest would be allowed. This is no doubt the ordinary rule of the Court, and it has been held that where a life policy forms part of the settled property, the premiums, which are

payable primarily out of income, are to be repaid to the tenant for life when the policy falls in, with interest at 4 per-cent (*Re* Morley, Morley *v.* Haig 1895 2 Ch. 738). The circumstances now under consideration, however, apparently come within the special rules relating to the principle of salvage, and it would appear therefore that such a rate of interest would be allowed as it was necessary to pay in order to raise the required loan. This view appears to be to some extent corroborated by the words of Kekewich, J., in Morley's case quoted above. He said: "I think the interest ought to be calculated "at 4 per-cent. I hesitate about the figures, but it is to be "observed that I am here dealing, not with the rate which "trustees, with restricted powers of investment, might have "earned, but with a rate which an absolute owner, free to "deal as she pleased, might have obtained for her money." This willingness of the Court to consider the special circumstances of the case in fixing the rate of interest seems to suggest that if, as is very likely, the loan could not be raised for this purpose on better terms than the ordinary rate for policy loans charged by the assurance company issuing the policy, such rate would probably be allowed.

Claims.

Under this head, I propose to notice one or two points that sometimes arise in connection with the settlement of claims, and, more particularly, death claims.

Paying claims without a grant of representation.

When the policy is for a comparatively small amount and constitutes practically the whole of the estate left by the assured, application is sometimes made to the assurance company to pay the claim without requiring a grant of representation to the estate. As a warning of the dangers incurred in acceding to such a request, the case of The Attorney-General *v.* New York Breweries Company Limited 1899 A.C. 62 may be considered. Here an American, named Clausen, domiciled in, and a citizen of, New York, held shares and debentures in the defendant company, which was an English company, incorporated under the Companies Acts, with a registered office in London. On Clausen's death, in 1893, his will was proved in New York by the executors there, but they did not take out a grant of representation in England. These executors requested the company to transfer Clausen's shares and debentures to them, and to pay them the interest and

dividends due, which the company did in part. The Attorney-General having filed an information against the defendants, claiming an account and payment of duty, judgment was in the first instance given in favour of the company, but this was reversed by the Court of Appeal, and this reversal was confirmed by the House of Lords. In delivering judgment, Lord Halsbury, L.C., said: "The property is in England, and belonged to the person "who is now dead. That property could only properly be taken "possession of, or administered by, a person who, by law, was "qualified to deal with it. . . . Undoubtedly the persons who "are concerned in this case, having the control and dominion "over the property in question, did an act whereby the title to "the property belonging to the deceased person became vested "in someone else; and they are not either executors or "administrators in the sense in which these words are used in "the Taxing Acts. *Primâ facie,* therefore, they are executors "*de son tort,* and *primâ facie,* as it appears to me, they have "taken possession of, and administered the property, in respect "of which this question arises." Lord Shand said: "I concur "with your lordships in thinking that this company, in agreeing "at their own hands to transfer to a third party, who had no "title to it as executors, the property of the deceased, were "taking possession of, and administering that estate . . . . I "think, therefore, that the result was, that they became executors "*de son tort.*" Lord Davey said: "If a man who owes money "to a deceased person, takes upon himself to pay that money "to some one who has no authority to receive it, I think he does "an act which is an appropriation in his own hands, and asserts "a right to exercise control and dominion over the debt." It is, I think, clear from the views expressed in these extracts from the judgment, that the decision is applicable to the case of an assurance company paying the amount due under a policy without the production of proper proof of title; and that a company doing so, in addition to any risks it may run in the way of subsequent claims by other parties, renders itself liable not only to the penalties imposed by the Stamp Act, 1815, section 37, but also for any death duties that would have been payable in respect of such policy monies if a grant of representation had been obtained in this country, and, probably, for double duty. It should be noted, however, as was pointed out by Lord Shand in the course of his judgment, that the decision in this case has no application where payment has

Payment to an executor before the will is proved.

been made to an executor here before he has proved the will, if there is no reason to suppose that he does not intend, ultimately, to prove it. Such a case is to be distinguished from the New York Breweries case, as in the latter instance, the company knew that the American executors did not intend to obtain a British grant of representation. In connection with this last-mentioned point, it may be noted that if an executor does an act in regard to the estate, and then dies before obtaining probate, the act will, nevertheless, be valid if the will is ultimately proved or administration granted with it annexed (Brazier *v.* Hudson 8 Sim. 67; Johnson *v.* Warwick 17 C.B. 516).

Payment of claim where assured was domiciled abroad at death.

Another question in connection with the liability of assurance companies for estate duty, when payment is made without a British grant of representation, arises under the provisions of the Revenue Act, 1889, section 19, which provides that, "where a policy of life assurance "has been effected with any assurance company by a person who "shall die domiciled elsewhere than in the United Kingdom, the "production of a grant of representation from a Court in the "United Kingdom shall not be necessary to establish the right "to receive the money payable in respect of such policy." It is clear from this that in such a claim, the company can pay without a British grant of representation. Nevertheless, the Inland Revenue Commissioners contend that such payment can only be made after due provision has been made for the payment of estate duty, either by the assurance company paying the amount of the duty direct to the Commissioners, or by their holding the policy moneys until such duty is paid. The contention is apparently based on section 8, subsection 4, and section 9, subsection 1, of the Finance Act, 1894, but it is difficult to see how the first-named subsection can support this view. It is admitted even by Mr. Soward in his work on Estate Duty, where the above-mentioned claim is made, that on the authority of Matthew *v.* Northern Assurance Company 1878 9 Ch. D. 80, an assurance company in the circumstances named does not come within the description of "trustee, "guardian, committee or other person in whom any interest "in the property so passing or the management thereof "is at any time vested", to whom that subsection applies. The Inland Revenue Commissioners are perhaps on firmer ground in so far as they base their claim on section 9, subsection 1, which enacts that "a rateable part of the estate duty on an estate, in

"proportion to the value of any property which does not pass to "the executor as such, shall be a first charge on the property in "respect of which duty is leviable." In view of the words of this subsection, it is perhaps not an unreasonable contention that any property coming within it is subject to a lien in favour of the Crown to the extent of the estate duty payable, and that if the policy moneys, in the circumstances in question, come within the subsection, the assurance company, having notice of a valid lien on the amount, cannot safely disregard that lien. As to whether the subsection reaches such policy moneys, at first sight it seems as if it did not, since they do pass to the executor as such, although it is true he is a foreign executor. Certain words of Lord Halsbury's, in the New York Breweries case throw some light, however, on this point. He said: "It is idle to suggest "that in the Taxing Acts, where they are dealing with English "finance, the words 'executor' or 'administrator' . . . are used "in those statutes in any other sense than as meaning an English "executor or an English administrator, dealing with our own "financial system." In the light of these words it may well be that the policy moneys in the case in question do not pass to the executor—that is, to an English executor, as such, and that, therefore, the Crown has a lien on them to the extent of the estate duty payable in respect of them. I must not, of course, be understood as agreeing with the contention of the Inland Revenue authorities in this matter, but only as indicating a point of view from which their demand does not seem quite so unreasonable and unfounded as one is at first inclined to regard it.

Paying claims under settlement policies direct to beneficiaries.

Before leaving the discussion of cases where the assurance company may be held liable for death duties, reference may be made to the question of paying settlement policies direct to the beneficiaries instead of to a trustee. Many companies are, I believe, prepared to do this where all the beneficiaries are *sui juris*, and there is no possibility of any addition to their number. There are, however, two risks to be borne in mind in adopting this course. In the first place, some of the beneficiaries may have alienated or encumbered their interests, and the alienee or encumbrancer may well take the view that such an alienation or encumbrance is not an assignment of a policy of life assurance within the meaning of the Act of 1867, and that notice of it should be given, not to the assurance company but to the trustee of the trust constituted by the policy. He may accordingly have given such notice to the

assured as statutory trustee, or be waiting to give notice to the trustee when the latter has been duly appointed. In the second place, where such a policy is taken out by the assured and, as is usually the case, the premiums are paid by him, there can be little doubt but that it comes within the provisions of section 2, subsection 1 (*c*) of the Finance Act, 1894, which, by reference to the Customs and Inland Revenue Acts of 1881 and 1889, makes chargeable with estate duty "money received under a "policy of assurance effected by any person dying after the first "day of August, 1894, on his life, where the policy is wholly "kept up by him for the benefit of a donee, whether nominee or "assignee, or a part of such money, in proportion to the "premiums paid by him, where the policy is partially kept up by "him for such benefit." The interpretation to be placed on the latter part of this section can be seen by reference to the case of The Lord Advocate *v.* Fleming, to which I have called attention in connection with the subject of bankruptcy. It would, therefore, appear that policies such as those referred to, are chargeable with estate duty, and if so, then on paying the amount due to anyone other than the trustee, either statutory or expressly appointed, the assurance company may, in addition to any other risks, find itself liable for any estate duty that may be payable in respect of the policy. In confirmation of the view that such policies are subject to estate duty, reference may be made to the case of The Attorney-General *v.* Dobree 1900 1 Q.B. 443. Here the assured, shortly before his marriage, effected an assurance with a French assurance company on his own life, the policy being made payable, on his death, to the lady he was about to marry. After the marriage, but in pursuance of an agreement made before that event, he assigned the policy to the trustees of a settlement under which his wife was a beneficiary, and covenanted to pay the premiums, which covenant he performed. On his death, the Inland Revenue Commissioners claimed estate duty, and their claim was allowed.

Are settlement policies liable to estate duty?

Somewhat akin to the foregoing questions is one that occasionally arises as to the power of a legatee to whom a policy of life assurance has been specifically bequeathed, to deal with it in the absence of an assignment of it to him by the executor of the testator. It is stated in Williams on Executors that "the interest in any "specific thing bequeathed vests at law in the legatee, upon "the assent of the executor", and it is pointed out in

Is an assignment necessary where a policy has been specifically bequeathed?

Wolstenholme's Conveyancing Acts that although under the Land Transfer Act, 1897, freeholds and leaseholds now vest in the legal personal representative, yet, "it would seem that the "assent of the legal personal representative can vest the legal "title in leaseholds or freeholds, part of a residuary gift, in the "residuary legatee or devisee." Since property of the description named would, like a policy of assurance, require to be transferred by a proper assignment or conveyance in ordinary circumstances, and yet can pass under a bequest or devise with the assent of the executor, it does not seem unreasonable to assume, that a policy of life assurance will, in like circumstances, vest completely in the legatee on the executor's assent. It is true that by a series of decisions, and ultimately by statute, it was decided that a special form of chose in action, in the shape of Goverment stock, vested in the executors, and that they must be parties to a transfer of it, and that Williams on Executors, in speaking of this on page 625, says that it "is, like all other personal property, assets "in the hands of the executor, and consequently that, although "specifically devised, it must in the first instance devolve upon the "executor." These words, however, with regard to choses in action other than Government stock, only imply that the assent of the executor is necessary, and not that there must be a formal assignment by the executor to the legatee; indeed, the express provisions with regard to such securities as Government stock would seem to imply that other forms of choses in action may vest in the legatee without assignment. This view is strengthened by the fact that while a policy has been held to form the subject of a good *donatio mortis causâ* (Amis *v.* Witt 33 Beav. 619), it was held, in the case of Moore *v.* Moore L.R. 18 Eq. 474, that railway stock, which would require the concurrence of the executors in the transfer, was not capable of passing in that way. It would appear, therefore, that an assignment by the executors is not really necessary, and that an assurance company, in such circumstances would be ill-advised in refusing payment to the legatee on production of the probate, with probate copy of will attached, and clear evidence of the executor's assent to the bequest. I must not, of course, be understood as suggesting that such a course is advisable from the legatee's point of view. Having regard to the difficulties that may arise in the future, in the matter of proving the executors' assent, it will always be more satisfactory to obtain an assignment from them, as this will be the most satisfactory evidence of their assent that

can be produced, and this is usually done. The absence of such an assignment, however, will not, I think, be a sufficient ground for refusing payment to the legatee. I have been considering more particularly the case where the policy is on the life of someone other than the testator. If it should be on the testator's own life, such a question will not often arise, as the sum assured will usually be paid before the executors are in a position to assent to the legacy; and in the absence of such assent, their receipt alone will be a sufficient discharge. Attention may also be called to the fact that where property vests in a person by reason of the assent of the legal personal representative of the deceased owner, the Inland Revenue Authorities claim that the instrument by which the assent is given should be stamped as a conveyance.

Powers of joint administrators. A few miscellaneous points referring to probates and letters of administration may now be noticed. The question is sometimes raised as to whether joint administrators have the same powers as joint executors, and can act singly. Some difference of opinion has existed on this point, and it is stated in Strahan & Kenrick's "Digest of Equity", 1905, page 475, that "It is doubtful whether, in any case, the act of one "only, of several administrators, is valid." It is, however, stated in Williams on Executors, 1905, 10th edition, page 720, that after the above view had been considered correct by Lord Hardwicke in Hudson *v.* Hudson 1 Atk. 460, in the subsequent case of Willand *v.* Ferm Selw. N.P. 767, note (8), 6th edition, it was held that one of several administrators stands on the same footing as one of several executors, and this view was recognized by Romilly, M.R., in Smith *v.* Everett 27 Beav. 454, when he expressed the opinion that the question was then settled. This statement may, therefore, be taken as correctly expressing the law on the point at the present time. It has been pointed out that the question is not of great practical importance, in view of the marked disinclination of the Court to grant a joint administration. As a matter of fact, however, I have had such grants before me on two or three occasions, and therefore thought it might be worth while to notice the matter.

Value of estate less than amount claimed under the grant. In my previous paper I referred to the case where, on the grant of representation being produced, the amount indicated on the grant, as representing the gross value of the estate, is found to be less than the amount claimed from the assurance company. It is stated in Walker and

Elgood's "Law of Executors", 1905, 4th edition, pages 114, 115, on the authority of Hunt *v.* Stevens 3 Taunt. 113 and Nail *v.* Punter 5 Sim. 563, that if the representative sues for a greater value than the gross amount of the deceased's effects, as shown by the certificate on the grant, he cannot recover. These cases, indeed, go a little beyond this and show that the amount claimed must not exceed that on which duty has been paid. Williams on Executors expresses this view, and says that: "Where an "executor or administrator produces the probate or letters of "administration, in proof of his representative character, and "his case shows that he sues for a greater value than is covered "by the probate or administration stamp, he cannot recover." I believe, however, as stated in my earlier paper, that this view is not generally acted upon by assurance companies, and that it is usual to disregard such a discrepancy between the amount appearing in the grant and the amount of the policy moneys. In view of certain practical difficulties that lie in the way, in attempting to give effect to these decisions, I cannot but think that such a course is a wise one to take, and that an assurance company would be ill-advised, in ordinary circumstances, in refusing payment on this ground.

Death of executor before taking out probate.

I have occasionally found some uncertainty existing as to what happens when an executor survives the testator but dies before taking out probate. I have already indicated that acts done by him in such circumstances are valid, if the will is ultimately proved or administration *cum testamento annexo* granted. It does not therefore follow, however, that his executorship is good to the extent of enabling him to transmit his powers to his own executor. The matter is dealt with in section 16 of The Court of Probate Act, 1858, which enacts that "Whenever an executor appointed in a will survives "the testator, but dies without having taken probate . . . the "right of such person in respect of the executorship shall wholly "cease, and the representation to the testator . . . shall . . . "devolve . . . in like manner as if such person had not been "appointed executor." It may also be noted in this connection, that where the executor of an executor becomes the representative of the original testator, no new probate of the original will is required (Wankford *v.* Wankford 1 Salk. 309).

Colonial probates.

The provisions of the Colonial Probates Act, 1892, are well known, providing as they do for the resealing in the United Kingdom of grants of representation

issued in any British possession to which the Act applies, or by a British Court in a foreign country. By section 18, subsection 2, of the Interpretation Act, 1889, the expression, "British possession," means any part of the King's dominions exclusive of the United Kingdom. The Act makes such a grant, when resealed here, equivalent to a grant originally made in this country, and now applies to nearly all the principal colonies, the most important British possession to which it does not apply being, apparently, India. A list of the colonies to which it applies will be found in Dr. A. E. Sprague's paper already referred to. The usefulness of the Act is not, however, so great as would at first sight appear, for the probate authorities at Somerset House some little time ago, in reply to some enquiries, advised that in many cases it was cheaper and more expeditious to obtain a power of attorney from the grantee of the colonial grant, in favour of some one in this country, and for such attorney to obtain a fresh grant here; and experience seems to indicate that this advice is sound. I have occasionally noticed a tendency among students to confuse this Act with the provisions of section 19 of the Revenue Act, 1889, already referred to, and it may, therefore, be worth while to point out that the latter Act is both wider and narrower than the former. Section 19 of the Revenue Act, 1889, applies to all countries outside the United Kingdom, whether British possessions or otherwise, provided only that the assured was domiciled outside the United Kingdom at the time of his death. On the other hand, it applies only to one particular form of personal property, namely, policies of life assurance, and if there is any other property to be dealt with here, a grant must still be obtained in this country. The Colonial Probates Act, 1892, makes no reference to domicil, and although only extending to grants issued by British possessions or British Courts in foreign countries, extends to property of every description that would be covered by a grant originally issued in the United Kingdom.

Resealing of Scottish and Irish grants.

In connection with the subject of resealing grants of representation, it may be noted that although provision has been made for the resealing in England of Scottish confirmations and Irish probates, such resealing is, I believe, the exception rather than the rule; indeed, after enquiries extending to several English assurance companies, I have been unable to find one where it is the practice to require such grants to be resealed. It is quite possible that some offices consider it advisable to do so, but it is not easy to see that there is much

real risk in disregarding the formality, while the difficulties likely to result from resisting payment on the ground of the absence of resealing are so great, that they may well make any company hesitate before adopting such a course. The law on the point is not perfectly clear or consistent, but leaving undiscussed the somewhat vexed question as to whether a policy of life assurance, issued by an English company to a person domiciled abroad, is English or foreign personalty, the position appears to be as follows: It is clearly established that in order to sue in an English Court in respect to the rights or property of a deceased person, a grant of representation, either originally issued in this country, or resealed here, must be obtained. It has been held that an executor who had obtained probate in Ireland could not sue here as executor, even to recover Irish assets (Carter *v.* Crofts, Godb. 33; Whyte *v.* Rose 3 Q.B. 493). In the last-named case, it was laid down by Tindal, C.J., that "no administrator could sue in the English "Courts, in respect of the personal estate, whenever it was found "at the death of the intestate, without an English administration." He further said that if the administrator, under an Irish administration, had received the debt (a bond which was in Ireland at the time of death), and given a release for it, it would have been a bar to any demand on the part of the administrator in England. It is further stated, in Williams on Executors, page 295, on the authority of the same case, that "an executor, "having clothed himself with an English probate, might, without "having obtained probate in Ireland, also sue in the Courts here "to recover a debt which was *bona notabilia* in Ireland." It would therefore appear probable that if an English company paid a claim in respect of a person domiciled in Scotland, under a Scottish confirmation not resealed here, such discharge would be a good answer to any claim made subsequently in England by a person claiming under an English grant. It also seems likely that if, in like circumstances, the insurance company insisted on the Scottish confirmation being resealed here, the executors could in many cases recover judgment in the Scottish Courts by means of their confirmation, and in the event of the company having no assets in Scotland, could bring that judgment to the English Courts and get it enforced against the company here, without the grant being resealed.

Grant of representation to a minor.

A further point of no great importance on the subject of the resealing of grants of representation may be

noticed here. In England the Probate Court will not knowingly issue a grant of representation to a person under the age of twenty-one, although I have seen cases where this has been done, no doubt through inadvertence. In Scotland, however, such grants would appear to be made knowingly in some circumstances, and a case of this description came under my notice recently. If such a grant is produced to the English probate authorities for resealing, it will apparently be resealed as a matter of course, without any enquiry being made as to the age of the executor; and the question arises as to whether such a grant, when resealed, is valid in England, although a grant would not originally have been made here in like circumstances. I do not know that this precise point has ever come up for decision in the Courts, but attention may be called to the case of *In re* the goods of the Duchess of Orleans 1859 1 Sw. and Tr. 255, where under similar conditions the Court refused to make a grant, although, had the law of the domicil been followed, such a grant could have been made. In this case it was laid down that, although the Court of Probate, in granting administration of the effects of a person who died domiciled abroad, will, in general, follow the law of the domicil, it will not grant administration to a person who, by reason of minority, is incapacitated from taking such a grant in England, although by the law of the domicil such minor may be entitled to administer the estate of the deceased. It should, however, be pointed out that in the case of *In re* the goods of Earl 1867 L.R. 1 P. and D. 450, where a different disqualification was involved, the law of the deceased's domicil was adopted by the English Court. It seems clear that such a grant to a minor as I have referred to will have no greater force or effect here than if it had originally been granted in England; and as it is probable that an assurance company would not be safe in paying to a representative known to be under age, to whom an English grant had inadvertently been made, it would seem that it could not safely act upon a similar Scottish grant resealed here. If, therefore, a discharge by a duly authorised English representative is required, it would appear that a grant of administration *durante minore ætate* should be obtained in this country. Of course, if payment would be made under the Scottish confirmation without requiring resealing, the question as to whether the representative is under age or not, can be disregarded, since there appears to be no doubt that such a grant can be validly made in Scotland.

Probate granted to some of the executors and not to all.

It is not unusual, where there are several executors, for the probate to be granted to one or more of them, the grant stating that power is reserved of making a like grant to the others. In such a case, the question sometimes arises as to whether payment can be made to executors to whom power is thus reserved, without the production of a fresh grant. Such a fresh grant is sometimes made, and is called a double probate. A further grant, however, appears to be unnecessary, the original grant enabling all the executors to act, whether it is expressly granted to them or not. The reservation is also unnecessary, they all having the right to obtain a grant, whether power of making such a grant be reserved or not (Williams on Executors, page 295).

Bonuses.

In concluding this section, attention may be called to two matters which cannot very well be classed with the other subjects discussed. The first of these is a case dealing with the question of bonuses, when the death occurs shortly before the declaration of bonus. Such a question, though not in its nature connected with the subject of claims, will usually be raised, if at all, in connection with a claim, and may therefore appropriately be considered here. The case referred to is that of Rosmead *v.* Norwich Union Life Insurance Society 1898 15 T.L.R. 9. Here the policies had been taken over by the defendants from the Reliance Society, and by the terms of the contract, the holders of all existing participating policies who should keep up their policies with the Norwich Society, were to be entitled to receive a reversionary bonus in respect of each succeeding quinquennium. A quinquennium terminated on 31 December 1897, and the assured died on 28 October 1897. The executors claimed a bonus in respect of the current quinquennium, but the society contended that as the bonus had not accrued at the date of assured's death, his executors could not recover such bonus or any part thereof, and this contention was upheld by the Court. Channell, J., in giving judgment in favour of the society, pointed out that a policy was entitled to a quinquennial bonus at the end of the quinquennium, provided it was in force, and that in this case it was not in force, and therefore was not entitled. The case is sometimes quoted to dissatisfied claimants as an authority upholding the general practice of making the title to a bonus depend on the survival of the assured until the completion of the bonus period. It must however, be borne in mind that the case was decided on the terms of the particular contract, and that contracts may exist which

would be open to a different interpretation. No doubt the usual practice would be upheld if it were contested, but it would not necessarily be so upheld on the authority of Rosmead's case.

Damages under Lord Campbell's Act.

The other matter to which I propose to refer, is the question of the allowance to be made in respect of assurances, in an action for damages under what is commonly called Lord Campbell's Act, 1846. Whenever the question arises in any particular case, great surprise is expressed that in estimating the damages to be paid under this Act, the amount of assurance should be deducted in the case of accident policies, and allowed for to a certain extent in the case of life policies. This is probably due to overlooking the fact that the Act was passed, not for the purpose of inflicting a penalty upon the party who is responsible for the accident, but to compensate certain of the relatives of the deceased for the actual pecuniary loss they have sustained by reason of his death. This is clearly seen from the case of Bradburn *v.* Great Western Railway 1874 L.R. 10 Ex. 1, where, in pointing out that when the action was by the injured party himself, insurances are not to be taken into account, it was stated that the rule in the case of Lord Campbell's Act was that "the damages were to be a compensation "to the family of the deceased, equivalent to the pecuniary "benefit which they might have reasonably expected from the "continuance of his life." From this point of view it is clear that any assurances that become payable by reason of the death must be taken into account, as otherwise the relatives might actually be better off, from a pecuniary point of view, in consequence of the death. The law on the point was laid down by Lord Campbell himself in the case of Hicks *v.* Newport Railway Company 1857 4 B. & S. 403 (note) where, in addressing the jury, he said: "You are not to look to the wants "of the family, but to the loss they have sustained by the father's "death . . . I think you should first consider what would be "the sum if there were no insurances . . . Then, if there be an "insurance for £1,000 against accidents by railways . . . it is "quite clear that there ought to be a deduction from the aggregate "amount in respect of that £1,000. Then with regard to the "policies upon his life independently of accident, if you allow "any deduction (and I think you will probably consider that "some deduction ought to be allowed), it will only be in respect "I should think, of the premiums that would be paid by the "family, or which would have been paid by himself if this fatal

"accident had not happened." Further light is thrown upon the latter part of this statement by the case of The Grand Trunk Railway Company of Canada *v.* Jennings 1888 13 App. Cas. 800, where it is stated that the proper allowance to be made in respect of ordinary life assurance, is to deduct from the estimate of what would have been the future earnings of the deceased, the amount of the premiums which, if he had lived, he would have had to pay out of his earnings for the maintenance of the policy.

As is well known, at least one company has, by private Act of Parliament, obtained an exemption from this rule in favour of its accident policies. This has given rise to some discussion as to why the rule should be maintained in respect of such policies issued by other companies. It seems more reasonable, however, that criticism should rather be directed to the peculiarity of our legislative system, which permits a rule of law to be relaxed in favour of an individual company, and thus places it in an advantageous position as compared with other companies carrying on a similar business. It certainly seems reasonable that such a rule as this should apply to all companies or to none.

Payment into Court under the Act of 1896.

Closely connected with the subject of claims, and, perhaps, strictly speaking, forming part of it, is the question as to the circumstances in which the policy moneys may properly be paid into Court under the Life Assurance Companies (Payment into Court) Act, 1896. There are very few reported cases dealing with this question, and this is to be expected, for in the nature of things, most of these cases are settled at Chambers and are never reported, the majority of payment into Court cases being the result of an amicable arrangement made between the assurance company and the claimant, in order to facilitate the ultimate settlement of the case. There are, however, a few cases to which attention may be called, and the most important of these is that of Harrison *v.* Alliance Assurance Company 1903 1 K.B. 184, which lays down the conditions on which a company can pay into Court, as distinguished from its liability to costs if it does so improperly. In this case the policy had been assigned, upon certain trusts, to trustees, of whom the plaintiff was the last survivor. Many years previous to assured's death the policy had been lost, and notice of this had been given to the assurance company. On the death of the assured, the company offered to pay the policy moneys

When can the policy moneys be paid into Court?

to the plaintiff on being furnished with a suitable indemnity, but the parties were unable to agree as to its terms. The plaintiff having commenced an action for the amount due, the defendants applied for leave to pay the money into Court under the Act of 1896. This leave was granted by the Judge at Chambers, and on appeal, was confirmed by the Court of Appeal. Collins, M.R., in delivering judgment in favour of the assurance company, after pointing out that the plaintiff's title could not be said to be absolutely free from doubt, and that the Act was passed for the purpose of meeting the case of such doubts, said: "It must be "borne in mind that the defendants, in seeking the protection of "the Act, do so at the risk of having to pay any costs to which "the plaintiff may, in consequence, be put if their own attitude "has been in any way unreasonable. If the plaintiff's title to "the policy money be as clear as his counsel alleges it to be, "then, no doubt, such order will hereafter be made against the "defendants with regard to the plaintiff's costs, as justice to him "may require." This case seems to be considered in some quarters as an authority for the proposition that the policy moneys can safely be paid into Court when the policy is lost and a satisfactory indemnity is not furnished. As a matter of fact, it is nothing of the kind. What, apparently, the case really decides is, that on an application for leave to pay into Court, the question of the reasonableness or otherwise of the grounds on which such payment is proposed will not be considered, and that, given any *primâ facie* evidence of a doubt as to the title, the only question that will then be considered is as to whether the provisions of the Act have been complied with, by the directors of the company stating that, in their opinion, no sufficient discharge can otherwise be obtained. The question as to whether the money was properly paid into Court will be considered when application is made for the money to be paid out. This view seems quite in accord with the preamble to the original Bill on which the Act was founded, and which states that there appears to be "no objection in "principle to empowering life assurance companies to pay "policy moneys into Court in cases of doubt and difficulty, of "course at their own risk as to payment of costs should the "proceedings be deemed by the Court to have been unreasonable." As a matter of fact, when the case of Harrison *v.* Alliance Assurance Company came before the Master in the Chancery Division for the payment of the money out of Court, the assurance company was ordered to pay the plaintiff's costs, and on the

matter being referred to Mr. Justice Buckley, at the instance of the company, he confirmed this order. In connection with this case, reference was made to the apparently unreported case of *In re* Policies on the life of John Rampling Neale Ward, which came before Cozens-Hardy, J., on 6 August 1901, and in which the facts were somewhat similar. Here, however, the assurance company was more fortunate as to their costs, which were ordered to be paid out of the fund.

Two reported cases, in which the policy moneys were paid into Court have already been referred to in connection with settlement policies, namely, those of *In re* Lavender's Policy and *In re* a Policy of the Scottish Equitable Life Assurance Society. In neither of these cases, however, does any dispute appear to have arisen as to the reasonableness, or otherwise, of paying the amount into Court; and they therefore throw no light on the question as to when an assurance company can properly do so. There are, however, two unreported cases, both decided in Ireland, which throw some light on this point. The first of these is that of Hawthorne *v.* Thomson, or, rather, this case and three others, in all of which the same plaintiff appears, and in which the defendants were certain Scottish assurance companies. The cases were heard in the King's Bench Court, in Dublin, in 1901, and the facts were as follows: The assured in 1895 assigned certain policies in trust for the benefit of his wife and two daughters. His wife pre-deceased him and he married again. In 1900 he made his will, appointing his second wife and another person executors. The second wife alone took out probate, and her solicitor thereupon wrote to the assurance companies, informing them that the assured had burned the assignment with the intention of revoking it, and accordingly she claimed the policy moneys. The assignees also claimed payment under the assignment, of which notice had been given to the companies. In these circumstances all the companies applied for leave to pay the moneys into Court, less the cost of doing so. The Court made an order for the moneys to be paid into Court, the question of payment of costs being reserved. I understand, however, that the matter of costs was settled privately between the parties, and it did not therefore come before the Court again.

The second case is the recent one of *In re* Alexander's Life Policy, decided by the Master of the Rolls, in Dublin, on 19 June 1906. Here the assured on 17 November 1905 instructed his solicitor to draw up a deed of assignment in respect

of a policy on his own life, and on 23 November following the deed was executed by him. On the next day he was certified to be of unsound mind, and two days later he was removed to an asylum, where he remained until 8 January 1906, when he died of general paralysis of the insane. In view of the doubt as to the condition of the assured's mind when he executed the deed, the assurance company paid the policy moneys into Court. The Master of the Rolls came to the conclusion, on the evidence, that the assured was of sound mind when he executed the deed, and therefore found in favour of it; but he held that, having regard to the circumstances of extraordinary suspicion in the case, the company were quite justified in paying into Court. He pointed out that the Act of 1896 was passed for the protection of assurance companies, and said that if the company pressed for their costs of appearance he would allow them.

FRIENDLY SOCIETIES AND INDUSTRIAL ASSURANCE COMPANIES.

The most important case to be referred to under this head, and indeed, in some respects, one of the most important cases noticed in the whole of this paper, is that of Cowling *v.* Topping 1906 1 K.B. 466. The facts of the case were as follows: A dispute arose between a claimant under an industrial assurance policy exceeding £20 in amount, and the assurance company issuing it; and the matter was taken before a court of summary jurisdiction under the provisions of section 7 of the Collecting Societies and Industrial Assurance Companies Act, 1896. This Court decided against the assurance company, and the latter appealed to a Divisional Court, on the ground that owing to the assurance in question not being for less than £20, the court of summary jurisdiction was not competent to try the case. By section 1, subsection (*b*) of the Act, an industrial assurance company, within the meaning of the Act, is one that, *inter alia*, grants assurances on any one life for a less sum than £20. The Divisional Court upheld the objection as to jurisdiction, and Lord Alverstone, C.J., in delivering judgment in favour of the assurance company, said: "It is, I think plain, from a consideration of a long course of "legislation upon kindred subjects, that the intention of the "Legislature, in conferring this jurisdiction, was to confine it to "claims on policies for small amounts, that is to policies for "amounts under £20. The analogy of other statutes shows "that a limit is always imposed upon jurisdiction of this nature.

Limits of the Collecting Societies and Industrial Assurance Companies Act, 1896.

"The language of section 1 (*b*) of the Act under consideration is "to apply to every body of persons, corporate or unincorporate, "'granting assurances on any one life for a less sum than twenty "'pounds.' . . . I prefer the construction of the section which "confines this jurisdiction to claims in respect of policies "originally granted for a less sum than £20. A question may "arise as to whether societies which grant policies for £20 and "higher amounts are not altogether outside this jurisdiction, but "it does not arise in the present case; and I am of opinion, "whatever view may be taken of that question when it is ripe "for decision, that this jurisdiction is, at any rate, limited to "policies for sums less than £20." No doubt this decision, as far as what it nominally decided is concerned, was in accordance with convenience and reason, as it certainly seems anomalous that a court of summary jurisdiction should be called upon to decide questions relating to policies assuring several hundreds of pounds; and this might easily happen had the decision been different, since policies of such amounts are not infrequently issued by all industrial assurance companies. Unfortunately, however, the only way of effecting this desirable end was by so interpreting the definition clause as to exclude policies of £20 and over from the definition of industrial assurance, and thereby exclude all such policies, not only from the operation of section 7, but also, apparently, from the operation of the whole Act, since, as far as I can see, in no case is a company that does not come within the definition as to amount of assurance, brought within the Act at all. The real effect of the decision appears, therefore, to be, as far as all policies of £20 and over are concerned, to nullify the Act of 1896 altogether, so that all the provisions as to notice before lapsing policies on account of non-payment of premiums, notice on transfer, &c., have no application to such policies, which constitute a considerable portion of the business of these companies. It seems not improbable, from the words of Lord Alverstone, that had the question been before him as to whether a company issuing policies for £20 and over came within the terms of section 1 (*b*) even in respect of policies for less than £20, he would have held that it did not. In that case, since every industrial assurance company issues policies of £20 and over, the whole Act, including the special provisions as to assurances on the lives of children, would in effect be nullified, as regards all industrial assurance companies, and such companies would stand on precisely the same

footing as ordinary assurance companies in all respects. In view of these facts, it is difficult to believe that the Court, in deciding as it did, realized the far-reaching effect of its decision; and, with great respect, it is submitted that in spite of the inconvenient results in connection with section 7, the more reasonable view was that advanced by the counsel for the claimant, and which was admitted by Lord Alverstone to be an interpretation which section 1 (*b*) would fairly bear, namely, that a company which, among its policies, issues some for less than £20, and complies with the other conditions of section 1, is an industrial assurance company in respect of all its policies, whether they exceed £20 in amount or not.

Nomination not revoked by subsequent will.

The other cases to which I have to refer under this head, deal with questions relating to the nomination and assignment of friendly society policies. The first case is that of Bennett *v.* Slater 1899 1 Q.B. 45. Here the assured under a policy of assurance effected with a registered friendly society, made a nomination in favour of the plaintiff, in accordance with the provisions of the Friendly Societies Acts. Subsequently he made a will, appointing the defendants executors, and disposing of his property otherwise. The total amount payable under the policy exceeded £100, the amount which could be nominated under the Acts. The executors contended (1) that the nomination was invalid by reason of the amount of the policy, and (2) that it was revoked by the subsequent will. Judgment was given in favour of the executors in the first instance, but on appeal, this was reversed by the Court of Appeal, and judgment given for the nominee. A. L. Smith, L.J., said: "The effect of the . . . Act . . . is, in my opinion, that if "the amount does not exceed £100, the society is, on satisfactory "proof of the death of the nominator, without more, to pay that "amount to the nominee, thus avoiding the necessity for "administration; but if the total amount exceeds £100, then "only £100 can be paid to the nominee, and with regard to the "excess over that amount, the general law would apply. . . "The second contention for the defendants was that the nomination "was revoked by the will. . . The subsection prescribes "a special manner in which the nomination may be made, and a "special manner in which it may be revoked; and it appears "to me that upon the true construction of the enactment, as a "nomination can only be made in the prescribed manner, so it "can only be revoked in the prescribed manner."

Assignment of friendly society policy.

The next case, dealing both with nomination and assignment, is that of Caddick *v.* Highton 1899 80 L.T. 527. Here a policy, issued by a registered friendly society, was assigned to a certain person, and subsequently a nomination was made in his favour. He pre-deceased the assured, and on a dispute arising as to the title to the policy moneys, it was held by Phillimore, J., that such policies were not assignable, but that the executor of the deceased nominee was entitled, on the ground that a nomination is not revoked by the death of the nominee, and that therefore the nomination never having been revoked, the executor was entitled to stand in the position of the nominee and receive the policy moneys.

Effect of nomination.

The result of these two cases, as far as a nomination is concerned, seems to be to decide that the effect of a nomination is that of a voluntary revocable assignment, but one revocable only in a certain way. The situation is perhaps best summed up in the words of Romer, L.J., in a subsequent case referred to below (*In re* Griffin), when he said: "It may "well be that when such questions (competition between nominee "and assignee) . . . come to be considered, that it may be held "that these nominees are really only in the nature of assigns— "assigns claiming under what is equivalent to an assignment "which is revocable in a particular way, or that they are in "the position only of assigns whose position, unless that position "is revoked, is conclusive as between them and the society."

The decision in the case of Caddick *v.* Highton, as to friendly society policies not being assignable otherwise than by nomination, was followed by Kekewich, J., in the case of *In re* Redman 1901 2 Ch. 471, as a matter of course, and no further light was thrown upon the subject until the decision in the case of *In re* Griffin 1902 1 Ch. 135. Here the policy was issued by a registered friendly society and assigned by the assured, who never made a nomination in respect of it. In a dispute between the assignee and the administratrix of the assured, Joyce, J., considering that he was bound to follow the previous decisions referred to above, decided in favour of the administratrix, on the ground that such policies were not assignable except by means of a nomination. On appeal, however, this decision was reversed, the Court of

Friendly society policies held to be assignable.

Appeal holding that the power of nomination conferred by the Friendly Societies Acts was in addition to, and not in substitution for, the ordinary power of

assignment. Romer, L.J., in delivering judgment in favour of the assignee, said: "The policy moneys in question are *primâ* "*facie* assignable. To make them non-assignable, you must find "some legislative enactment to that effect, either express or "arising by necessary implication. I can only say, after "considering the various Acts to which our attention has been "called, that I can find no such legislative enactment."

Stamps.

Mortgages to secure current accounts.

The first point under this head to which I wish to call attention, is that of the stamping of mortgages to secure current accounts where no limit is fixed to the amount secured. Such mortgages are of common occurrence in practice, since nearly all mortgages to banks are in this form. I referred to this matter in my previous paper, but in view of the present practice of the Inland Revenue authorities with regard to such mortgages, my previous remarks require some modification. I there stated, on the authority of Alpe's "Law of Stamp Duties", that such a mortgage is sufficiently stamped if the actual amount owing when the security is enforced does not exceed the amount for which the deed is stamped, even though greater amounts have been owing during the currency of the mortgage. Apparently this view is not now accepted by the Inland Revenue authorities, and I am indebted to Dr. A. E. Sprague for information as to their present attitude, as contained in a letter from them dated 8 June 1904. From that letter, it appears that the Board of Inland Revenue consider that an assurance company, when called upon to pay under such a deed an amount not exceeding that covered by the stamping, can safely make such payment without any enquiries as to the amount owing under the mortgage, and are, indeed, bound to do so. If, however, the amount payable under the policy exceeds that covered by the stamping, payment of such larger amount cannot safely be made until the assurance company is satisfied that the highest amount at any time secured under the deed during its currency, does not exceed that for which it is stamped, or until the stamping of the deed has been increased to cover the amount of the policy moneys. Apparently, however, even where the amount payable under the policy exceeds that covered by the stamping, payment up to the amount so covered can safely be made. It sometimes happens in such cases that the bank is prepared to accept payment of the amount so

covered, and to renounce any claim it may have to the balance in favour of the legal personal representatives of the assured. There does not appear to be any objection to this course being followed in such circumstances, the two amounts being paid to the bank and assured's representatives respectively. A case which sometimes presents itself in connection with such mortgages, and which is not free from difficulty, is where the deed includes policies of different companies, and payment is claimed from one of the companies, of an amount within that covered by the stamping, although the sums due under all the policies together will apparently exceed that amount. It would seem that the proper course in such a case is for the assurance company from whom payment is required, to call upon the bank to show that either the total amount claimed under the security, or the highest amount at any time due under it, does not exceed that covered by the stamping. I believe, however, that many companies in such circumstances will pay the amount due under their policy, if that amount is within the stamping of the deed; and indeed, they would be placed in a position of some difficulty if they refused, seeing that as far as the claim made against them is concerned, the deed is apparently in order.

Current account mortgages including policies of different companies.

Turning to the case of ordinary mortgages for a definite amount, when a further advance is made, and other property is brought into the security, it is common to carry out the transaction in the form of a mortgage of the new property in consideration of the further advance, and also to make the new property further security for the original advance, so that the new property cannot be redeemed without repayment of both the original and further advance. Such a deed, however, only attracts mortgage duty in respect of the further advance, and does not require any further stamp in respect of the further security for the original advance, provided the first mortgage deed is properly stamped. This is provided for under section 87, subsection 3, of the Stamp Act, 1891. The words of this subsection are not very clear, but there is no doubt that their meaning is as stated above, and this has been confirmed by adjudications on such deeds which have come before me.

Further advance with further security.

Mortgage deeds occasionally contain covenants and powers other than those immediately connected with the subject of the mortgage, and the question then arises as to whether such covenants and powers are so far accessory

Covenants and powers contained in mortgage deed.

to the leading object as to be covered by the ordinary mortgage stamp; or whether they are so far independent as to attract additional duty as covenants or powers. It is of course impossible here to describe and classify the various powers and covenants which may be included in a mortgage deed, but two illustrations, one of either class, may be mentioned. A power of attorney, empowering the mortgagee, on a sale of the premises, to sign or execute any deed necessary, on behalf of the mortgagor, has been held not to attract any additional duty, the deed being sufficiently stamped with the ordinary mortgage duty. On the other hand, a mortgage of a public-house containing a covenant on the part of the mortgagor to purchase beer, &c., exclusively from the mortgagee, has been held to attract a duty of 10*s*. in respect of the covenant, in addition to the ordinary mortgage duty. I ought, perhaps, to mention, however, that since I first wrote the above statement, a case has come under my notice in which the security consisted of a policy of assurance, and there was added a similar covenant to that referred to, by which the mortgagor, who was a publican, covenanted to obtain his trade supplies from the mortgagee, who was a brewer. Nevertheless, on the deed being submitted to the Inland Revenue authorities for adjudication, it was adjudged to be properly stamped, although only bearing the ad valorem mortgage duty. It is a little difficult to say, therefore, whether they have changed their practice in regard to this particular covenant, or whether the new adjudication is an illustration of the fact that even the Commissioners of Inland Revenue are not exempt from that liability to error which is supposed to characterize the whole human race.

Assignment by trustee in bankruptcy.

Perhaps, to this last-mentioned cause, may be attributed another peculiar decision on the subject of stamping, to which my attention was recently called. Under section 144 of the Bankruptcy Act, 1883, certain exemptions from stamp duty are given. It is not necessary to give them in detail, but it may be said that the object of the exemptions appears to be to save expense to the bankrupt's estate; and, therefore, such deeds and documents as would have to be stamped at the expense of the estate are, in general, exempt. It is, however, expressly stated in Alpe's "Law of Stamp Duties" that a conveyance upon sale by the trustee, of all or any part of the estate of a bankrupt, is liable to ad valorem conveyance duty; and this is evidently so, since the cost of stamping such a document will fall on the purchaser, and not on the bankrupt's

estate. Nevertheless, when, a short time ago, a deed by which the trustee in bankruptcy assigned a policy on the bankrupt's life to the bankrupt, was objected to on the ground that it was unstamped, a letter from the Inland Revenue authorities was produced, stating that they considered the assignment in question to be within the exemption conferred by section 144 of the Bankruptcy Act, 1883, and that it did not therefore require stamping.

Collateral security where the primary security is a promissory note.

It sometimes happens that a mortgage is produced purporting to be by way of collateral security and stamped at the collateral rate, and when the alternative is offered of having the denoting stamp affixed in accordance with the provisions of section 11 of the Stamp Act, 1891, or of producing the primary security, the latter is produced, and proves to be only a promissory note. It is difficult to believe, in view of the difference between the duty on a mortgage and that on a promissory note, that any real doubt could exist as to the insufficiency of the collateral duty in such a case; but I have more than once seen it contended that the collateral rate was applicable in these circumstances. It may, therefore, be worth while to point out that in a case recently submitted to the Inland Revenue Commissioners for adjudication on this point, they held that the collateral duty was insufficient, and that the deed must be stamped with the full mortgage duty.

Variation of proviso for redemption.

Occasionally, on a transfer of mortgage, the proviso for redemption is varied, and some difference of opinion has existed in the past as to whether such variation operated as a reconveyance, and attracted reconveyance duty in addition to the transfer duty. The matter came before the Court in the case of Humphreys *v.* The Commissioners of Inland Revenue 1900 (81 L.T. 199). Here a mortgage, originally for £1100 had been paid off to the extent of £100, and the balance was, at the request of the mortgagor, assigned to transferees, and the mortgaged property was conveyed to them, discharged from the proviso for redemption in the original mortgage, but subject to a new proviso for redemption on payment of the sum of £1000 and interest. The Commissioners of Inland Revenue assessed the duty at the sum of 10*s*. 6*d*., being 5*s*. in respect of a transfer of mortgage for £1000, and 5*s*. 6*d*. in respect of a release of a security for £1100. On appeal, the Court held that the deed was a transfer of the mortgage debt for £1000 within the meaning of the Stamp Act, 1891, and was

liable to the duty on such transfer; but was not a release of the £1100, the amount secured by the original mortgage, and was, therefore, not liable to the second duty assessed upon it as a release.

Mortgages to building societies.

One other point in reference to the stamping of mortgages may be noted. While there is no exemption in favour of a building society incorporated under the Building Societies Act, 1874, in respect of its mortgages, there is still an exemption in existence in respect of mortgages to benefit building societies, which were in existence before the passing of the Building Societies Act, 1874, and have not been incorporated under that Act. The exemption, however, only extends to a mortgage by a member of such a society for securing the repayment to the society of an amount not exceeding £500.

Reconveyances by building societies.

Although there are no exemptions in favour of societies incorporated under the Building Societies Act, 1874, in respect of mortgages, there is an exemption in respect of reconveyances, as can be seen by reference to the case of The Old Battersea Building Society *v.* The Commissioners of Inland Revenue 1898 2 Q.B. 294. This was a case of dispute as to whether a reconveyance by a society incorporated under the Act of 1874, attracted reconveyance duty. By section 41 of the Act, certain exemptions from stamp duty are given. By section 42 it is provided that a mortgage to such a society may be vacated by a statutory form of receipt, without any reconveyance. For the society, it was contended that, "a "reconveyance, and a receipt endorsed on the mortgage, are "merely alternative forms of procedure, and the latter being "admittedly exempt from stamp duty, it would be anomalous if "the former required a stamp; the liability does not depend upon "the alternative adopted in a particular case." The Court held that such a reconveyance was exempt from duty, but did so on somewhat different grounds from those advanced by the society. It held that, by reason of the provisions of section 42, the reconveyance endorsed on the mortgage was "a document "authorized to be signed, made or produced in pursuance of the "Act or by the rules of the society", within the meaning of section 41, and, as such, was exempt from reconveyance duty.

Reconveyance by registered friendly society.

This decision apparently extends to the case of a registered friendly society, since the exemptions given in the matter of stamping, and the power to vacate a mortgage by means of a statutory form of receipt, conferred by

the Friendly Societies Act, 1896, are almost identical with those conferred upon building societies by the Building Societies Act, 1874.

It is important to note the grounds on which the above decision is based, and to observe that even the statutory form of receipt is apparently exempt from stamping, not on the ground that it is endorsed upon a duly stamped deed, but on the ground that it comes within the description of a document executed in pursuance of the Act, and is therefore exempt under the special exemptions conferred by the Acts on such documents. If it were otherwise, the decision might be extended to the case of societies registered under the Industrial and Provident Societies Act, 1893, which also have the power to vacate their mortgages by means of a statutory form of receipt having the effect of a reconveyance. A special exemption from stamp duty in favour of the statutory form of receipt, in the case of mortgages of such societies in Scotland, is given by section 44, subsection 5, of the last-named Act, but with this exception, there appear to be no exemptions from stamp duty, even the statutory form of discharge given by such a society being apparently liable to reconveyance duty. That this is so may be inferred from the decision in the case of Firth & Sons, Limited, *v.* The Commissioners of Inland Revenue 1904 2 K.B. 205. Here, on a deed securing certain redeemable debenture stock, a receipt was endorsed by the trustees, acknowledging that all the debenture stock and interest had been "redeemed, paid off, and satisfied." The Commissioners contended that this was liable to reconveyance duty as a discharge, but on appeal, judgment was given against them by Channell, J., who pointed out that the characteristic of all documents coming under the general head of "Reconveyance", was that they operated to put an end to some existing security. He went on to say: "The document in question, I think, has "not any force as an instrument putting an end, either to any "obligation or to any security. It is merely evidence that the "obligation or security no longer is in effective existence, and it "comes, therefore, quite clearly under the heading 'Receipt.' ". . . I construe the word 'discharge', in reference to the other "documents with which it is associated (in the Stamp Act, 1891), "and I think, therefore, that it means . . . a document which "in itself actually discharges the obligation." He therefore held

Discharge of mortgage when the society is registered under the Industrial and Provident Societies' Act, 1893.

Receipt endorsed on duty stamped deed.

that the receipt in question was not properly described as a discharge, and thus, in any case, only attracted receipt duty. As however it was endorsed on a duly stamped instrument, it did not, in this instance, require to be stamped at all. It seems evident from the grounds on which this judgment is based, that a receipt operating as a reconveyance attracts reconveyance duty, whether endorsed on a duly stamped deed or not, in the absence of any statutory exemption; and that the statutory form of receipt in the case of societies registered under the Industrial and Provident Societies Act, 1893, requires therefore to be stamped with such duty.

When a policy has been reassigned, can defects in stamping be waived?

If the holder of a policy assigns it, and by one or more subsequent assignments it becomes revested in such holder, the question has been discussed as to whether all or any of such deeds may be passed as in order, although unstamped or insufficiently stamped. It was pointed out in my previous paper, on the authority of Alpe's "Law of "Stamp Duties", that the Inland Revenue authorities did not regard such deeds as coming within the provisions of section 118 of the Stamp Act, 1891, provided they were executed before 16 May 1888. This concession has now been extended to all such deeds whenever executed, and it is stated in Alpe's "Law of "Stamp Duties", 1905, 10th edition, page 198, that "In practice "insurance companies are only required to insist upon the "payment of duty upon assignments where such assignments are "necessary links in the assignee's title; in cases where there is "an assignment from, and back again to, the original insurers or "any assignee, it is considered that the claimant does not claim "under such assignments, inasmuch as they merely record a "transaction which is completed and closed." This is confirmed by the statement in Highmore's "Stamp Laws", 1902, 2nd edition, page 212, where it is stated, in reference to section 118, that "the provision is not regarded as extending to an "assignment of a policy followed by a reassignment to the "assured prior to the commencement of any transaction which "forms part of the title of the claimant." It will be noted that the concession does not merely extend to the case of a mortgage and reassignment, but apparently extends to deeds of any nature, and to any number of such deeds, provided they have the effect of revesting the policy in the original assignor, and thus form what may be termed a closed loop in the chain of title.

It is important to bear in mind that the concession referred to above only represents the present practice of the Inland Revenue authorities, and does not profess to be based on a strict interpretation of section 118. If, therefore, they should alter their practice in this respect, a contingency by no means beyond the bounds of possibility, deeds which can now be passed as being in order may have to be regarded as not in order when dealt with at some future date. For this reason, apart from the question as to whether such deeds would be admissible as evidence when unstamped or insufficiently stamped, many would still insist on such deeds being put in order as to stamping where the transaction was not a final one in respect of the policy. It may, therefore, be well to point out another matter in connection with the stamping of reassignments which not infrequently occurs in practice. If part of the mortgage debt is paid off, and in consideration thereof a part of the mortgaged property is reconveyed, such reconveyance is often stamped only with reconveyance duty in respect of the amount paid off. This, however, is incorrect, as can be seen by reference to the case of Munro *v.* The Commissioners of Inland Revenue 1895 23 R. 232. Here a bond in security was executed for £5,000, of which £2,000 was first paid off upon the execution of a deed of partial discharge, stamped with 10*s*. On a question as to the duty to be paid on the final discharge, the Inland Revenue authorities contended that the 10*s*. was not ad valorem duty on the amount paid off, but the fixed duty of 10*s*. as a deed other than one described in the schedule as attracting ad valorem duty, and that this was the proper duty in the circumstances. They accordingly held that the ultimate reconveyance attracted the full reconveyance duty in respect of the whole £5,000, namely, £1. 5*s*. The Lord President, in giving judgment in favour of the Commissioners, said: "I think "the Commissioners rightly read the 5th section (relating to "reconveyance duty) as applying, and applying only, to such "discharges as have the effect of wholly freeing the subjects of "the security from that security, and that the duty is to be "calculated by the maximum of the burden which was ever "incumbent by virtue of the security. It results, first, that the "duty is payable equally, on a discharge of the balance of the "money, with a discharge of the whole money; and, second, "that this section does not apply to a discharge which lifts off "from the security subjects, only part of the security, but

Reconveyance on repayment of part of mortgage debt.

"leaves it in part, still incumbent. . . . It follows that such "(partial) discharges are liable to an ordinary deed stamp; for "they are certainly deeds, and they are not, in the words of "this Act, 'described in this schedule.'" It is pointed out in a footnote to the case, that where the total amount at any time secured is less than £2,000, so that the ad valorem duty on reconveyance could not exceed 10*s*., the practice of the Inland Revenue authorities is to consider a reconveyance of part of the property, executed on a partial discharge, as sufficiently stamped with reconveyance duty in respect of the total amount at any time secured.

Payment of premiums as consideration.

It sometimes happens that the premiums on a policy of life assurance are paid for many years by someone other than the assured, usually by a relative, and on a question arising as to the position of the parties, an assignment of the policy is arranged, the consideration as stated in the deed being the past payment of premiums. In such a case, some difference of opinion has existed as to whether a deed of this nature required to be stamped with ad valorem conveyance duty in respect of the whole amount paid in premiums, or whether it could be passed as in order in this respect if the stamp covered the ad valorem duty on the surrender-value of the policy at the date of assignment, or even if it were stamped with 10*s*. as a conveyance for other than valuable consideration, in view of the fact that such payment of premiums may, in itself, give no lien on the policy. I have recently seen some correspondence with the Inland Revenue authorities on this point, and they held that such a deed required to be stamped with ad valorem conveyance duty to the extent of the full amount paid in premiums.

Receipt for policy moneys endorsed on policy.

I pointed out in my previous paper that the Commissioners of Inland Revenue did not regard the exemption from duty in favour of receipts endorsed upon duty stamped instruments, as extending to the case of a receipt endorsed upon a policy of assurance for the amount payable thereunder; and since then I have seen a case in which a penalty of £5 was imposed upon stamping such a receipt after execution. It is true that in this particular instance the penalty was, on application, subsequently remitted; but the fact of the penalty being imposed indicates the view that the Commissioners then took of the matter. This view has, however, now been altered, and in Alpe's "Law of Stamp Duties", 1905, 10th edition, page 198, it is stated, I understand upon the authority of

the Commissioners of Inland Revenue, that a receipt endorsed upon a policy of assurance for monies payable thereunder, is exempt under the provisions of exemption (11) under the general head of "Receipt."

Deed securing periodical payments.

Provision is sometimes made in a deed, inter alia, for periodical payments to be made by one party to the other. The commonest case of this description is that of a deed of separation between husband and wife. Such payments may be weekly, monthly, quarterly, half-yearly or yearly, and since the duty in respect of this consideration comes under the heading of "Bond, Covenant or Instrument", for securing an annuity, and is chargeable in respect of the sum periodically payable, a difference of opinion has existed as to whether the sum chargeable is the actual amount of the periodical payment, or the total amount of such payments due in the course of a year. It was held in the case of Clifford *v.* The Commissioners of Inland Revenue 1896 2 Q.B. 187, where the consideration consisted of certain weekly payments to be made, that the amount of the weekly payments fixed the stamp duty, and not the total amount of such payments in a year. In the case of Jackson *v.* The Commissioners of Inland Revenue 1902 50 W.R. 666, a separation deed was executed between husband and wife, and it was agreed that if the wife should continue to observe the covenants entered into by her, the husband would pay to the wife "the clear weekly sum of £1." It was held that the duty was assessable on the periodical payment of £1, and not on £52, as contended by the Crown. It is, however, to be noted, that where there is any indication in the deed that the payments are really instalments of an annual payment, the duty is payable on the total amount receivable in the year. This can be seen from the case of Lewis *v.* The Commissioners of Inland Revenue 1898 2 Q.B. 290, where in a separation deed between husband and wife, the former undertook to pay a certain sum by quarterly payments on the usual quarter days. This was held to attract duty in respect of the total amount payable in a year under the deed.

Stamping of separation deed.

This last-mentioned case is also an authority for the statement that a deed of separation such as that referred to, attracts not only ad valorem duty in respect of the periodical payments, but also the ordinary duty of 10*s.* as a deed.

Duty attracted under more than one head.

Reference has incidentally been made, as in the case last mentioned, to instances where a deed attracts duty

under more than one head. Attention may here be called to some other cases of this description which frequently occur in practice, and in which the necessity for more than one stamp is often overlooked. As is well known, settlement duty is at the rate of 5*s*. per £100 of the property settled. This, however, does not apply to property of every description, but only to such property as money, stocks and securities, including policies of assurance. If, however, as is usually the case, the settlement also comprises other property such as realty and furniture, or a covenant to settle after-acquired property, a duty of 10*s*. will be payable in respect of such property or covenant, irrespective of its value, in addition to the ad valorem settlement duty in respect of the other class of property.

Settlements.

Appointment of new trustee.

An appointment of a new trustee is properly stamped with 10*s*.; but if, as is frequently the case, the deed contains a conveyance of the trust property to the new trustee, or a vesting declaration under section 12 of the Trustee Act, 1893, whereby the trust property is vested in the new trustee, such conveyance or declaration attracts a further duty of 10*s*.

Double statutory declarations.

A case coming under this head, which has excited considerable interest in view of the action of the Inland Revenue authorities, is that of a statutory declaration made by two or more persons. For some time past, the Commissioners of Inland Revenue have contended that such a declaration is really to be considered as a separate declaration in respect of each declarant, and consequently attracts as many times the duty of 2*s*. 6*d*. as there are parties to it. The matter came before the Court recently in the case of The Reversionary Interest Society (Limited) *v*. The Commissioners of Inland Revenue 1906 22 T.L.R. 740. Here a statutory declaration was made by two persons, one of them declaring alone as to a portion of the contents of the declaration, and both as to the remainder of it. The Commissioners held that this was chargeable with a duty of 5*s*., as being a declaration as to two distinct matters, within the meaning of sections 3 (2) and 4 (*a*) of the Stamp Act, 1891. On appeal, however, it was held that it only constituted one declaration, and was therefore only liable to a duty of 2*s*. 6*d*. Walton, J., in delivering judgment in favour of the appellants, said that: "The instrument was, in ordinary "language, one declaration, but it was one declaration in which

"two persons joined . . . The whole of the declaration was "made by one declarant, and as to part of it, he was corroborated "by the other declarant . . . If the other declarant had joined "in the whole declaration, it would not have required more than "one stamp, because it would have been a declaration made by "two persons, both declaring the same facts. He thought that "this was a single declaration made for one purpose. Must it be "treated as if it was a declaration as to distinct matters, or was "it really two declarations on one piece of paper? He had "come to the conclusion that this was one declaration, and "therefore the appeal must be allowed." This case is not quite on all fours with the one that frequently occurs, where, although the declaration is made for one purpose, two or more declarants each declare as to different and distinct facts. The words of the judgment quoted seem, however, to extend to this case also, and presumably, therefore, no difficulty will now be raised by the Inland Revenue authorities in such circumstances.

Stamping of joint indemnity.

Before the decision of the foregoing case, a question had been raised as to whether the ordinary form of joint and several indemnity, with two or more sureties, came within the contention of the Inland Revenue Commissioners and attracted duty in respect of each surety. It is, however, difficult to understand how any serious doubt could exist as to this, in view of the case of Ramsbottom *v.* Davies 4 M. & W. 584, decided as long ago as 1839, which seems decisive on this point. In that case, each of three sureties made himself severally liable to the extent of £50 in respect of any loss the plaintiffs might sustain by reason of their paying a certain debt. The indemnity was stamped with only one stamp, and in an action upon it, an objection was raised to its production in evidence, on the ground that it was insufficiently stamped. The Court overruled this objection, and in holding that it was correctly stamped, Parke, B., said: "I am of opinion that this is only one "transaction, and that one stamp only is necessary. Here each "of the parties entered into one agreement by which each bound "himself to a certain extent, in consideration that the others "would do the same. It is very similar to the case of Bowen *v.* "Ashley 1805 1 Bos. & P. (N.R.) 274, where it was held that "if several persons bind themselves in a penalty by one bond, "conditional for the performance by each and every of them, of "the same matter, such bond required only one stamp." It will be noted that in this case the indemnity involved was several

only, and not joint and several as is usually the case with the indemnities taken by life assurance companies where more than one surety is required. The case of these latter is stronger than if they were merely several, and it may therefore be considered as established beyond doubt that they only attract one duty.

I will conclude this section with a few miscellaneous cases of stamping which frequently occur in practice, and which are often overlooked by those responsible for the execution and stamping of the deeds. In the conveyance of the equity of redemption of mortgaged property, not only is conveyance duty payable on the amount owing for principal and interest under the mortgage, in addition to the consideration actually paid for the equity of redemption, under section 57 of the Stamp Act, 1891; but if, as sometimes happens, the mortgagee is a party to the conveyance, and the purchaser covenants to pay the mortgage debt, the deed is liable, in addition, to collateral security duty at the rate of 6*d*. per £100 on the amount of the mortgage debt. Thus, in an adjudication on a deed of this description, where the consideration for the assignment was £95, and the amount owing under the mortgage was £594, the duty payable was held to be conveyance duty on £689, and collateral security duty on £594.

Conveyance of equity of redemption, with covenant for payment of debt.

Conveyance of equity of redemption for other than valuable consideration.

Where the equity of redemption in mortgaged property is assigned for other than valuable consideration, and there is a covenant by the assignee to pay the mortgage debt and interest, such assignment is chargeable with ad valorem conveyance duty on the amount due for principal and interest, and not with the fixed duty of 10*s*. Where, however, the covenant referred to is absent, and the equity of redemption is conveyed solely as a gift, the ad valorem duty is not considered to be payable, and such a deed is, apparently, sufficiently stamped with 10*s*.

Consideration partly valuable and partly otherwise.

A case that sometimes occurs, and which is practically identical with the first of the foregoing cases, is where an assignment is expressed to be made partly for valuable consideration and partly for other reasons. In such a case, the practice of the Inland Revenue authorities apparently is to require duty only in respect of the valuable consideration. Thus, in a recent case which came under my notice, the consideration was stated to be £20 and also the natural love and affection of the assignor for the assignee; and the deed was adjudged to be duly stamped with 2*s*.

Declaration against merger.

When a mortgagor releases the equity of redemption to the mortgagee, the deed sometimes contains an express declaration that the mortgage is not to be considered as merged in the equity of redemption, the object of such declaration being to protect the assignee of the equity of redemption against any intermediate encumbrances. The fact that the mortgage in such a case is thus kept alive does not, however, prevent the release from attracting conveyance duty under section 57 of the Stamp Act, 1891, in respect of the amount due under the mortgage for principal and interest, in addition to the consideration for the release.

Release of equity of redemption in consideration of cancelling of debt.

It was held, in the case of The Scottish Equitable Life Assurance Society *v.* The Commissioners of Inland Revenue 1894 22 R. 85, that where the equity of redemption in mortgaged property is released to the mortgagee in consideration of the cancelling of a certain portion of the debt, ad valorem conveyance duty is payable in respect of the amount of debt so cancelled. If, however, the amount so cancelled exceeds the value of the property, the Commissioners of Inland Revenue will accept duty on the value of the property, on being satisfied as to such value. A deed of this description should, however, bear the adjudication stamp.

Mortgage to protect surety.

It was held, as long ago as 1852, in the case of Lord Canning *v.* Raper 1 E. & B. 164, that if one person becomes surety for another, and the latter assigns property to the former as security for any amount that the surety may have to pay, such an assignment is to be regarded as a mortgage, and therefore as attracting ad valorem mortgage duty on the total amount which may become payable under the bond into which the surety has entered.

Record of transfer attracts conveyance duty.

Another old case, that of Horsfall *v.* Hey 1848 2 Ex. 778, shows that an instrument, to be a conveyance within the meaning of the Stamp Act, need not actually purport to convey. In that case it was held that any instrument which operates as a record of the transfer of property is a conveyance for this purpose, and that the fact that the past tense is used is immaterial from this point of view.

Maximum collateral duty.

By section 7 of the Revenue Act, 1903, it is provided that the whole amount of duty payable in respect of an instrument which comes within the description of a collateral or auxiliary or additional or substituted security, or by way of

further assurance, shall not exceed ten shillings. It may be noted that the reconveyance of a collateral security is also liable to a maximum duty of ten shillings, but if the ad valorem duty is less than that amount, it will be sufficiently stamped with the less amount. In the case of the transfer of a collateral security, the duty is, in practice, also limited to ten shillings, but if the ad valorem duty would exceed this amount, the adjudication stamp should be obtained.

Consideration part present and part contingent.

When the consideration for a conveyance is in part a sum to be paid at the time of execution, and in part a sum to be paid on the happening of some contingency which may or may not happen, conveyance duty is payable in full on both portions of the consideration.

Sale of mortgaged property by mortgagor and mortgagee.

When mortgaged property is sold, and the mortgagor and mortgagee join in the conveyance, the mortgage debt being paid to the mortgagee out of the purchase-money, such a conveyance, although in a sense operating also as a discharge of the mortgage, only attracts ordinary conveyance duty, and not also reconveyance duty in respect of the amount of the mortgage.

Nominal consideration.

When in a conveyance for valuable consideration, a nominal consideration is also introduced, as where one of the parties joining in it is expressed to do so for a consideration of ten shillings, this nominal amount is apparently treated by the Inland Revenue authorities as part of the valuable consideration, and duty is charged in respect of it. Thus a conveyance of land subject to a mortgage of £300, in consideration of 10*s.*, has been held to attract a duty of £1. 15*s.*; and a conveyance of part of some mortgaged property, where a consideration of £150 was paid to the mortgagor, and 10*s.* was stated to be paid to each of three mortgagees, has been held to attract a duty of 17*s.* 6*d.*

Property taken in discharge of legacy.

It sometimes happens that when a pecuniary legacy is bequeathed, the legatee, instead of receiving the amount of the legacy in money, takes some portion of the estate in discharge, either in whole or in part, of the legacy; and such a deed of transfer as that required to carry out the transaction may thus become part of the title to a policy of assurance. It may therefore be worth while to notice that a deed of this description attracts, not the fixed duty of 10*s.*, but ad valorem conveyance duty on the amount of legacy discharged by such transfer.

Foreclosure order.

In connection with the subject of mortgages, reference has been made to an order for foreclosure absolute. Such an order was held, in the case of Huntingdon *v.* The Commissioners of Inland Revenue 1896 1 Q.B. 422, to be a conveyance on sale within the meaning of the Stamp Act, 1891, section 54, and therefore to be chargeable with ad valorem conveyance duty on the amount owing for principal, interest and costs. It is also held, as might be expected, that a conveyance by the mortgagor, or by an officer of the Court or other person appointed to convey to an equitable mortgagee, in pursuance of an order for foreclosure of an equitable mortgage, attracts similar conveyance duty. The matter has now been dealt with by statute, and section 6, of the Finance Act, 1898, provides that: "For the "removal of doubts with reference to the effect of sections 54 and 57 "of the Stamp Act, 1891, it is hereby declared that the definition "of 'Conveyance on Sale' in the said section 54 includes a decree "or order for, or having the effect of, an order for foreclosure." It is also provided that the duty shall not exceed that chargeable on a sum equal to the actual value of the property, and that if the value is stated in the order, such value shall be conclusive for the purposes of duty; and that any conveyance following upon such order or decree shall be exempt from ad valorem conveyance duty.

Conclusion.

In concluding this paper, which has grown to a far greater length than I anticipated at the outset, I may perhaps be permitted to make one or two remarks by way of forestalling some at least of the many criticisms to which it is open. Even with the limitations which I imposed upon myself in regard to subjects discussed elsewhere, and to which I have referred in my introductory remarks, I am painfully aware that my paper is very incomplete. Probably everyone who reads it will at once call to mind some points which manifestly come within its scope, and which I have left unnoticed. Many such omissions are no doubt

Reasons for certain omissions.

due to sheer forgetfulness or ignorance on my part, but there may, perhaps, be a somewhat better excuse with regard to others. I have confined myself, at any rate to a great extent, to matters that have come under my own personal notice in the course of my official duties, and in connection with efforts to assist students in their difficulties, or, at least, to such matters and to those which have naturally arisen from a

consideration of the legal problems so presented. It is no doubt possible, and even probable, that by making enquiries among other members of this Institute, I might have collected a much larger number of questions for consideration; but had I done so I should, I fear, have tended to limit the discussion which, I hope, will follow the reading of this paper. That discussion will, I feel sure, bring to light many points not referred to by me; but the great advantage of leaving them to be introduced then, rather than seeking to refer to them myself, is that they will then be dealt with by those who have already had to consider them as concrete cases in everyday business life, and who, for that reason, are able to present them in a more practical and forcible manner than is possible to one who approaches them from an outside, theoretical point of view, as interesting legal problems.

Life Assurance Companies Acts 1870 to 1872.

There is one omission, however, which will at once strike everyone, and which therefore, perhaps, calls for some further explanation. I refer, of course, to questions connected with the Life Assurance Companies Acts, 1870 to 1872. I need hardly remind the Institute that this matter some years ago formed the subject of a most valuable paper by Mr. George King, who then dealt very fully with it; but it may be thought by some that, in view of the great interest taken in it at the present time, a further discussion of the subject would be opportune. I am not altogether disposed, however, to take this view myself—at least as far as a discussion at a public meeting of the Institute is concerned. It is generally understood that proposals for amending the Acts and rendering them more suitable and effective under present conditions, are likely to be brought forward in the near future; and if on such an occasion the opinion of this Institute is to exercise that weight and authority that it should, in influencing any proposed legislation having for its object the regulation of the conduct of life assurance business, that opinion should be clear and unanimous. It is, I believe, the practice of the Judicial Committee of the Privy Council, with regard to any case coming before it, that whatever differences of opinion may exist among the members, these differences are not allowed to appear in the final decision; the judgment, in the form of advice to the Crown, being expressed as the judgment of the Judicial Committee as a whole, although in reality it may only be the opinion of a majority of the members.

Reasons for avoiding public discussion at present time.

I cannot but think that in regard to the very important question of any alteration in the Acts of 1870 and 1872, a similar method should be adopted by this Institute; and that whatever differences of opinion may exist as to proposed amendments, such differences should be discussed privately rather than publicly, and that any advice to be tendered to the Government on the subject should appear as the advice of the Institute as a whole, and not as the advice or opinion of any individual members or majority of members.

Other reasons why the Acts of 1870 and 1872 are not discussed in this paper.

Even if it should be deemed advisable to have a public discussion on the subject at the present time, it will, I think, be readily admitted that the importance of the matter demands that it should be dealt with in a paper by itself, and not as a mere incident among others, in a miscellaneous paper of the present description. Moreover, all will probably agree in thinking that it should be dealt with by one of our senior members who could speak from a long experience of the working of the present Acts, and who would, perhaps, for that reason, be less likely than one of the younger members, to make suggestions for amendment which, however excellent they may appear from a theoretical point of view, might lead to serious difficulty in practice. For these and other reasons, I considered it inadvisable to discuss the Acts of 1870 and 1872, or to refer to them in such a way as should lead to a discussion of them; and have contented myself with referring to such legal subjects as arise in the ordinary routine of office practice, and concerning which a free interchange of opinion is likely to be altogether beneficial.

TABLE OF CASES CITED.

For facility of reference, each case is given under the name of either party to it.

INDEX.

Abstract of the Discussion.

Mr. J. R. HART said the paper indicated in a very striking manner the large number and peculiar nature of legal questions connected with life assurance. The author had not, however, dealt with the whole range, as, besides those discussed, there were the numerous points connected with life office investments, and the Life Assurance Companies Acts. As there were about 110 cases discussed in the paper, there was ample evidence that the legal side of the assurance business was a very important one. The paper might be divided roughly into two parts, one dealing with the forms, contracts, and methods employed in a life office, and the other with the large number of questions that arose in connection with title. The first case was that raised by the Commissioners of Inland Revenue, as to whether a double endowment policy was a life policy for purposes of stamp duty. While it was satisfactory to learn that the contention of the insurance company was upheld, it would be noticed that the Judge dealt with the policy contract as a whole, taking the death portion with the endowment portion. The question naturally arose as to whether, for the purpose of stamp duty, the Inland Revenue authorities would admit that a pure endowment, with or without return of premiums, would also come under that decision. He had made enquiry, and had found that the authorities had admitted that. Then the question arose whether they would admit it also for income tax purposes. It was difficult to see how it could be otherwise, because the premiums for deferred annuities were exempt, and deferred annuities were simply a series of endowments.

There were two points with regard to insurable interest, as to which some doubts might be raised. First, as to whether a policy could be assigned in such a way as to get over the want of insurable interest, and secondly, in the event of a company successfully contesting a claim for want of insurable interest, whether the premiums were returnable or not? The author showed that such an assignment had been held to be just as much in contravention of the Act as if the assignee had effected the policy. As bearing on that and the question of the premium, he might mention a case decided in the Scottish Courts only last year, and not mentioned in the paper, "McDonald *v.* The National Mutual of Australasia." An elderly man, in very impecunious circumstances, effected a policy and assigned it for £5 to another person, who paid the first premium and who evidently took up the insurance as a speculation. The company refused to pay the claim on the ground of want of insurable interest, and their action was upheld by the Court. It was important to notice that, although the agent was as much responsible as in the cases quoted by the author, the company did not have to return the premium. As to questions arising in connection with the proposal, that afforded an illustration of the results of loose methods of business. However carefully the actuary might prepare the forms, it was left to the

agent or inspector to secure the proposal, and he thought it had to be admitted that the exactitude and care employed in the office were not always exercised outside. That was borne out by the cases quoted by the author. In all such cases, disputes arose through the agent improperly filling up the proposal form. It would be noticed, however, that two of the instances quoted in the paper related to accident, and one to fire. Fortunately in life business there was a medical examination, as an important check on the proposal; otherwise it seemed quite clear, from the way in which business was sometimes done in the outside field, that there would be a very large number of disputes on similar points. As to how far the company was bound by the knowledge of its agent, the author discussed on pages 122–125 the conflict of opinions and decisions on the question, in the light of the important "Bawden" and "Biggar" cases, and feared that the more unfavourable decision might be followed in future cases. It was satisfactory, however, to find in the suit previously mentioned, "McDonald *v.* The National Mutual of Australasia", that, although it was urged that the company's agent knew that the life had been declined, the Judge followed the "Biggar" case, where it was held that in filling in false answers the agent was not acting as the agent of the company, but as the agent of the applicant.

Conflict of laws in relation to contract was a difficult and complicated subject, which, so far as he knew, had not been brought before the Institute on a previous occasion, and therefore the members owed a great deal of gratitude to the author for discussing it so fully and clearly. One of the most common ways in which it arose in an office was as to the manner in which policy loans should be carried out, when an English office was dealing with a domiciled Scotsman. Scottish solicitors had advised that the form of the deed should be Scottish, but in the light of the author's arguments he failed to see why that should be so. The policy would be an English contract, although the company had a branch office in Scotland, according to the decision in "Parken *v.* Royal Exchange." Then the loan appeared to be a separate contract, while it was also apparently an English one. He should very much like to have the author's views on that important point. He also desired to learn the practice of other offices in respect to the question, as it seemed that if the company was content to ignore the personal covenant it would be safe in using an English form in all cases.

Passing on to title questions, an example of a question where practical considerations might, he thought, be taken into account was where one was dealing with the requirement for formal re-conveyance discussed by the author of the paper. In dealing with a first class bank, it was perhaps the general practice to accept merely a letter from the bank, stating that they had no further claim on the policy. In dealing with private persons he should accept in most cases a similar disclaimer. The author mentioned some transactions in which he said it was generally considered

advisable for the company to require that the persons with whom they were dealing should show a title in law and equity. He would be glad if the author would mention the nature of such transactions. On the question whether production of the deed was notice under the Act of 1867, he could not share the opinion of the author. It appeared to him that the intention of the Act was to postpone the investigation of any deed by the company till some payment had to be made under it; and to provide that in the meantime the company should have some written document to keep. If a deed was merely produced and taken away again, the company would have nothing. For that reason, a certified copy of the deed under the Scottish Act of 1862 would be sufficient, because the company could keep it, but the mere production of the deed itself was insufficient. When a deed was produced, with an application for payment under it, the case was different; the office had to investigate the deed and keep it, and a notice seemed unnecessary. In that connection the phrase "Withdrawal of notice", so commonly used, seemed to be objectionable. The notice being on the company's books, the deed relating to it must be investigated. The claimant under the deed might withdraw his claim but not the notice. Two interesting cases regarding notices to trustees of reversions illustrated the risks attaching to dealing with those securities. In the case of "*In re* Wasdale", where all the trustees died and the executrix of the last surviving one carried on the trust, one would like to know why the notices given to those trustees were not handed on to the executrix and by her to the new trustees. The result was that an encumbrancer enquiring of the new trustees was misled, and it would appear that if it was not incumbent on new trustees to see that notices were handed on to them, one ought to enquire into the history of the trusteeship in every case where one was purchasing or lending upon a reversion. He should be glad to have the author's views on that point. It seemed to him to illustrate more than ever the risks of dealing with reversions, and pointed to a higher rate of interest than was obtained being called for. It was only necessary to compare the method an office adopted for safeguarding its notices, and the method adopted in a solicitor's office, to see the additional risks attaching to reversions.

With regard to claims, on the question of paying to a person domiciled abroad, under the Revenue Act, 1889, the author stated that the Commissioners contended that such a payment could be only made after due provision had been made for estate duty. He thought, however, the members could be satisfied that the point had been disposed of, in view of the clear statement made by Mr. Denmead, of the Estate Duty Office, in a paper read before the Faculty of Actuaries last month. Mr. Denmead stated that the Act did not exempt moneys from liability for estate duty, and that under section 9 of the Finance Act the Crown had a charge on the monies to the extent of its claim, but the persons accountable for the duties were the persons to whom the moneys were payable by

the company. The Crown might obtain an injunction if they heard of the proposed payment, but otherwise the company appeared to be quite safe in paying over the moneys. The next point in that section was as to the specific bequest of a policy where it vested in the beneficiary with the assent of the legal personal representative. It was useful to learn from the paper that the Inland Revenue claimed that the assent should be stamped as a conveyance. He had had his attention called to a case decided in 1905, "Kemp *v.* the Commissioners of Inland Revenue", where it was decided that if the assent was merely under hand no stamp was required.

With regard to bankruptcy, one of the most important cases discussed was that of the "Scottish Union *v.* Fairley", where the office had lent to the assured in ignorance of his sequestration, and it was held that the claim of an assignee of the trustee in bankruptcy was in priority to that of the office. Following the author's reasoning, it seemed quite clear that an English office was in the same position in dealing with a domiciled Scotsman as a Scottish office, and that it was necessary to search for bankruptcies in Scotland. In criticizing that decision, the author stated that it was hard to understand how the bankruptcy of 1856 could override the provisions of the Policies of Assurance Act, 1867, which declared that the date on which written notice was received should regulate the priority of all claims. But he noticed that even in the English case, "*In re* Wallis", quoted by the author, it was clearly decided that a trustee in bankruptcy was not in the position of an encumbrancer for value; he was merely a statutory assignee, and did not get the same rights under the Policies of Assurance Act as an ordinary encumbrancer for value. Further, the author did not mention the clause in the Scottish Bankruptcy Act which protected the debtor who made a *bonâ fide* payment in ignorance of sequestration. The effect of that provision was fully discussed by Mr. W. J. Crosbie, in a paper on "Daily Questions of Legal Title", read before the Glasgow Actuarial Society five years ago, from which it appeared that in virtue of that clause a company paying a claim would be protected, though not in granting a loan or paying a surrender.

In conclusion, with regard to the paper as a whole it seemed to him that one or two of the subjects, like bankruptcy or conflict of law, might perhaps have been better dealt with in separate papers. Very little was known of such matters, and they might well be treated by Mr. Barrand. Secondly, there was a need for a thorough index of the subjects and cases discussed, in order that the paper might be made available for ordinary use. There was also a necessity for keeping it up to date, because cases had occurred since the paper was written which slightly affected some of the conclusions. Apart from that, he thought it was difficult to over-estimate the value of the paper; and whether considering it as an aid to the student, or as a work for every day office use, he thought few contributions to the proceedings would meet with more wide-spread or thorough appreciation.

MR. E. J. MACGILLIVRAY (a visitor) said that he should like to refer to that portion of the paper on pages 134–138, dealing with a policy as evidence of contract. The author cited two recent cases, which were of very considerable importance, one being the case of "Horncastle *v.* Equitable Life Assurance Society of the United States", and the other the recent case in the House of Lords, "Bailey *v.* British Equitable Assurance Company, Limited", 1906 A.C. He thought, himself, that those two cases had a very much greater significance than the author attributed to them. They were really authorities, or perhaps it would be better to say examples, of a very broad legal principle, namely, that, where there was a written contract, extrinsic evidence must not be admitted to vary that contract or to add to it. Of course, apart from that, there were instances where extrinsic evidence was admitted, where the Court was asked to rectify the contract, and that might be done on the grounds that the written contract did not embody the real contract between the parties previously entered into, and that there had been some mistake in the drawing up of the formal document, but that was apart from questions of the kind under consideration. The question of rescission of the contract on the ground of misrepresentation, again, was apart from the question raised in the paper, which was how far extrinsic matter could be read into a contract that had been completed, the contract being contained in a policy. The first question really was whether extrinsic matter could be read into the policy at all, so as to vary the policy or add to the policy. He thought the clear answer was that it could not, and that if the matter was not expressly embodied in the policy, either in the actual words of the policy or by reference, as in the case of the proposal, no other matter could be read into it, and it was not possible to connect any other matter with the policy by parol evidence. That was a doctrine which was perfectly clear. The next question was whether, if extrinsic matter could not be read into the policy to vary or add to the policy, the extrinsic matter could be given the effect of a separate contract, a contract collateral to the contract contained in the policy. There was no doubt that in many cases which came before the Courts on other matters, cases, for instance, of landlord and tenant, extrinsic matter was read as containing a collateral contract, not as varying the written contract or adding to the written contract, but as being something separate and another contract altogether. (The speaker then referred in some detail to the reported landlord and tenant cases of "De Lassalle *v.* Guildford", 1901, 2 K.B., 215, and "Henderson *v.* Arthur", 1907, 1 K.B.)

In applying suits like "De Lassalle *v.* Guildford" to insurance cases, it was extremely difficult to find that they could be made applicable at all. Lord Bowen laid down very clearly the principles upon which it was possible to consider some matter that was stated orally or extrinsically to the written contract as a separate or collateral agreement, and give effect to it as such. He said: "Suppose two parties should make an oral contract, with the intention that it

should be afterwards reduced into writing, and that that which is reduced into writing shall be the only contract, then of course one cannot go beyond it; but if they intend, as they might, that there should be something outside such contract, they might agree that that should exist, notwithstanding it was not in the contract which was put into writing." Therefore, when attempting to set up some extrinsic matter as collateral to the written contract, it was incumbent upon the person attempting to set it up to show, first, that the parties entering into the agreement did not intend that the written document should cover the whole transaction between them; and, secondly, that the alleged collateral agreement was not inconsistent with what was contained in the written document. He did not think it was ever possible in the case of an insurance policy to show either of those two things, because it was perfectly obvious that it always was the intention of the parties, in the case of an insurance contract, to embody the whole agreement in the policy, and to leave nothing outside the policy, and consequently any extrinsic matter which might be set up was almost certain to be in some way or other inconsistent with the terms of the written policy, and when it was set up it was almost invariably an attempt to alter what was in the written document. It was perfectly clear that was so in the two cases referred to. In the Horncastle case it was clear it was an attempt to vary what was written in the contract, and it was also so in the case before the House of Lords. The author suggested it must not be too hastily assumed that on the authority of those cases no effect would be given to representations made by an official of the company prior to the issue of the policy, and that the authority of the cases was weakened by the case of "De Lassalle *v.* Guildford", but he himself submitted that those cases did represent a very broad legal principle, which was applicable not only to insurance cases but to all cases which came before the Courts, and that was, that extrinsic evidence could not be admitted to vary the written agreement.

Mr. J. H. BARNES said there was one small point, dealing with the question of evidence of age, not noted in the paper. In the anxiety to secure business, there was a great temptation on the part of offices to leave out of the proposal forms anything that was likely to alarm persons who were filling them up, and with that object in view the requirement with regard to evidence of age was in many cases softened down, so that in the proposal forms there was merely a statement that it was desirable that evidence of age should be furnished at an early date. In other cases there was actually no reference at all to the necessity of furnishing evidence on that particular point. Then, when the premium was paid, and the assured received his policy, he found that one of the conditions of the contract was that he should furnish evidence of his age. While to all insurance officers the questions were perfectly simple and clear, it was possible to understand an outside man saying, "If I am called upon to furnish evidence on this point why not on any other point, such as the age of my father at the date of his death,

or the cause of his death, and so forth"? In the actual case which came under his notice, the company was advised that it was quite impossible to impose a restrictive condition after the premium had been paid. The man came and said he was absolutely unable to furnish any evidence of age whatever, that there was nothing drawing his attention to the fact on the proposal, and that it was not until he got the policy that he found the condition, and he therefore asked that the condition should be waived in his particular case. It would be particularly interesting to have some information from the author on that subject, because, unless the advice received was wrong, it appeared necessary to have in the proposal form a distinct statement that evidence would be required on this point. The office of course had the right, if the declaration embodied in the proposal was properly drawn, of avoiding the contract or calling for its adjustment, if it could prove that the age was wrongly stated, but what was required was, not that the company should be in a position to avoid the contract after having obtained evidence that the statement was wrong, but that they should throw the onus on the insured of proving that his statement was right.

MR. W. C. SHARMAN desired to allude to the case of the "Scottish Union *v.* Fairley." The author took the view that if the case had referred to an English company granting a loan, the decision of the Courts would have been the same as in the case of a Scottish company granting the loan. If the matter were tried in the Scottish Courts that would certainly seem to be the case, as it was a principle, fairly well established, of international law that all questions relating to priority of creditors in bankruptcy should be referred to the *lex domicilii*, which in that case would be Scots law. If however the case came before the English Courts, possibly a different view might be taken. There was another principle of international law also fairly well established, which laid down that the form of procedure, and the evidence required to prove a contract, must be always according to the law of the country in which the case was tried, *i.e.*, the *lex fori*. It was distinctly laid down in the Policies of Assurance Act that no assignment should confer on the person therein-named any right to sue, unless written notice of the assignment had been given to the insurance company. In these circumstances it would seem that a Scottish trustee coming before the English Courts would, if he had not given notice, be unable to produce sufficient evidence to satisfy the Court of his right to sue, and the title of a subsequent mortgagee who had no notice of a prior charge would therefore prevail. He mentioned the point with deference, in view of the high legal attainments of the author of the paper, but it seemed that it might have some bearing on the question.

MR. C. R. V. COUTTS said there was one small point arising out of the remarks of a previous speaker with regard to extrinsic or collateral contracts which he should like to mention, namely, the liability to pay surrender-values. Did that count as a collateral or extrinsic contract, when it was not embodied in the policy? He

knew it was the custom in more recent years in many cases to provide for surrender-values, and sometimes to put a scale of surrender-values on the policy, but the majority of the older policy forms contained no definite contract to pay any surrender-values at all. As a rule, there was a statement in the prospectus at the time of issue giving a scale, and the question was whether the office was bound by that scale as a collateral contract, or whether it could adopt a new scale or new conditions, say a non-forfeiture condition, by which the value would be gradually absorbed in payment of the premiums if the policy was allowed to lapse. That was rather an important point that might arise, and he did not think the author had touched upon it in his paper.

Mr. W. T. MAY observed that there was a small point on page 153 of the paper, with reference to the question as to whether a mortgagee who had a power of sale in his deed could come to the office and claim to surrender. He had been advised very often by solicitors that they could not safely accept a surrender in such a case. It was held that the essence of a sale was that it should be at a sale price. It was well known that the Commissioners of Inland Revenue, adjudicating on surrender-values of policies for estate duty purposes, added a good deal in order to get the sale price. If the mortgagee actually sold the policy to the company under a power of sale, and the real consideration was the surrender-value, he thought very likely the Courts might enquire and say that the transaction was a surrender and not a sale, and that there was no power to surrender. There was a case where refusal to accept a surrender would press rather hardly on the mortgagee, and that was where the mortgagor, who was also the assured, had disappeared. In that case, the policy had really no sale value; it could not be sold at the mart, because the assured had disappeared, and under these circumstances it was clear that the surrender-value was a good consideration. The point was not of much consequence, but it occurred so often in practice that, if the specific power to surrender could be dispensed with, it would save a good deal of trouble in dealing with clients.

Mr. H. E. W. LUTT understood Mr. Hart to say, with regard to an English policy on which a loan was made to an assured domiciled in Scotland, that the deed should be executed in accordance with English law. One English company had, however, been advised by a firm of solicitors that where they made a loan on a policy to a man domiciled in Scotland it required to be witnessed by two persons in conformity with the Scottish practice. Perhaps there was reason for that. If executed in England, the policy was an English contract, but the mortgage deed made by the insured in Scotland was possibly interpreted as a Scottish contract. On page 213 the author mentioned a case of repayment of a small proportion of a loan, and a transfer of the remainder. Although the case had not actually come to a decision, he took it that when that loan was paid off and a re-conveyance of the security made to the original mortgagor, the re-conveyance would require stamping in respect of

the whole amount originally secured. Perhaps the author would be able to give some information on that point. Questions had been asked as to the practice of searching in bankruptcy. It might be of interest to the meeting to know that one company made a practice of searching in every case, whether the title was English, Scottish or Irish; and, before they made payment of a surrender-value, even in an ordinary English title, it was their practice to make a search, to see that nothing had been registered against the policyholder.

Dr. A. W. Findlay, referring to the case of "Gedge *v.* Royal Exchange Assurance Corporation", discussed by the author on page 121, with reference to the question of insurable interest, and the rights of defendants other than the insurer to plead the Gambling Act as a defence, stated that the Gambling Act only went to avoid the contract of assurance, and consequently could only be pleaded by one of the contracting parties, which in the case of a claim for the policy moneys would be the insured or persons claiming through him. It seemed clear that, although the Gambling Act might be pleaded, and would, although not pleaded, avoid the contract if the facts showed that it was clearly illegal—as in "Gedge *v.* Royal Exchange",—it would not be applicable in any question arising among the parties beneficially entitled to the policy moneys after they had been paid by the company to the administrator, or whoever might have the distribution of them; the administrator would be a quasi-stakeholder, any claims against him would depend, not upon the legality of the assurance, but upon the rights of the claimant, however acquired. The author rather suggested a doubt as to this, and cited "Gedge *v.* Royal Exchange" as weakening the decision in "Hadden *v.* Bryden", whereas the decision in Gedge's case was on another point altogether.

The author, in referring to "Bawden *v.* the London, Edinburgh and Glasgow Assurance Company"—as to the authority of an agent to bind the company,—dealt with a case in which the assured had only one eye. This naturally was known by the agent when he effected the insurance, and it was his duty to pass the knowledge on to the insurance company, but he failed to do so. The other cases dealing with the authority of an agent were cases in which the agent fraudulently or ignorantly filled up the proposal form which the insured signed. He would suggest to the author that the decision in the case of "Bawden *v.* the London, Edinburgh and Glasgow Assurance Company" was quite right, because in that case the insurer took upon himself the knowledge of the physical defect of the loss of the eye, and the case could be supported on that ground. This distinguished the case from the other cases cited, in which the assured must be taken to have known that he was signing an incorrect statement, and could not subsequently be heard to say that the statement which he had signed, and which formed the basis of the contract, was not his statement.

Dealing with the question of notice, he observed that the author had not dealt with the effect of the Judicature Act on the question of notice to insurance companies. He himself had always been

under the impression that the Judicature Act, 1873, had enlarged the powers of assignment of policies given by the Policies of Assurance Act, 1867, and that the equitable doctrine of notice must be considered in investigating the titles to policies before payment of the moneys assured thereby, and that it was not necessary that there should be a formal notice in writing to establish priority of title. The question of conflict of laws had been dealt with very exhaustively by the author, and it must probably be taken that the *lex loci contractus* would apply to intermediate dealings with the policy, quite independently of the law of the original contract embodied in the policy (see "Lee *v.* Abdy", 17 Q.B.D., p. 309), and that mesne assignments, mortgages, and other dealings with the policy, must be considered in relation to the law of the place in which those dealings were made, unless there was some expressed or implied intention of the parties that some particular law should apply; still, for the purpose of the actual contract, the policy alone must be looked to, and the law governing that, which in most cases would be the law of the place in which the contract was entered into.

Mr. DOUST SMITH referred to the case mentioned on page 159, "Swanley Coal Company *v.* Denton." In connection with that case the author discussed the question as to the effect on a Bill of Sale of properties being included in the schedule to the bill other than those coming within the definition of a personal chattel. With regard to that point, the speaker would like to know in what circumstances such a document would come before the notice of insurance companies. Undoubtedly, it would be as evidence of title, in that there had been some notice, direct or constructive, of the fact that the policy had been dealt with. Therefore, when the author dealt from an assurance company's point of view merely with the abstract question as to the document being void as a bill of sale, he did not sufficiently emphasize the fact that the document might be valid in other respects. In the case under discussion, the author did not refer to the fact that the Court was mainly concerned with the circumstances under which the muniments of title were handed over; in point of fact there was no evidence to show the actual intention of the parties. But when an assurance company is dealing with a case where a policy has been included in a bill of sale, there is generally evidence before them to show the intention of the parties to transfer an interest in the policy, and therefore it was essential to remember, not so much the doctrine laid down in the case quoted by Mr. Barrand, as the doctrine laid down in such a well-known case as "*In re* Burdett", 20 Q.B.D., 31, in which a chattel other than a personal chattel was included, and the bill of sale was held valid as an equitable assignment of the other chattel, although invalid as a bill of sale. There was also a more analogous case to the one discussed by the author, namely, "*In re* O'Dwyer", 19 L.R. Irish, 19, where a bill of sale included a mortgage of chattels real. The Court held that, as there was evidence before it, apart from the mere recital of the deed of mortgage in the schedule, of an intention to create

a security on the chattels real, although invalid as a bill of sale, the document might be valid in so far as it was a mortgage of the chattels real. Therefore, he thought that when an office was dealing with a bill of sale, it should consider more particularly the question whether the document did or did not operate as a good equitable assignment of the policies included in the schedule thereto.

There was another case, which he understood had been decided after the author had written his paper, involving a question of importance to offices, when considering payment to an assured who was an undischarged bankrupt. Some offices held that it was desirable to require the concurrence in the receipt of the Official Receiver or Trustee in Bankruptcy, even if the Trustee had not intervened. If that line was taken, it was essential to study the case mentioned by the author on page 169, namely, "*re* A. Bennett *ex parte* the Official Receiver", which was an attempt to extend the doctrine laid down in the case of "Cohen *v.* Mitchell", namely, that any person dealing with an undischarged bankrupt *bonâ fide* and for value before the Trustee intervened obtained a good title. There was an attempt in that case to extend this protection to a legatee, but it was held by the Court that the doctrine was laid down purely for the protection of the *bonâ fide* holder for value. Therefore, if a company which held the opinion mentioned had an assignment which partook of the nature of a voluntary assignment, they should see that they obtained the concurrence of the Trustee in Bankruptcy with the voluntary assignee in the receipt.

Mr. J. E. FAULKS associated himself with previous speakers in expressing his gratitude to Mr. Barrand for his able and instructive paper. The author had the highest qualifications for the task he had undertaken. In addition to his knowledge of law and his great industry, of both of which the paper afforded ample evidence, he had opportunities, which were practically unique, of becoming acquainted with the many points of title which might conceivably arise in connection with life policies. He had understood from Mr. Barrand that something like 6,000 titles passed through his hands in the course of a year, and therefore it was obvious he must have had occasion to consider many points that in the course of a more limited experience would occur only once perhaps in an official lifetime.

He proposed to deal with just one or two points that were mentioned in the paper, or that had arisen in the course of the discussion, and then say a few words on the paper as a whole. One point had been raised in the discussion with regard to a Power of Sale, and in that connection he might say that some years ago the question arose whether deeds containing a power of sale only could be acted upon in the case of a surrender to the office; counsel was consulted on the subject, and his opinion was, "Certainly not with the ordinary form of surrender receipt; you must have a properly drawn receipt under a power of sale, and that must be stamped with a conveyance stamp on the amount of the

surrender-value." On that opinion the office acted for some years, but presently a solicitor, acting in the case of a large surrender, said that he would not pay the *ad valorem* duty, and he eventually got the stamp duty adjudicated at 1*d.* The position therefore now was that a slightly different form of receipt was taken, where there was only a power of sale, from that taken where there was a strict power of surrender.

With regard to the question of estate duty in the case of payment of claims under the Revenue Act, 1889, Mr. Hart had referred to a statement by Mr. Denmead, in a recent paper read before the Faculty of Actuaries, to the effect that in such cases the office was under no liability to see to the payment of the estate duty. It was, however, only last year that that point had arisen in his own personal experience, and the authorities were distinctly under the impression that the company was liable to see to the payment of the duty. The history of the point was rather curious. When the Finance Act of 1894 first came into force, a member of the Institute obtained an opinion from the Inland Revenue authorities that in such cases the office was under no liability to see to the payment of estate duty, but subsequently the Inland Revenue raised an entirely different contention. In that connection he had noticed one omission in the list of cases cited by the author, that of the "Attorney-General *v.* Wack", reported in *The Times* of the 4th June, 1898 or 1899, a case on which he believed the authorities relied, in stating that the office was liable to see to the payment of estate duty.

A somewhat surprising thing in the paper was the extract the author gave from a modern text-book stating that there was no right of foreclosure of a *chose in action.* He had not seen the context, but he could not help thinking that there must be some grave misapprehension. He could not see why there should be any difference, as regarded a policy of insurance, from cases of foreclosures of reversions, and, speaking roughly, he thought he had seen something like twenty of the latter. He was quite in agreement with the author's criticism of the extract, although he thought it might have been put a little more strongly. He would like to call attention to the expression which occurred in a case about a policy which was granted "for behoof of" somebody else, issued in Scotland. He should like the author to say whether he agreed with him in thinking that no such policy should be issued, in England at any rate. In England there was, as far as he knew, no way in which such a policy could in general be issued. A could insure the life of B, if he had a proper insurable interest. A wife might insure her life for the benefit of her husband, or a husband his life for the benefit of his wife, or either for the benefit of a child, but otherwise he did not know that in English law there was any way in which a person could insure his own life for the benefit of another.

The author suggested that "Notes" of cases should be inserted in the *Journal.* In the course of some work he recently had the

pleasure of doing for the Institute, he referred to a volume of the *Journal*, and, quite by chance, came across a letter from a then member of the Institute suggesting a similar course, written as long as thirty-two years ago (*J.I.A.* xviii, 297). He thought some of the members would welcome such an arrangement, not only for its immediate usefulness, but also because it would, to some small extent, tend to secure for the Institute the same position on the practical and legal side of their business that it undoubtedly occupied on the more theoretical and scientific side. Speaking quite unofficially, and also with due regard for the good work done elsewhere, he for one looked forward to the time when all such matters as notes on legislation, the obtaining of counsel's opinion, the circulation of reports of legal decisions, and so on, should be part of the work of the Institute, when in fact the whole of the corporate work of the actuarial profession should be done by or under the *ægis* of the Institute itself.

THE PRESIDENT, in moving a hearty vote of thanks to Mr. Barrand for his paper, hoped that the author would add a copious index, because he felt that the essay would become a text-book. With regard to the proposal which Mr. Barrand had called attention to and Mr. Faulks had referred to, he had very much pleasure in stating that the Council at the last meeting decided, on the recommendation of the Editor, to make arrangements for statements as to law cases to appear periodically in the *Journal*. The members would be also glad to hear that Mr. Barrand had been asked to undertake that very onerous duty, and it was hoped he would accept the position.

The motion was carried with acclamation.

MR. A. R. BARRAND, in reply, thanked the members for the kind reception they had given to the paper. With regard to the question raised by Mr. Hart, as to the formalities to be observed when a contract was made in England and the assured lived in Scotland, it was pointed out in the paper that if the contract was to be deemed to be made in Scotland there was no doubt at all that the *lex loci contractus* would apply, and the contract executed there must be executed in accordance with the formalities of Scots law. But the difficulty arose where the contract from its nature must be assumed to have been made in England, but at the same time the actual document embodying the contract was executed in Scotland. Mr. Hart had mentioned the matter to him in conversation a little time previously, and he had endeavoured to find some authority for it, but regretted to say that the law on that point was not quite clear. He had consulted the treatises of Foote and Dicey and Westlake, the chief authorities, and Dicey referred to a remark in Nelson's "Private International Law", where it was stated that *primâ facie*, and as a general rule, the formalities of a contract relating to movables required by the law of the place where it was made were necessary and sufficient for its external and formal validity in England, but, if the contract was intended by the parties thereto to be an English contract and

transaction, it would be a good contract and enforceable in England if it complied with the formalities required by the law of England; and Story in his "Conflict of Laws", in a note said: "The place where a contract is signed is not necessarily the place where the contract in contemplation of law is made, for it may not be the intention of the parties that it should go into effect there." The real answer, therefore, to the question appeared to be that, if from the nature of the transaction the contract must be assumed to be made in England, or, if not made in England, it was, from its form, the evident intention of the parties to make what the Courts would call an English contract, it would be sufficient if it was executed in English form. It depended to some extent on the actual way in which the transaction was carried out.

Mr. Hart had also raised the question as to whether a pure endowment was a policy of life assurance. If the first case he quoted was referred to, that of the "Prudential Assurance Company *v.* the Commissioners of Inland Revenue", it would be seen there that, although Mr. Justice Channell in deciding the case said the contract must be considered as a whole, yet he first of all analyzed the contract into its two constituents and said, "If it were to be split up and treated as two separate contracts, I should incline to the view that even the old age endowment portion of it would satisfy the definition." Those words alone appeared to be sufficient to show that a pure endowment was considered, at least by the Court in that instance, as coming within the definition of a policy of life assurance. Mr. Hart appeared to think that in the paper he expressed some fear that the decision in Bawden's case would be followed in future cases. If that impression had been conveyed by the remarks in the paper, he would like to say that it was not his intention so to do. The concluding words he had written on the subject would show, he thought, what his view was: "In view of the conflict of legal opinions and decisions on this point, perhaps all that can safely be said is that, while there seems to be a tendency to distinguish Bawden's case from others resembling it, and to confine its authority to cases standing exactly on all fours with it, nevertheless, while hoping for the best, we must be prepared for the worst as represented by that case." From one or two cases that had been decided lately, the tendency seemed to be to follow Biggar's case rather than Bawden's case, but at the same time he had quoted a very recent case in which Mr. Justice Grantham took the other and perhaps the worse course. With regard to the question of enquiries being made, not merely from the existing trustees, but from all previous trustees who might have received notice of any transaction affecting the trust, apparently from the decision in Wasdale's case that was the only safe course to adopt. Whether it was practicable or not was another matter; he was inclined to think it was not. It was, however, not wise to shut one's eyes to the fact that in not doing so some sort of risk was being taken. Mr. Hart had also raised a question as to whether the production of the deed was a sufficient compliance with the Act

of 1867, and differed from his own opinion on that point. There was of course room for difference of opinion on that as on most other points, and in matters of law as in other things he was not prepared to consider himself infallible. In that connection he might mention the case of "Derry *v.* Peck", where on a point of pure law the Court of Appeal unanimously reversed the decision of the Court below, and the House of Lords unanimously reversed the decision of the Court of Appeal. With regard to Horncastle's case, he was sorry if any words of his should have suggested that he had not fully considered the question as to when extrinsic evidence could be given in reference to a contract and when it could not. He had endeavoured to state the necessary conditions laid down on this point by the learned Judge in dealing with that case. On page 137 he had pointed out that Horncastle's case was distinguished from the other case, "De Lassalle *v.* Guildford", on the grounds that in the former (1) the representation set up was not a collateral agreement, (2) the policy was intended to cover the whole contract, and (3) the agreement and policy were inconsistent. He thought that was quite sufficient to indicate as to when they should take notice of a previous representation, and when it was safe to ignore it.

Mr. Sharman had referred to Fairley's case, and had suggested that the question of notice was a matter of form, and would be dealt with by the *lex fori*, rather than the *lex domicilii.* In that connection he might quote the words of Lord Watson where he said: "It would, in my opinion, be contrary to the elementary principles of international law, and, as far as I know, without authority, to hold that the right of a Scottish creditor when so perfected can be defeated by a transaction between his debtor and the citizen of a foreign country, which would be, according to the law of that country, but is not according to the law of Scotland, sufficient to create a real right in the goods." That was a distinct contribution to the question, and seemed to indicate that in the opinion of a distinguished judicial member of the House of Lords, in such a matter it was the *lex domicilii* that would prevail rather than the *lex fori*, in spite of the marked inclination of courts, wherever practicable, to apply the *lex fori.*

Mr. Lutt had referred to a case where a part of the loan was paid off, and had asked whether, when subsequently the remainder of the loan was repaid, the re-assignment must be stamped for the whole amount. The revenue authorities in such a case demanded a stamp in respect of the whole amount at any time secured. With regard to Mr. Doust Smith's remarks on the question of a bill of sale, the paper was dealing rather with present day cases, and assumed a knowledge of cases decided some time ago. The point he was dealing with, which he trusted had not led to any misunderstanding was that of the effect on the bill of sale itself. As he had said in his paper, a bill of sale could not even be used as evidence to prove that the policy was deposited with the intention of creating a charge on it; to the extent to which the bill showed this, it would itself be void altogether, as creating a charge on

something other than a personal chattel. Perhaps if he had said, "It would itself be void as a bill of sale", it would have been less free from any doubt, because that was what was intended. Granted that it was void as a bill of sale, it was still valid as an equitable mortgage of the property which ought not to have been included in the schedule. That was decided in the case of "*In re* Burdett" nearly twenty years ago.

With reference to the payment of estate duty in connection with policies paid under the Revenue Act, 1889, his attention had been called to the case of the "Attorney-General *v.* Wack." He was bound to say that he had left that case out of the paper with "malice aforethought." Some time ago he had occasion to know that the revenue authorities were quoting that case. They quoted it first with a wrong date, and he had some trouble in finding it, and when he found it he could not see that it had any very clear bearing on this particular subject, and therefore he had left it out. If the Inland Revenue authorities desired to set out a fairly good claim to estate duty in such circumstances, he might perhaps say, without egotism, that he thought they could make out a much stronger case by taking the line of argument he had discussed in the paper.

With reference to foreclosure and a recent text-book, he could assure Mr. Faulks that he had not misunderstood the author in any way. So far from that being the case, he had discussed the point with the writer of the book in question himself, and he appeared still to adhere to the views there expressed. Mr. Faulks had raised a point as to whether there could be a settlement policy apart from the Acts. With regard to that he would call attention to some words he had quoted on page 186, by Mr. Justice Swinfen Eady, when he was dealing with the question as to whether the policy in the case he had before him actually came under the Act or not. In connection with the suggestion that the policy, by reason of its terms, was not within the Statute, Mr. Justice Swinfen Eady said that the result would have been the same if it were held that the policy was not within the Statute, for in that case it was simply an agreement between the assured and the assurance society that the institution on his death would pay the money to the person appointed.

An Investigation into the Mortality among Scandinavian Emigrants to the Congo. By Paul Bergholm.

["Undersökning angående Dödligheten bland Skandinaviska Congofarande." Stockholm, 1906.]

In September 1905, at the request of Director Sven Palme, I began an investigation into the mortality among Scandinavian emigrants to the Congo, based upon a portion of the Statistical Reports contained in a work entitled "Scandinavians in the Congo", published in the summer of 1905 by the Danish Lieut.-Colonel

H. Jenssen-Tusch. I submit herewith the following account of the manner in which the investigation in question was carried out.

The material is gathered from the "Chronological Record "respecting 922 Scandinavian Congo Emigrants 1878–1904" contained in "Supplement No. 1" to "Scandinavians in the Congo." This record gives the number, nationality, name, occupation and date of entry for all Scandinavians emigrating to the Congo in each of the years from 1878 to 1904. The records as to date of birth and date of exit from the material for observation have been obtained partly, and to the greater extent, from "Scandinavians in the Congo", further from the "Swedish Mission Society's" Register of Missionaries sent out, and, finally, from official statements prepared ministerially by the General Secretariat of the Congo State.

The whole material is thus constituted of 922 persons, of whom 236 died during the period of observation—of these, 24 during the journey home from the Congo or later on as a result of residence in the Congo. From this material the whole number of entrants in the year 1904 has been separated—altogether 60 persons—because for these, with the exception of information as to the date of entry, the other necessary data were wanting in nearly every case.

The remaining 862 persons have been entered on cards on which are given the number, nationality, name, occupation, and date of entry direct from the "Chronological Record", relying on the data found as above as to the date of birth and entry.*

In cases where exit is caused by death the cause of death when known has been given on the card.

All the dates are given in years and months. Each person is assumed to have been born and to have died in the middle of the month. The ages at exit and entry and the periods of observation which are given on the cards are obtained by subtracting one date from the other, hence the ages and periods of observation are also given in years and months.

From the 862 cards thus filled up 39 have been excluded on

* As it is customary for an emigrant to the Congo, after two years of residence there, to seek some period of rest in his native land before returning thither, and as it would have made the investigation unnecessarily difficult to take account of such periods of absence from the Congo, the date of the last exit which was not followed by any re-entry has, as a rule, been taken as the date of exit. Only in a couple of cases, when the interruption in the residence in the Congo occupied a disproportionately long time and this time was exactly given, has it been taken into account, and the time of observation consequently divided into the earlier and later periods. The same person has thus been entered on two cards—that is, treated as two separate persons. For the persons who remained in the Congo at the close of 1904 the date of exit has been treated as 31st December 1904.

the ground of incomplete information respecting dates of birth and exit, leaving, for the mortality investigation, a body of 823 persons, with 230 cases of death, and with a total period of observation of 30,421 months. The average period of observation thus amounts to 37 months.

From this material there have been produced altogether seven tables. Table 1 gives a general survey of the entire material as it appeared before any eliminations had been made. When the numbers had been entered in columns 2 and 3, column 4 was obtained by adding the difference between the entries and exits for each year to the number found at the close of the preceding year. The numbers for column 5 were obtained by taking for each year the average of the numbers of persons at the close of the year in question and the preceding year. Column 6 was obtained direct from the original statements, and column 7 by dividing for each year 100 times the number in column 6 by the number in column 5. The figures thus obtained give the percentage of mortality among emigrants to the Congo for each year from 1878 to 1904. By dividing 100 times the total of the numbers in column 6 by the total of the numbers in column 5, the average percentage of mortality is obtained for the entire material for the period from 1878 to 1904.

For the following investigations the material has been divided into four different groups, with the occupation as the basis of division. As there are few different occupations represented, and these are of quite a different character one from the other, it has been deemed proper to try and discover the effect which different occupations would exercise on the mortality. The material has, in accordance herewith, been divided into

	Group 1.	Soldiers . .	141	persons with	65	deaths.
	„ 2.	Missionaries .	104	„	41	„
	„ 3.	Seamen . .	506	„	107	„
	„ 4.	Others . .	72	„	17	„
and	„ 5.	Total . . .	823	„	230	„

This division into groups has been carried out in Tables 2, 3 and 7.

Table 2 gives the exposed to risk and the mortality in different years of observation. The numbers in columns 2, 5, 8, 11 and 14, and the numbers in columns 3, 6, 9, 12 and 15 have been obtained direct from the original statements. By dividing 100 times the last named figure by one-twelfth of the former, the figures in columns 4, 7, 10, 13 and 16 were obtained. They give the percentage of mortality in different years of observation.

Table 3 forms an abridgment of Table 2. It shows the

percentage of mortality in different periods of duration, grouped so as to include approximately the same number of months of exposure. By dividing 100 times the total of the deaths by one-twelfth of the total of the months of observation, the average percentage of mortality is obtained for each group.

Table 4 gives the mortality of all those observed for different ages. Column 2 was obtained by adding together for each age the months of exposure to risk. The numbers in column 4 were obtained by dividing 100 times the numbers in column 3—which were taken direct from the original statements—by one-twelfth of the numbers in column 2. This gives a measure of the mortality among the observed at different ages. In order to facilitate comparison with other mortality tables this "observed force of mortality" has been graduated graphically for the ages 23 to 40, where the material is most complete. The graduated force of mortality is given in column 5.

Table 5 contains a "comparison between the actual deaths "and those calculated from the male mortality table of the "Swedish population for the years 1881 to 1890." Column 2 is identical with the corresponding column in Table 4, and the numbers indicated by q_x in column 3 are taken from the graduated male mortality table for the whole kingdom for the years 1881 to 1890. Column 4 has been obtained by multiplying for each age one-twelfth of the numbers in column 2 by the numbers in column 3, and column 6 results from dividing the numbers in column 5 by those in column 4. By dividing the sum of the numbers in column 5 by the sum of the numbers in column 4, the number 12·7 was obtained, which thus gives the proportion between the observed mortality and the mortality which might be expected to be found among an equally large number of men of the same age in Sweden.

Table 6 gives a graphic representation of the observed and graduated force of mortality (as set forth in Table 4). For the sake of comparison it also shows the mortality according to the male Swedish Mortality Table for the years 1881 to 1890 employed in the preceding table.

Table 7 exhibits the deaths divided into different occupations, and causes of death. As the number of deaths from unknown causes—altogether 39—is deducted from the total number of deaths according to Table 1, there remains a total of 197 cases of death from known causes.

The causes of death given in the first column of the table have

been arranged according to the apparent importance of their influence upon the mortality in the Congo. Column 7 was obtained by dividing 100 times the number in column 6 by the whole number of cases of death from known causes, 197. The numbers in column 7 thus show the percentages due to different causes of death.

In the tables explained above, the mortality among Scandinavian emigrants to the Congo has been set forth from somewhat different points of view.

In Table 1 it has been shown how the mortality varied in the course of the years from 1878, up to and including 1904. Four different periods worthy of consideration are noticeable with regard to mortality.

The first period includes the years 1878 to 1884; here the data are still scanty, and as a consequence the mortality fluctuates considerably: from 100 per-cent one year to 0 per-cent another.

The second period, the years 1885 to 1889, shows more stable conditions as regards mortality, with a relatively low mortality, averaging 5 per-cent.

The third period includes the decade 1890 to 1899. This shows an extremely high mortality, the rate varying between 9·2 and 15·5 per-cent. This decade forms an interval differing greatly from the preceding and following periods.

In the fourth period, years 1900-1904, the mortality percentage is relatively low, varying between 3·7 and 8·5 per-cent. For the year 1904 it was 5·6 per-cent.

If contrasted with the adverse conditions of mortality in the decade commencing with 1890, the last period thus shows a decided improvement. The experience in the other periods warns us, however, against entertaining too great hopes of the permanence of the improvement which has occurred. Nor do there exist any perceptible guarantees against a relapse into the enormous mortality of the period 1890-99.

In Tables 2 and 3 an endeavour has been made to show the effect on the mortality of the length of the period of observation, that is, of the duration of residence in the Congo. It is clear from both tables that the heaviest mortality arises during the first years of observation. In Table 3 it is seen also how, for all the observed periods, the mortality percentage falls with the length of the period of observation from 13 per-cent in the first year to 6·3 per-cent in the 6th to 10th years. An increase is certainly noticeable in the twelve last years of observation, but too much importance must not be ascribed to this. With the small material

which is available at these high observation periods, a single death more or less varies the rate of mortality appreciably.

Table 3 shows how the mortality is affected in different groups of occupation. The military group divides itself from the rest in a specially remarkable manner, with a mean mortality of 15·2 per-cent. It shows a mortality more than twice as great as that of the missionary group, whose corresponding figure is 7·5, and nearly double that of the seamen group with 7·7 per-cent. The mean percentage of mortality for the whole observed material, 9·1, is also far below the figure of the military group. As there is every reason to believe that the selection among entrants into the Congo is much more stringent among soldiers who are in the Government service than, for example, among missionaries working for private institutions, this great difference in favour of the latter is the more noteworthy.

From Table 7 it appears clearly that about 20 per-cent of the cases of death among soldiers arise from sunstroke, accident and injuries in battle, while among missionaries no case of death is noted from these causes.

This circumstance contributes in a measure to explain why the mortality in the military group is so much heavier than in the missionary group. That the seamen group, in which accident likewise plays a prominent *rôle* as a cause of death, has, nevertheless, such a decidedly lower percentage of mortality than the military group seems, however, difficult of explanation. No other explanation than the heavier claim upon the individual of military service, which does not at all times afford opportunity for due regard to personal safety, can be given here.

Table 4, Mortality Table, sets forth, as already stated, the mortality at different ages among all those observed. The graduated values of the force of mortality for the ages most numerously represented here show a feature worthy of note. At the youngest ages the force of mortality decreases with increasing age. Age 32 marks a turning point; thereafter an increase in the force of mortality is apparent. It seems thus as if the ages about 32 years were most capable of resistance to the ravages of the Congo climate.

Tables 5 and 6 set forth the mortality in the Congo in comparison with the normal Swedish mortality. As already pointed out the comparison shows that the mortality among Scandinavians in the Congo is nearly 13 times as great as the normal Swedish mortality.

Finally, as regards mortality as depending on different causes of death, it appears from Table 7 that *fever* is, beyond comparison, the most important cause of death, since nearly three-fourths of all cases of death are ascribed to this cause. It is malaria and its complications, hæmaturic fever and dysentery, which, in most cases, make the white man's grave. Tuberculosis seems, on the contrary, not to be of special importance, probably owing to the rigid selection before entry into the Government service. It is also significant that, of the five cases of death from tuberculosis which are recorded, not less than 3 were among missionaries who, thanks to their rigidly ascetic and generally quiet life, show themselves, in other cases above referred to, among those most capable of resistance.

As a general measure of the mortality among Scandinavian travellers in the Congo, this investigation fixes the mean mortality percentage at 9·1.

It is of special interest to compare this result with what is known from previous investigations concerning mortality in the Congo.

The author of "Scandinavians in the Congo" mentions in his work some officially prepared tables showing the mortality from 1880 to 1890 among white persons employed in the Government service or in private enterprises now abandoned.

These include a table of the average rate of mortality in the years 1890-1896, amongst Whites employed in the Congo Railway Company, given as 9·9 per-cent, of which 8·5 per-cent is due to sickness.

Another table gives the mortality among officials of the Department of the Interior—including soldiers, seamen, engineers, and artisans—from the years 1885-1896, with 352 cases of death for 5,068 years of observation. What is meant by a year of observation is, however, not identical with 12 months of exposure, but must, on an average, be assumed to be three-fourths of a year. The tables would thus give 352 deaths for 3,801 actual years of observation, or a mean mortality of 9·2 per-cent.

Both these tables are, however, misleading, in so far that they only deal with deaths in the Congo itself, and take no account of deaths which occur on the homeward journey or subsequently as a result of residence in the Congo. The mortality given is, therefore, to a great extent, too low.

Of more value in this comparison is the result of an investigation "On the rates of mortality in certain parts of

Africa", which was undertaken in the year 1897 for the Institute of Actuaries by Dr. A. E. Sprague. This investigation, which included among others 971 officials in the Congo State, gave as a result a mean rate of mortality of 9·4 per-cent for Whites in the Congo.

Having regard to the fact that Dr. Sprague's investigation was carried out at a time when the mortality in the Congo was considerably heavier than now, it must be assumed that the result of the foregoing investigation agrees much more nearly with the result at which he arrived.

This indicates that the mortality among Scandinavians in the Congo does not differ appreciably from the mortality among other Whites there. If any difference can be shown, it seems to be in favour of the Scandinavians.

TABLE 1.

Synoptical Table.

Year	No. of Entrants		No. of Exits (including Deaths)	No. of Persons at close of year	Mean No. observed during the year	No. of Deaths in the year	Mortality per-cent during year
(1)	(2)		(3)	(4)	(5)	(6)	(7)
1878	3	* − 2	...	1	1	0	0
79	2	...	...	3	2	0	0
80	2	− 1	2	2	2	2	100·0
81	2	...	1	3	2	0	0
82	2	...	1	4	4	0	0
83	20	...	3	21	12	2	16·7
84	13	...	8	26	24	4	16·7
85	3	...	9	20	23	2	8·7
86	11	− 1	8	22	21	1	4·8
87	20	− 2	5	35	28	1	3·6
88	14	...	3	46	40	2	5·0
89	29	...	9	66	56	2	3·6
90	33	− 2	19	78	72	7	9·7
91	55	− 2	22	109	94	13	13·8
92	43	− 1	41	110	110	11	10·0
93	38	...	33	115	112	13	11·6
94	42	...	37	120	118	17	14·4
95	24	...	35	109	114	14	12·3
96	49	...	29	129	119	11	9·2
97	81	− 1	40	169	149	17	11·4
98	79	− 2	53	193	181	28	15·5
99	73	...	62	204	198	21	10·6
1900	68	− 1	51	220	212	14	6·6
01	46	...	37	229	224	19	8·5
02	65	...	48	246	238	13	5·5
03	45	...	53	238	242	9	3·7
04	60	..	69	229	234	13	5·6
1878–1904	922	−15	678	...	2,632	236	9·0

* The figures in this column give those persons who entered during the year the date of whose exit is unknown, and who cannot therefore remain the subject of observation.

TABLE 2.

Complete years elapsed from entry	Soldiers			Missionaries			Seamen			Other Occupations			Total			Complete years elapsed from entry
	No. of months observed	No. of Deaths	Mortality per-cent	No. of months observed	No. of Deaths	Mortality per-cent	No. of months observed	No. of Deaths	Mortality per-cent	No. of months observed	No. of Deaths	Mortality per-cent	No. of months observed	No. of Deaths	Mortality per-cent	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
0	1,500	26	20·8	1,166	9	9·3	5,219	49	11·3	692	9	15·6	8,577	93	13·0	0
1	997	21	25·3	1,071	5	5·6	3,650	22	7·2	413	2	5·8	6,131	50	9·8	1
2	758	2	3·2	895	7	9·4	2,639	11	5·0	290	1	4·1	4,582	21	5·5	2
3	501	6	14·4	636	5	9·4	1,362	5	4·4	182	1	6·6	2,681	17	7·6	3
4	432	1	2·8	568	3	6·3	1,025	9	10·5	154	1	7·8	2 179	14	7·7	4
5	374	3	9·6	493	4	9·7	711	3	5·1	107	0	0	1,685	10	7·1	5
6	260	2	9·2	379	1	3·2	475	1	2·5	75	1	16·0	1,189	5	5·0	6
7	127	1	9·4	291	2	8·2	387	3	9·3	60	0	0	865	6	8·3	7
8	71	1	16·9	241	1	5·0	289	...	0·	60	0	0	661	2	3·6	8
9	48	0	0	214	1	5·6	225	1	5·3	59	1	20·3	546	3	6·6	9
10	32	1	37·5	195	0	0·	162	2	14·8	31	0	0	420	3	8·6	10
11	12	0	0	165	2	14·5	112	...	0	13	1	92·3	302	3	11·9	11
12	12	0	0	107	0	0	91	1	13·2	...	...	...	210	1	5·7	12
13	7	1	100	63	1	19·0	73	...	...	...	...	...	143	2	16·8	13
14	...	...	...	28	...	...	52	...	...	...	...	...	80	0	0	14
15	...	...	...	24	...	...	32	...	...	...	...	...	56	...	0	15
16	...	...	...	24	...	...	12	...	...	...	...	...	36	...	0	16
17	...	...	...	19	...	...	12	...	...	...	...	...	31	...	0	17
18	...	...	...	4	...	...	12	...	...	...	...	...	16	...	0	18
19	...	...	...	...	...	...	12	...	...	...	...	...	12	...	0	19
20	...	...	...	...	...	...	12	...	...	...	...	...	12	...	0	20
21	...	...	...	...	...	...	7	...	...	...	...	...	7	...	0	21

TABLE 3.

Complete years elapsed from entry	Soldiers			Missionaries			Seamen			Other Occupations			Total			Complete years elapsed from entry
	No. of months observed	No. of Deaths	Mortality per-cent	No. of months observed	No. of Deaths	Mortality per-cent	No. of months observed	No. of Deaths	Mortality per-cent	No. of months observed	No. of Deaths	Mortality per-cent	No. of months observed	No. of Deaths	Mortality per-cent	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(2)	(13)	(14)	(15)	(16)	(17)
0	1,500	26	20·8	1,166	9	9·3	5,219	49	11·3	692	9	15·6	8,577	93	13·0	0
1	997	21	25·3	1,071	5	5·6	3,650	22	7·2	413	2	5·8	6,131	50	9·8	1
2–4	1,691	9	6·4	2,099	15	8·6	5,026	25	6·0	626	3	5·8	9,442	52	6·6	2–4
5–9	880	7	9·5	1,618	9	6·7	2,087	8	4·6	361	2	6·6	4,946	26	6·3	5–9
10–21	63	2	38·1	629	3	5·7	589	3	6·1	44	1	27·3	1,325	9	8·2	10–21
0–21	5,131	65	15·2	6,583	41	7·5	16,571	107	7·7	2,136	17	9·6	30,421	230	9·1	0–21

TABLE 4.

Mortality Table.

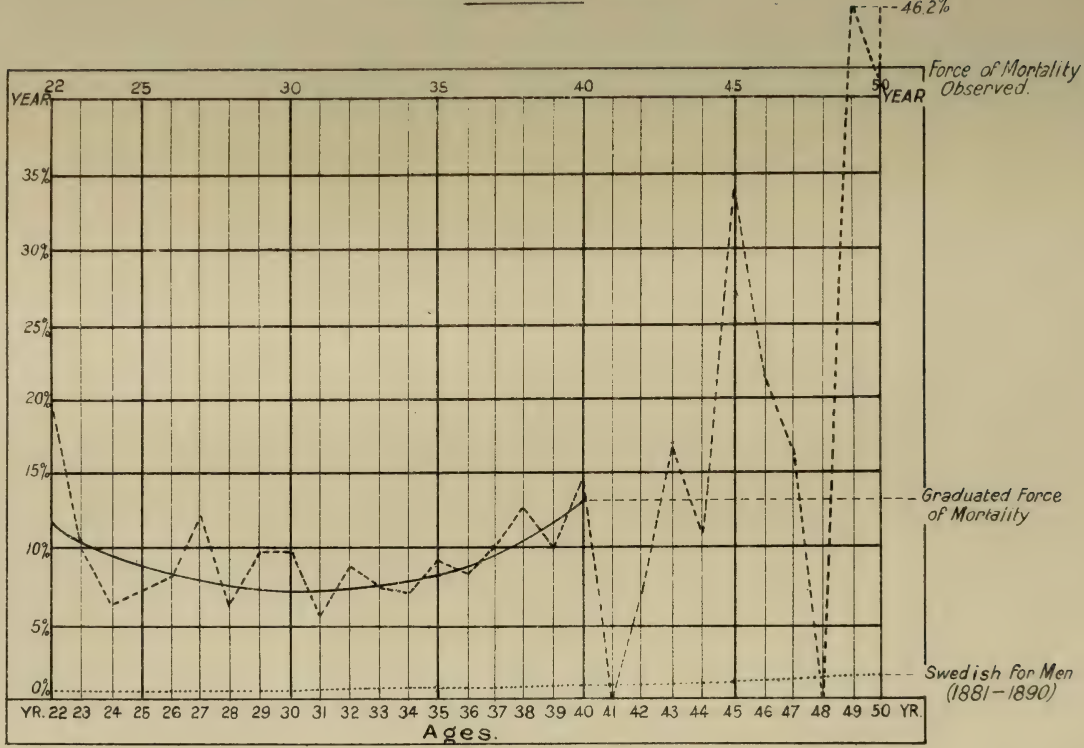
Ages	Months of observation	No. of Deaths	Force of Mortality Observed %	Force of Mortality Graduated %
(1)	(2)	(3)	(4)	(5)
17–20	109	1	11·0	...
21	366	1	3·3	...
22	693	11	19·0	11·6
23	959	8	10·0	10·2
24	1,409	8	6·8	9·5
25	1,976	12	7·3	9·0
26	2,372	16	8·1	8·6
27	2,402	24	12·0	8·3
28	2,332	12	6·2	8·0
29	2,103	17	9·7	7·9
30	1,971	16	9·7	7·8
31	1,710	9	5·7	7·7
32	1,771	13	8·8	7·6
33	1,488	9	7·3	7·7
34	1,380	8	7·0	7·9
35	1,228	9	8·8	8·3
36	1,128	8	8·5	8·8
37	954	8	10·1	9·7
38	830	9	13·0	10·6
39	725	6	9·9	11·7
40	593	7	14·2	13·0
41	439	0	0·0	...
42	346	2	6·9	...
43	284	4	16·9	...
44	210	2	11·4	...
45	140	4	34·3	...
46	110	2	21·8	...
47	72	1	16·7	...
48	53	0	0·0	...
49	26	1	46·2	...
50–53	29	1	41·4	...

TABLE 5.

Comparison between the actual deaths and those calculated from the male Mortality Table of the Swedish population for the years 1881 *to* 1890.

Ages x	Months of observation $=12l_x$	q_x	Calculated Deaths $q_x \cdot l_x$	Actual Deaths T_x	$\frac{T_x}{q_x \cdot l_x}$
(1)	(2)	(3)	(4)	(5)	(6)
17–20	109	0·532	0·05	1	20·0
21	366	0·644	0·20	1	5·0
22	693	0·664	0·38	11	28·9
23	959	0·672	0·54	8	14·8
24	1,409	0·678	0·80	8	10·0
25	1,976	0·674	1·11	12	10·8
26	2,372	0·665	1·31	16	12·2
27	2,402	0·665	1·33	24	18·0
28	2,332	0·677	1·32	12	9·1
29	2,103	0·681	1·19	17	14·3
30	1,971	0·673	1·11	16	14·4
31	1,910	0·670	1·07	9	8·4
32	1,771	0·678	1·00	13	13·0
33	1,488	0·692	0·86	9	10·5
34	1,380	0·694	0·80	8	10·0
35	1,228	0·711	0·73	9	12·3
36	1,128	0·743	0·70	8	11·4
37	954	0·773	0·62	8	12·9
38	830	0·794	0·55	9	16·4
39	725	0·827	0·50	6	12·0
40	593	0·875	0·43	7	16·3
41	439	0·913	0·33	0	0·0
42	346	0·938	0·27	2	7·4
43	284	0·973	0·23	4	17·4
44	210	1·019	0·18	2	11·1
45	140	1·062	0·12	4	33·3
46	110	1·100	0·10	2	20·0
47	72	1·144	0·07	1	14·3
48	53	1·203	0·05	0	0·0
49	26	1·261	0·03	1	33·3
50–53	29	1·374	0·03	1	33·3
...	...	...	18·01	229	12·7

Table 6.


46.2%
Force of Mortality Observed.
Graduated Force of Mortality
Swedish for Men (1881–1890)
YEAR
22
25
30
35
40
45
50
35%
30%
25%
20%
15%
10%
5%
0%
YR. 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 YR.
Ages.

CORRESPONDENCE.

MORTALITY ON THE CONGO.

To the Editor of the Journal of the Institute of Actuaries.

15 *March* 1907.

Dear Sir,—In the foregoing valuable investigation by Paul Bergholm into this subject, the date of death has been taken as the date of exit of those who have died. That is to say, in finding the "exposed to risk", the year of death has not been treated as a complete year, but the life has been held as exposed to risk only up to the date of death. This procedure is different from that usually followed by British Actuaries, and its effect is to diminish the "exposed to risk", and consequently to increase the resulting rates of mortality. In fact the rates deduced by this method are not the true "rates of mortality" as defined in the *Text-Book*, but are approximations to the "force of mortality." This difference should accordingly be borne in mind when making any comparison between M. Bergholm's results and those which have been published in this country. Assuming the deaths to be evenly distributed throughout the year, the effect of M. Bergholm's method will be to diminish the "exposed to risk" by six months in the case of each death. On this assumption, therefore, the true rates of mortality can be deduced from his figures by adding to the number of years of life exposed to risk, as found by him, one-half of the number of deaths. Making this adjustment, we find that the average rate of mortality over the whole experience as shown in Table 3 is reduced from 9·1 per-cent to 8·7 per-cent. The average rate deduced by me ten years ago (*J.I.A.*, xxxiii, 288) was 9·4 per-cent, which is considerably higher; but this difference may be due to a difference in the average ages in the two experiences. In order to test this I have divided the figures in M. Bergholm's Table 4 into five groups, representing the same ages as the groups in the table given by me on p. 290 of vol. xxxiii. Applying the above-mentioned adjustment in respect of the year of death, the following figures are obtained:

Ages	Sprague				Bergholm			
	Years of Life	Actual Deaths	Rate of Mortality per-cent	Expected Deaths	Years of Life	Actual Deaths	Rate of Mortality per-cent	Expected Deaths
17–25	408	29	7·1	34	480	41	8·5	34
26–28	288	23	8·0	24	618	52	8·4	49
29–33	362	29	8·0	28	802	64	7·9	64
34–37	151	16	10·6	12	407	33	8·1	43
38 & up.	138	16	11·6	15	341	39	11·4	39
...	...	113	...	113	...	229	...	229

Having regard to the limited extent of the data, the two sets of rates correspond as closely as could have reasonably been

anticipated. The columns headed "expected deaths" are obtained by multiplying the exposed to risk in each set of data by the rates of mortality deduced from the other; and it will be seen that (omitting fractions) they coincide in each case with the actual deaths. This shows that, on the whole, the two experiences were subject to the same rates of mortality, and I therefore do not think that the figures indicate any superiority in the mortality of Scandinavians in the Congo over other Europeans.

M. Bergholm writes as if it were an established fact that the rate of mortality in the Congo has appreciably diminished in recent years, and it would be interesting to know his authority for this. His own investigation is, as he points out, not a sufficient proof, and the above comparison seems to throw doubt upon it.

The following figures, deduced from Tables 3 and 7, bring out the relative importance of various causes of death, and show clearly that sailors suffer less from fever than any other class, this being the principal reason why they experience a much lighter death-rate than soldiers. As Table 7 relates to 236 deaths, while Table 3 (from which the "exposed to risk" are taken) relates to 230 deaths only, these figures are rather too large in some cases. The extent of the errors therein can be measured by a comparison of the average rate of mortality in each class with the corresponding rate shown in Table 3:

Approximate Rates of Mortality per-cent per annum from various causes.

Cause of Death	Soldiers	Missionaries	Sailors	Other Occupations	Total
Fever . . .	10·1	6·0	4·3	5·1	5·7
Other Diseases . .	1·9	·7	1·3	1·7	1·3
Accident and violence	1·6	0·	·8	·6	·7
Unknown . . .	1·6	1·1	1·5	2·8	1·5
...	15·2	7·8	7·9	10·2	9·2

It may be that the lighter mortality from fever among sailors is due to their occupation keeping them more or less out of the reach of mosquitos, and so rendering them less liable to be infected with malaria.

In this connection I may mention having once seen it stated that the crews of the vessels on one of the great African lakes had suffered much less from malaria ever since they had adopted the plan of mooring at some distance from the shore instead of close in; but, unfortunately, I cannot now trace the source of that statement.

Yours faithfully,

A. E. SPRAGUE.

22, *George Street,*
Edinburgh.

THE INSTITUTE OF ACTUARIES.

ROYAL PATRIOTIC FUND CORPORATION.

The following correspondence has recently passed between Colonel J. S. Young, Secretary of the Corporation, and the President of the Institute of Actuaries:

Seymour House,
17, Waterloo Place, S.W.,
19 *December* 1906.

Sir,—His Royal Highness the Duke of Connaught, President of this Corporation, and the Executive Committee are very sensible of the patriotic services rendered by the President and Council of the Institute of Actuaries in connection with actuarial valuations of the funds administered by this Corporation.

In recognition of such services, I am directed to ask that you will be pleased to move the Council of the Institute of Actuaries to allow the President for the time being to be nominated for co-optation as a member, for the three years, 1907, 1908 and 1909, of the Royal Patriotic Fund Corporation.

The limitation of the years named is imposed by the provisions of the Act of Parliament for the constitution of the Corporation.

It is recognized that the President of the Institute of Actuaries might not be able to give attendance at all the Committees of the Corporation. I am, therefore, to say that attendance of the President would only be expected at the meetings of the Executive Committee which, in the past three years, have taken place at this office at 11.30 a.m. on the first Wednesday in February, March, May, July, November and December in each year.

I am to add that a special meeting of the Royal Patriotic Fund Corporation will be held on 8 January next for the purpose of co-opting six members for the three years 1907, 1908 and 1909.

I am, therefore, requested to ask the favour of a reply to this letter not later than 30th instant, so that timely arrangements may be made for the proceedings of the Special Meeting of the Corporation on 8 January.

I remain, Sir,
Yours very faithfully,
(Signed) J. S. YOUNG, Colonel,
Secretary.

The President of the Institute of Actuaries.

The Institute of Actuaries,
21 *December* 1906.

Sir,—I beg to acknowledge receipt of your letter of the 19th instant, in which you are good enough to inform me that His Royal Highness the Duke of Connaught, President of the Royal Patriotic

Fund Corporation, and the Executive Committee are graciously pleased to ask the Council of the Institute of Actuaries to allow the President of that body for the time being to be nominated for co-optation as a member of the Corporation for the three years 1907, 1908, and 1909.

In reply I beg you to inform His Royal Highness and the Executive Committee that the Council of the Institute of Actuaries feel greatly honoured by the invitation extended to their President for the time being, and that they accept it with the fullest appreciation.

Allow me to thank you for your information as to the meetings of the Executive Committee, which I have no doubt our President for the time being would be able to attend.

I remain, Sir,
Yours very faithfully,
(Signed) FRANK B. WYATT,
President.

SEYMOUR HOUSE,
17, WATERLOO PLACE, S.W.,
8 *January* 1907.

SIR,—I have the pleasure, by direction of His Royal Highness the Duke of Connaught, President of this Corporation, to acquaint you that, at a special meeting of the Corporation held this day, the President of the Institute of Actuaries for the time being was co-opted to be a member of the Corporation and of its Executive Committee for a period of three years ending 31 December 1909.

I am, therefore, directed to request your attendance at this office, on Wednesday, 16th instant, at 11.30 a.m., for the purpose of constituting Sub-Committees for carrying on the work of the Corporation.

I have the honour to be, Sir,
Your most obedient Servant,
(Signed) J. S. YOUNG, Colonel,
Secretary.

THE PRESIDENT OF THE INSTITUTE OF ACTUARIES.

THE INSTITUTE OF ACTUARIES,
10 *January* 1907.

SIR,—I beg to acknowledge, with gratification, the receipt of your letter of the 8th instant, in which, by direction of His Royal Highness the Duke of Connaught, President of the Royal Patriotic Fund Corporation, you acquaint me that, at a special meeting of the Corporation, held on the 8th instant, the President of the Institute

of Actuaries for the time being was co-opted to be a member of the Corporation and of its Executive Committee for a period of three years ending 31 December 1909.

I have, further, to thank you for informing me of the meeting to be held on the 16th instant, which I shall have the honour of attending.

I am, Sir,
Faithfully yours,
(Signed) FRANK B. WYATT,
President.

THE INSTITUTE OF ACTUARIES *v.* "THE INSTITUTE OF ACCOUNTANTS."

It may interest our readers to know that, in an action brought by the Institute of Actuaries against certain persons carrying on, or advertised as carrying on, a business known as the "Institute of Accountants" (not to be confounded with the well-known Institute of Chartered Accountants in England and Wales), and whose Fellows and Associates had begun to use the letters F.I.A. and A.I.A. respectively, after their names, an Order was, on the 1 February 1907, made by the Honble. Mr. Justice Joyce (with the consent of the Defendants) perpetually restraining the Defendants and all other the persons constituting or carrying on business as the Institute of Accountants from using, sanctioning, or authorizing the use by the Fellows and Associates of such Institute of Accountants of the letters F.I.A. and A.I.A. respectively, after their names, either alone or in conjunction with any other letter or word, or from carrying on business in any such manner as to deceive or lead to the belief that such business is that of or is in any way connected with that of the Institute of Actuaries.

MESSENGER PRIZE ESSAY.

In January 1905, the Council offered Two Prizes for the two best Essays on the following subject, namely—

"*The Methods of ascertaining the Rates of Mortality amongst the general Population of a Country, District, or Town, or amongst different classes of such population, by means of Returns of Population, Births, Deaths, and Migration.*"

(For detailed Syllabus, see vol. xxxix, pp. 127-8.)

The President, Mr. F. B. Wyatt, announced at the Sessional Meeting, held on 25 March 1907, that the Council had, on the recommendation of the Adjudicators, awarded a Prize of 25 Guineas to Mr. C. H. Wickens, A.I.A., of the Commonwealth Bureau of Census and Statistics, Melbourne, Australia, for his Essay on the above subject.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Comparative Bonuses under Whole Life and Endowment Assurances. By Hermann Julius Rietschel, F.I.A., *of the Sun Life Assurance Society.*

[Read before the Institute, 25 February 1907.]

It is now some years since a paper was presented to the Institute dealing with the comparative bonus-earning powers of whole life and endowment assurances. In view of the importance of the subject I have ventured to submit this paper, which I hope may lead to a useful and interesting discussion. The matter has been so ably dealt with by the late Mr. Sunderland, Mr. Lidstone and Mr. Chatham that I have the greatest hesitation in presenting my own views, and I must therefore beg the Members of the Institute to receive this paper in the same kindly spirit which they have always manifested towards new contributors.

Object of Paper.

My object in the paper has been to show the relative bonuses which should be allotted to whole life and endowment assurance policies having special regard to

(1) The expenses of the office;
(2) Variations in the rate of interest earned; and
(3) Mortality.

Office Premiums.

In Table A are given the office premiums which have been employed in this investigation. They are based on the $H^{[M]}$ 4 per-cent Table, with a loading for a £1 per-cent compound reversionary bonus, together with 10 per-cent and a constant of 3*s*. per-cent for expenses and commission—

Table A. *Sum Assured* £100.

Age at Entry	Whole-Life Premiums	Endowment Assurance Premiums				
		Term 30 years	Term 25 years	Term 20 years	Term 15 years	Term 10 years
25	2·323	3·402	4·040	5·046	6·772	10·291
35	2·925	3·640	4·220	5·169	6·855	10·336
45	3·952	4·285	4·732	5·568	7·149	10·538
55	5·692	...	...	...	...	...

Sources of Profit.

Equity demands that the whole of the profits should be divided as far as practicable among the policyholders in the proportions in which they have contributed to them. The profit of an office is derived from the following sources:—

(1) Loading.
(2) Interest earned on the Funds over and above that assumed in ascertaining the reserves.
(3) Mortality.
(4) Miscellaneous.

(1) *Loading Profit.*—Each year's office premium should provide the net premium according to the valuation basis adopted by the office, together with a true share of the expenses and a balance which would be the surplus loading, and which should be returned to the policyholder.

As we are now instituting a comparison of the relative bonus-earning powers of whole life and endowment assurances it is most important that each class should bear its due share of the office expenditure. This expenditure may be divided between initial and renewal expenses. I have made a rough analysis of the expenses of several offices and I find that the initial expenditure of an average office may be assumed to be about £3 per-cent of the sum assured and the renewal expenditure to be $7\frac{1}{2}$ per-cent of the office premiums. It would be inadvisable, however, to

assume that the renewal expenses entirely take the form of a percentage of the premiums as part of this expenditure should be made to depend upon the sums assured in force, *e.g.*, the item "salaries" should be divided between "new sums assured", "premiums", and "total sums assured in force." For the purpose of the following investigation I have made the assumption (A) that the initial expenditure is £3 per-cent of the sum assured, and that the renewal expenses can be represented by a percentage of 5 per-cent of the premiums and a constant of 2*s*. per-cent of the sum assured.

At the end of this paper I have endeavoured to show the very great influence that the apportionment of the renewal expenditure will have upon the relative rates of bonus to be allotted to whole life and endowment assurances.

(2) *Interest Profit.*—This is derived from the surplus interest earned on the moneys in the possession of the office, *i.e.*, the reserves, and should therefore be returned to the policyholders in proportion to the reserves held in respect of their policies.

(3) *Mortality Profit or Loss.*—If the mortality profit be allotted to those policies that have earned it, *i.e.*, by the formula $(q_{x+n}-q'_{x+n})(1-\text{V})$, where $q =$ rate of mortality expected according to the valuation basis, and $q'=$rate actually experienced, the effect would, owing to selection, be to allocate practically the whole of the mortality profit made by the office to the policies in their first quinquennium of duration, so that the bonuses in the first quinquennium would be large as compared with those immediately following. Such a result, while correct in theory, would be objectionable in practice as in the early years of a policy the office has to bear the heavy expenses incurred in placing the business on the books. With a view to meeting this objection, I have later in my paper modified the contribution method sketched out above.

In order to show the *relative* mortality profit earned by whole life and endowment assurances I have assumed that the office will experience $\text{O}^{[\text{M}]}$ Mortality in the case of both classes. I am aware that it is generally held that under endowment assurances the mortality is lighter than that experienced under whole life assurances, but I do not think that this difference is likely to continue in the future. The greater proportion of the new business of an office now consists of endowment assurances and these would in former years have been effected as whole life policies so that endowment

assurances as a class will tend to become more and more a mixed body of lives similar to the whole life class. A well-managed office may possibly experience a lighter rate of mortality than is shown by the $O^{[M]}$ Table, and the examples given must therefore be regarded not as results likely to obtain in the future but merely as showing the *relative* proportion in which these bonuses due to mortality profit are earned by whole life and endowment policies of various terms. A point to be remarked is that the item of mortality profit will be considerably reduced by the use of the O^{M} Tables in valuations, as a large part of what was formerly mortality profit will now be thrown up in the loading profit.

Miscellaneous Profit.—The principal items coming under this head are profit from surrenders, lapses, and the non-participating business, as well as profit or loss on investments. The participating policyholders have not in any way contributed to the profit from the first three items, and, therefore, no questions of equity arise in considering the method to be adopted in distributing it. In an average office its amount would be comparatively small, and its effect on the bonuses trifling.

Those offices which bring into account in their revenue accounts profit or loss from investments and divide such profit or loss between the policyholders introduce an element of instability into their bonus results. It is consequently advisable not to regard profit or loss on investments as being one of the usual sources of profit, and I have, therefore, not considered it necessary to take it into account in the results given.

Resulting Bonuses.

Bonuses derived from contributions exclusive of Mortality Profit.—Tables B and C give the reversionary bonuses to be allotted to whole life and endowment assurances at the end of successive quinquenniums in accordance with the above contribution method (excluding any allowance for mortality profit) in the case of two offices both valuing by the O^{M} 3 per-cent Table, one earning a 4 per-cent rate of interest and the other $3\frac{1}{2}$ per-cent. As the above method of distribution is not one which I would recommend without modification, I have for the present refrained from commenting upon the results.

Table B.

Reversionary Bonuses per £1,000 of Assurance earned by the Office making a rate of interest of 4 per-cent.

O^M Loadings. Valuation Basis O^M 3 %

Age at Entry	End of Quinquennium	Whole-Life Assurance	Endowment Assurances				
			Term 30 years	Term 25 years	Term 20 years	Term 15 years	Term 10 years
25	1st	£71	58	56	54	53	55
	2nd	77	66	64	65	66	76
	3rd	81	72	71	75	80	...
	4th	84	78	80	84	...	...
	5th	89	85	87	...	...	...
	6th	94	91	...	...	...	...
35	1st	£55	49	47	46	45	48
	2nd	62	57	57	57	61	70
	3rd	67	65	65	68	74	...
	4th	74	71	73	78	...	...
	5th	79	79	81	...	...	...
	6th	84	85	...	...	...	...
45	1st	£49	47	45	42	42	44
	2nd	57	56	54	55	57	66
	3rd	65	64	63	66	70	...
	4th	70	70	72	75	...	...
	5th	79	77	80	...	...	...
	6th	85	83	...	...	...	...
55	1st	£44	...	...	...	...	...
	2nd	54	...	...	...	...	...
	3rd	63	...	...	...	...	...
	4th	70	...	...	...	...	...
	5th	79	...	...	...	...	...
	6th	84	...	...	...	...	...

TABLE C.

Reversionary Bonuses per £1,000 of Assurance earned by the Office making a rate of interest of $3\frac{1}{2}$ per-cent.

O[M] LOADINGS. VALUATION BASIS O[M] 3 %

Age at Entry	End of Quinquennium	Whole-Life Assurance	ENDOWMENT ASSURANCES				
			Term 30 years	Term 25 years	Term 20 years	Term 15 years	Term 10 years
25	1st	67	54	51	48	47	46
	2nd	67	56	53	51	50	53
	3rd	66	56	54	53	55	...
	4th	65	56	54	55	...	...
	5th	65	57	55	...	...	...
	6th	64	57	...	...	...	...
35	1st	51	45	43	41	39	39
	2nd	52	47	45	44	45	48
	3rd	53	49	48	47	49	...
	4th	55	50	49	50	...	...
	5th	55	51	50	...	...	...
	6th	56	51	...	...	...	...
45	1st	46	42	40	37	36	36
	2nd	48	45	42	41	41	44
	3rd	50	48	46	45	46	...
	4th	51	49	47	48	...	...
	5th	53	50	49	...	...	...
	6th	54	50	...	...	...	...
55	1st	40	...	...	...	...	...
	2nd	43	...	...	...	...	...
	3rd	47	...	...	...	...	...
	4th	50	...	...	...	...	...
	5th	52	...	...	...	...	...
	6th	55	...	...	...	...	...

Bonuses derived from Mortality Profit only.—In Table D are shown the reversionary bonuses per £1,000 assured obtained from what is usually regarded as mortality profit by an office experiencing $O^{[M]}$ Mortality but valuing by the O^{M} Table.

TABLE D.

Age at Entry	End of Quinquennium	Whole-Life Assurance	Endowment Assurances: Term 30 years	Term 25 years	Term 20 years	Term 15 years	Term 10 years
25	1st	+ £5	+ £3	+ £3	+ £3	+ £2	+ £2
	2nd	− 9	− 6	− 5	− 4	− 3	− 1
	3rd	− 9	− 5	− 4	− 3	− 1	...
	4th	− 5	− 2	− 2	− 1	...	...
	5th	− 3	− 1	− 0	...	...	...
	6th	− 2	− 0	...	...	...	...
35	1st	+ £22	+ £18	+ £17	+ £15	+ £12	+ £10
	2nd	+ 1	+ 1	+ 1	+ 1	+ 1	+ 0
	3rd	− 4	− 3	− 2	− 1	− 1	...
	4th	− 2	− 2	− 1	− 0	...	...
	5th	...	...	...	...	...	...
	6th	...	...	...	...	...	...
45	1st	+ £33	+ £32	+ £30	+ £27	+ £23	+ £19
	2nd	+ 6	+ 6	+ 5	+ 4	+ 3	+ 2
	3rd	− 2	− 2	− 1	− 1	− 0	...
	4th	...	...	...	...	...	...
	5th	...	...	...	...	...	...
	6th	...	...	...	...	...	...

It will be observed that the reversionary bonuses to be allotted to the short term endowment assurances are much smaller than those to be allotted to whole life assurances. Table D has been given merely in order to show the relative effect on the bonuses produced by the experience of a rate of mortality different from that assumed in the valuation. The very much larger "mortality profit" derived from the policies effected at the older ages at entry should also be noted. An unsatisfactory feature is the large bonuses allotted to the policies of short durations, and, in view of the initial expenditure incurred by the office in connection with its new business, it is desirable to alter this.

It must now be pointed out that for an office valuing on the O^{M} Table the generally accepted idea of mortality profit is, perhaps, somewhat misleading. An aggregate table is avowedly only used because it gives an approximation to the

select reserves. Therefore, regarding individual policies, it would appear probable that the select reserves should be assumed made, and under these circumstances it will be seen that if select mortality is experienced, no profit from mortality will arise. The increased reserves in the early years will be compensated for by decreased reserves in the older years with the result that the surplus will be practically the same as that shown by an O^M valuation, but the profit will all have arisen from loading and interest. The select net premiums should therefore be employed in ascertaining the surplus loading margins.

Bonuses derived from the use of $O^{[M]}$ *Loadings.*—In Tables E and F I have stated the reversionary bonuses resulting from the use of $O^{[M]}$ Loadings. An examination of these two tables leads to the following conclusions:

(1) In the case of the office earning 4 per-cent—

(*a*) The whole life bonuses can be very nearly expressed in the form of a compound reversionary bonus of the same rate for all ages at entry.

(*b*) The reversionary bonuses under endowment assurances are in the first quinquennium smaller than the corresponding whole life bonuses, but they increase rapidly with the duration of the assurances until they finally equal or exceed the whole life bonuses. The total reversionary bonuses on maturity at the more important ages at entry differ by less than 5 per-cent from the corresponding total whole life bonuses. If, therefore, the same rate of bonus be given to endowment assurances as to whole life assurances the resulting bonuses will not differ widely from those produced by the contribution method.

(2) In the case of the office earning 3½ per-cent—

(*a*) The whole life bonuses can be very nearly expressed in the form of a uniform simple reversionary bonus of the same rate for all ages at entry.

(*b*) The endowment assurance bonuses are all smaller than the corresponding whole life bonuses. They also can, except for the shorter terms, be very nearly expressed in the form of a simple reversionary bonus of the same rate for all ages at entry and

terms of assurance; such rate would, however, be about ten per-cent lower than the corresponding whole life rate.

(3) If the same rate of bonus is to be allotted to both classes the office must provide for a surplus interest margin in excess of one-half per-cent, or, alternatively, the endowment assurance premiums must be loaded more heavily in proportion than the whole-life premiums.

TABLE E.

Reversionary Bonuses per £1,000 of Assurance earned by the Office making a rate of interest of 4 per-cent.

$O^{[M]}$ LOADINGS. VALUATION BASIS O^{M} 3 %

Age at Entry	End of Quinquennium	Whole-Life Assurance	Endowment Assurances					Compound Reversionary Bonus	
			Term 30 years	Term 25 years	Term 20 years	Term 15 years	Term 10 years	Rate	
25	1st	67	56	54	53	53	56	27*s.* %	67
	2nd	73	64	62	64	66	77		72
	3rd	78	70	70	74	80	...		77
	4th	81	76	79	83	...	...		82
	5th	86	84	86	...	...	...		88
	6th	92	90	...	...	...	...		94
35	1st	59	53	51	51	50	54	25*s.* %	63
	2nd	66	61	61	61	65	75		66
	3rd	70	68	68	72	78	...		71
	4th	77	74	76	81	...	...		75
	5th	82	82	84	...	...	...		80
	6th	87	87	...	...	...	...		85
45	1st	59	57	55	52	52	55	26*s.* %	65
	2nd	69	65	63	64	66	76		69
	3rd	75	74	72	74	78	...		74
	4th	79	79	80	83	...	...		79
	5th	89	86	88	...	...	...		84
	6th	94	91	...	...	...	...		89
55	1st	66	...	...	...	...	...	28*s.* %	70
	2nd	75	...	...	...	...	...		74
	3rd	85	...	...	...	...	...		80
	4th	92	...	...	...	...	...		86
	5th	101	...	...	...	...	...		92
	6th	105	...	...	...	...	...		98

Table F.

Reversionary Bonuses per £1,000 of Assurance earned by the Office making a rate of interest of $3\frac{1}{2}$ per-cent.

$O^{[M]}$ Loadings. Valuation Basis O^M 3 %

Age at Entry	End of Quinquennium	Whole-Life Assurance	Endowment Assurances				
			Term 30 years	Term 25 years	Term 20 years	Term 15 years	Term 10 years
25	1st	63	52	49	47	47	47
	2nd	63	54	51	50	50	54
	3rd	63	54	53	52	55	...
	4th	62	54	53	54	...	...
	5th	62	56	54	...	...	...
	6th	62	56	...	...	...	...
35	1st	55	49	47	46	44	45
	2nd	56	51	49	48	49	53
	3rd	56	52	51	51	53	...
	4th	58	53	52	53	...	...
	5th	58	54	53	...	...	...
	6th	59	53	...	...	...	...
45	1st	56	52	50	47	46	47
	2nd	58	54	51	50	50	54
	3rd	59	58	55	53	54	...
	4th	60	58	55	56	...	...
	5th	62	59	57	...	...	...
	6th	62	58	...	...	...	...
55	1st	62	...	...	...	...	...
	2nd	63	...	...	...	...	...
	3rd	67	...	...	...	...	...
	4th	69	...	...	...	...	...
	5th	71	...	...	...	...	...
	6th	73	...	...	...	...	...

Before leaving these tables I would draw attention to the differences caused by the use of $O^{[M]}$ instead of O^M Loadings. The employment of O^M Loadings gives to the policies effected at the younger ages at entry too large a share of the surplus and to those taken out at the older ages too small a share. These differences in the amounts of the reversionary bonuses at the younger and older ages at entry are almost as marked in the case of endowment assurances as in the case of whole life assurances.

Expenses of Management and Commission.

Table G. *Sum Assured* £100.

Deductions made from the Office Premiums in order to meet the Expenses of the Office and the Commission.

Age at Entry	Whole-Life Assurance	Endowment Assurances				
		Term 30 years	Term 25 years	Term 20 years	Term 15 years	Term 10 years
25	·349	·432	·481	·558	·692	·964
35	·397	·452	·497	·570	·701	·971
45	·477	·502	·538	·602	·725	·988
55	·612	...	...	...	...	...

From Table G it will be seen that a large proportion of the premiums is required in order to provide the expenses of the office, and no definite conclusion as to the relative rates of bonus under the two classes could very well be arrived at without some investigation into the effect produced on the bonuses by variations in the rates of expenditure.

Assumption B.—In order to deal with this point, I will assume that the expenses may be represented by an initial expenditure of £3 per-cent, as before, and $7\frac{1}{2}$ per-cent of the office premiums for renewal expenses. This assumption would, assuming an average office premium of £4 per-cent, produce the same expense ratio as that adopted in our investigation. The results are shown in Table H side by side with those produced by our original assumptions.

Table H shows that if the renewal expenditure be assumed to take the form of a percentage of the office premiums instead of a percentage and a constant, the relative bonuses under whole life and endowment assurances would be very considerably affected. The whole-life bonuses are increased for ages at entry 25 and 35 and for age at entry 45, where the office premium is about £4 per-cent, no change is produced. In regard to endowment assurances of terms of less than 25 years the bonuses are decreased; for term 10 years by as much as 20 per-cent in the first quinquennium of duration. If, therefore, the present assumption as to the allocation of the expenses is justifiable, the conclusion would seem to be that endowment assurances of terms less than 25 years should obtain smaller bonuses than whole life assurances.

TABLE H.

Reversionary Bonuses per £1,000 of Assurance earned by the Office making a 4 per-cent rate of interest:

(*A*) *assuming Expenses are £3 per-cent of the Sum Assured for Initial Expenses and 5 per-cent of the Office Premiums and 2s. per-cent of the Sum Assured for Renewal Expenses.*

and (*B*) *assuming Expenses are £3 per-cent of the Sum Assured for Initial Expenses and $7\frac{1}{2}$ per-cent of the Office Premiums for Renewal Expenses.*

Age at Entry	End of Quinquennium	Whole-Life Assurance (A)	Whole-Life Assurance (B)	Endowment Assurances: Term 30 years (A)	Term 30 years (B)	Term 25 years (A)	Term 25 years (B)	Term 20 years (A)	Term 20 years (B)	Term 15 years (A)	Term 15 years (B)	Term 10 years (A)	Term 10 years (B)
25	1st	67	73	56	58	54	54	53	51	53	48	56	46
	2nd	73	78	64	65	62	62	64	62	66	62	77	68
	3rd	78	83	70	71	70	70	74	72	80	76	...	
	4th	81	86	76	77	79	79	83	82	...		...	
	5th	86	90	84	85	86	86	...		...		...	
	6th	92	96	90	91	...		...		...		...	
35	1st	59	62	53	54	51	51	51	48	50	45	54	44
	2nd	66	69	61	62	61	61	61	59	65	60	75	66
	3rd	70	73	68	69	68	68	72	70	78	74	...	
	4th	77	79	74	75	76	76	81	79	...		...	
	5th	82	84	82	83	84	84	...		...		...	
	6th	87	89	87	88	...		...		...		...	
45	1st	59	59	57	56	55	53	52	49	52	46	55	44
	2nd	69	69	65	64	63	62	64	61	66	61	76	67
	3rd	75	75	74	73	72	71	74	71	78	74	...	
	4th	79	79	79	79	80	79	83	81	...		...	
	5th	89	89	86	86	88	87	...		...		...	
	6th	94	94	91	91	...		...		...		...	
55	1st	66	63	...		...		...		...		...	
	2nd	75	72	...		...		...		...		...	
	3rd	85	82	...		...		...		...		...	
	4th	92	89	...		...		...		...		...	
	5th	101	98	...		...		...		...		...	
	6th	105	102	...		...		...		...		...	

Assumption C.—In Table K are given the results deduced on the assumption that the initial expenditure is £2 per-cent of the sum assured instead of £3 per-cent. The results show that the endowment assurance bonuses are affected by this alteration to about the same extent as the whole life bonuses so that it would appear that any practical variation in the rate of initial expenditure would not appreciably alter the relationship of the bonuses under the two classes.

Table K.

Reversionary Bonuses per £1,000 of Assurance earned by the Office making a 4 per-cent rate of interest.

(*A*) *assuming the Initial Expenditure is £3 per-cent and the Renewal Expenses 5 per-cent and 2s.*

and (*C*) *assuming the Initial Expenditure is £2 per-cent and the Renewal Expenses 5 per-cent and 2s.*

Age at Entry	End of Quinquennium	Whole-Life Assurances		Endowment Assurances: Term 25 years		Endowment Assurances: Term 15 years		Endowment Assurances: Term 10 years	
		(*A*)	(*C*)	(*A*)	(*C*)	(*A*)	(*C*)	(*A*)	(*C*)
25	1st	67	73	54	60	53	59	56	64
	2nd	73	79	62	67	66	71	77	84
	3rd	78	83	70	74	80	85	...	
	4th	81	85	79	83	...		...	
	5th	86	90	86	89	...		...	
35	1st	59	65	51	57	50	56	54	62
	2nd	66	71	61	66	65	71	75	82
	3rd	70	75	68	73	78	83	...	
	4th	77	81	76	80	...		...	
	5th	82	86	84	87	...		...	
45	1st	59	65	55	61	52	59	55	63
	2nd	69	74	63	69	66	72	76	83
	3rd	75	80	72	77	78	83	...	
	4th	79	83	80	84	...		...	
	5th	89	93	88	92	...		...	

To summarize the results of the investigation into the effect produced on the bonuses by variations in the rates of expenses we have—

(1) If the renewal expenses are assumed to take the form of a percentage of the premiums instead of a percentage and a constant, the whole life loading margins (for all policies effected at the principal ages at entry) would be increased, whereas in the case of endowment assurances, except for the longer terms, the margins are reduced. In other words the whole life bonuses are increased whereas the endowment assurance bonuses are decreased, the result being to make the bonuses for the latter class smaller than the whole life bonuses.

(2) If the initial expenditure be increased the loading margins under both classes are decreased, with the result that the reversionary bonuses are in each class reduced by about the same amount. The relative rates of bonus remain therefore unaffected.

(3) If the renewal expenditure be assumed to take the form of a percentage of the premiums and a constant dependent on the sum assured, a decrease in the percentage would increase the loading margins both for whole life and endowment assurances, and, on trial, it was found that the effect on the relationship of the bonuses under the two classes was very small.

The division of the renewal expenditure between sums assured and office premiums is a task of extreme difficulty, and depends almost entirely upon the judgment of the assessor. This being so, and the division having so large an effect on the relative bonuses to be allotted to endowment assurances and whole life assurances, it follows that some latitude must be allowed in fixing the relative bonuses under the two classes.

Reduction in the Valuation Rate of Interest below 3 per-cent.

The subject under review could not be considered to have been fully discussed unless some reference were made to the modifications in the contribution method that would become necessary if the office adopted a lower rate of interest than 3 per-cent, say $2\frac{1}{2}$ per-cent, in its valuation. If the contribution method were adopted without modification the surplus loading margins would for some ages and terms be reduced to vanishing point and, as the profits to be allotted to the new policies are almost entirely derived from this source, their bonuses would be very greatly reduced, whereas on the other hand the bonuses on the older policies would be greatly increased. This is due to the fact that the valuation is in effect not a $2\frac{1}{2}$ per-cent valuation at all, as the office has to rely on its surplus interest margin to make up the deficiencies in the loadings.

There can be only two reasons for the reduction in the valuation rate of interest—

(1) The office is now earning a rate of interest which will not enable it to maintain its rate of bonus in the future. In order to retain the same surplus interest

margin as formerly, the reduction in the valuation rate of interest has become necessary. The rate of profit derived from this interest source is not therefore increased, and in order to maintain its bonuses and pay its expenses, the office will still require to allow for the same loading margins as formerly.

For such an office, the basis of valuation should therefore be

$$A_{x+n}^{\mathrm{OM}2\frac{1}{2}\%} - \pi_x^{\mathrm{OM}3\%}\mathbf{a}_{x+n}^{\mathrm{OM}2\frac{1}{2}\%}.$$

If this basis be adopted, the contribution method would not require any modification, as the annual surplus loading margins would be unaffected and the rate of surplus interest would remain the same.

(2) The office is still earning the same rate of interest as formerly, but, in spite of this, it desires to strengthen its reserves by employing a valuation rate of interest of $2\frac{1}{2}$ per-cent instead of 3 per-cent. As explained in the case of office No. 1 the valuation cannot be considered a true $2\frac{1}{2}$ per-cent valuation unless the office still retains the same loading margin as hitherto.

In order to find the relationship of the new valuation reserve to the 3 per-cent reserves, it will be necessary to find the value of i' from the equation

$$\begin{aligned} A^{\mathrm{OM}2\frac{1}{2}\%} - \pi^{\mathrm{OM}2\frac{1}{2}\%}\mathbf{a}^{\mathrm{OM}2\frac{1}{2}\%} &= A^{\mathrm{OM}i'} - \pi^{\mathrm{OM}3\%}\mathbf{a}^{\mathrm{OM}i'} \\ &= (A^{\mathrm{OM}3\%} - \pi^{\mathrm{OM}3\%}\mathbf{a}^{\mathrm{OM}3\%}) + \{(A^{\mathrm{OM}i'} - A^{\mathrm{OM}3\%}) \\ &\qquad - \pi^{\mathrm{OM}3\%}(\mathbf{a}^{\mathrm{OM}i'} - \mathbf{a}^{\mathrm{OM}3\%})\} \end{aligned}$$

From this equation it follows that the reserves may be divided into two parts:

(1) The ordinary 3 per-cent valuation reserves which provides the loading and surplus interest necessary to maintain the bonuses on the old and new policies; and

(2) An additional reserve over and above this.

From this division of the reserves it is seen that, assuming the same conditions to prevail, as formerly, as to expenses, rate of interest, and mortality, the office will obtain (*a*) the same rate of interest and loading profit as in former years when it valued on a 3 per-cent basis, and (*b*) a further profit due to the extra reserves made. The items included in (*a*) should obviously be

allotted to the policies on the same plan as was employed before the change in the valuation basis. The question now to be solved is how the additional profit derived from the extra reserves is to be allotted. The formula originally used for the division of the surplus was $(P'_x-\pi-\epsilon)(1+i')+(i'-i)V$, where P'_x is the office premium, ϵ is the amount to be deducted on account of expenses, i' and i are the experience and valuation rates of interest respectively and V the reserve made. In order to form the extra reserve a certain proportion of the profit must have been held up representing, say, m per-cent of the contributions, so that the office had to keep back from each policy $\frac{m}{100}\{(P'_x-\pi-\epsilon)(1+i')+(i'-i)V\}$ of cash surplus. That is to say both the loading profit and the surplus interest have had to bear their share in providing the extra reserve. The extra profit derived from such reserve should therefore be applied towards increasing the cash surplus to be allotted to each policy by a percentage, *i.e.*, the reversionary bonuses should be increased by the same percentage.

If, therefore, the valuation basis of the office were $2\frac{1}{2}$ per-cent instead of the 3 per-cent which has been assumed as the basis of valuation in making the comparisons between the bonuses earned by whole life and endowment assurances the bonuses under both classes would be increased by the same percentage, and, therefore, they would still bear the same proportions to each other as would have been the case if the office had valued on a 3 per-cent basis.

NOTE.—The question may be raised as to the reasons for employing the 3 per-cent net premiums in ascertaining the loading margins. The defence lies, I think, in the fact that if the conditions obtain which have been assumed in the calculation of the office premiums, then the loading margins, together with the interest profit, will produce bonuses approximately of the form allowed for in the formula upon which the office premiums are based.

GENERAL CONCLUSIONS.

(1) The apportionment of the expenses incurred by the office between whole life and endowment assurances will have a very considerable effect upon the relative bonuses to be allotted to the two classes. Before any conclusions can be arrived at a careful analysis of the expenses must be made as the formula to be employed for the purpose of apportioning the expenses will cause considerable differences in the relative rates of bonus to be allotted.

(2) Favourable mortality, as a source of profit, is probably of much less importance than has usually been considered to be the case, and its influence on the relative bonuses, apart from its effect on the loading profit, may probably be ignored. For an office valuing on a true aggregate table, mortality will not consistently yield a profit or loss, and such profit or loss will only arise from accidental deviations from the Select Mortality Table, due to the smallness of the experience of an individual office.

(3) It would appear that for an office earning a rate of interest one per-cent in excess of its valuation rate the reversionary bonuses under both whole life and endowment assurances can be very nearly expressed in the form of a compound reversionary bonus of the same rate for all ages at entry and terms—always assuming the office premiums to have been loaded approximately for bonuses of this form.

(4) If only one-half per-cent in excess of the valuation rate is earned the reversionary bonuses under both classes can be expressed in the form of a uniform simple reversionary bonus, the rate for endowment assurances, on the assumptions made in the paper, being somewhat less than that for whole-life policies. Under these conditions, if endowment assurances and whole-life policies are both to receive the same rate of bonus the premiums under the former class should be loaded more heavily in proportion than those under the whole-life class.

(5) In order to maintain the same rate of bonus under both classes it is vital that a rate of interest more than one-half per-cent in excess of the valuation rate should be earned. The larger interest profit derived from the endowment assurance policies will then compensate for the heavier annual payments required to meet the expenditure of the office.

Abstract of the Discussion.

Mr. H. T. ADLARD said that the question of the relative bonuses which were due to whole-life and endowment assurances did not, owing to the varying conditions in different offices, admit of a general answer that would be applicable to all cases; so that any investigation into the subject based on new data was both interesting and valuable, especially as endowment assurances now formed so large a proportion of the new business of life assurance offices. The author's numerical results depended, of course, upon the particular scale of premiums adopted in Table A, but they seemed to be fair premiums although rather above the average, especially in the case

of the whole-life premiums and at the older ages. But the most important point was the method employed for the distribution of profit. That must, of course, be some form of contribution method, and the author had divided the interest profit in proportion to the reserves, and the whole of the remaining profit in proportion to the $O^{[M]}$ loading less expenses, calculated at the valuation rate of interest. He quite agreed that it was more equitable to divide the loading profit in proportion to the select, rather than the aggregate loadings, as select tables gave more accurate risk premiums than aggregate tables, and there seemed to be no objection to dividing the profit from surrenders, lapses, and from the working of non-profit business, in that way, as it did not seem to belong to the with-profit policyholders in any definite proportions; but the profit from mortality required a little consideration.

The author had said that assuming select reserves, if select mortality were experienced no profit from mortality would arise, and that if a lighter mortality were experienced it could only arise from accidental deviations from the select mortality table due to the smallness of the experience of the office. He thought that was rather too sweeping a statement, as it was quite possible for a lighter mortality than the $O^{[M]}$ Table to be experienced, and in such a case the mortality profit should, strictly speaking, be divided in proportion to the difference between the expected and actual death strains. The effect of that would be to give a larger share of the surplus to policies of short duration than to policies which had been in existence for a long period. But as the expenses and commission pressed most heavily on the new business, it would be sufficiently accurate—unless the mortality experienced was very much lighter than that shown by the table used in the valuation—to divide the mortality profit in proportion to the surplus loadings. So that the method of distribution adopted in the paper would seem, except in very special cases, to be sufficiently accurate. The figures in Tables E and F confirmed the results obtained by the late Mr. Sunderland, Mr. Lidstone and Mr. Chatham, but the author had carried his analysis a step further, and shown the effect on the relative bonuses of whole-life and endowment assurances of a reduction in the interest margin, and of variations in the incidence of the expenses.

With regard to the variations in the interest margin, the author had shown in Table E that on the assumptions made in the paper as to premiums, valuation basis, &c., with a margin of 1 per-cent, the results of the contribution method could be very nearly expressed by a compound reversionary bonus both for whole-life and endowment assurances, the rate being slightly higher for whole-life policies than for endowment assurances, and that was consistent with Mr. Lidstone's result that, with the assumptions made in his paper, with a margin of $1\frac{1}{8}$ per-cent, a compound reversionary bonus could be given to whole-life and endowment assurances, the rate being slightly lower for whole-life than for endowment assurances. The difference was accounted for partly by the fact that the reserves in proportion to which the interest profit was divided were greater in

the case of endowment assurances than whole-life assurances, and partly by the fact that the author's whole-life premiums were larger in proportion to the endowment assurance premiums than Mr. Lidstone's. Mr. King had also obtained the same result by discounting a certain amount of bonus and finding that there was a slightly larger margin of loading left in the case of endowment assurances as compared with whole-life cases. Mr. Chatham had also shown that with an interest margin of 1 per-cent, endowment assurances were entitled to a compound reversionary bonus.

The author had shown in Table F that if the interest margin were only $\frac{1}{2}$ per-cent, both whole-life and endowment assurance bonuses, except under the shorter-term policies, could be very nearly expressed in the form of a uniform simple reversionary bonus at the same rate for all ages at entry and terms of assurance, but that the rate for the endowment assurances would be about 10 per-cent lower than the corresponding rate for the whole-life policies; and that was consistent with Mr. Chatham's results that, with an interest margin of $\frac{3}{4}$ per-cent, endowment assurances were entitled to a slightly increasing simple reversionary bonus. Tables H and K showed the effect on the relative bonuses on whole-life and endowment assurances of variations in the incidence of the expenses, and were very interesting. Those tables showed that an alteration in the initial expenses did not materially affect the relative rates of bonus; also, that if the renewal expenses took the form of a percentage of the premiums and a constant depending on the sum assured, an alteration in the percentage only affected the relative rates of bonus to a small extent; but that if the renewal expenses took the form of a percentage of the premiums instead of a percentage plus a constant, the whole-life bonuses were increased and the endowment assurance bonuses were decreased. From this it would appear that, other things being equal, an office not paying commission should be able to give larger relative bonuses to endowment assurances than a commission-paying office.

Mr. S. J. H. W. ALLIN said the subject of the paper was one which especially lent itself to discussion, owing to the fact that the data necessary for comparison were very limited, especially with regard to expenses and mortality. The author had divided the paper into three heads—expenses, interest and mortality. With respect to expenses, the author suggested that the renewal expenses should be distributed as a percentage of the sums assured and a percentage of the premiums, instead of a percentage of the premiums only. Personally, he was not quite sure that that was correct. The correct distribution of expenses was most important, as affecting different classes of assurances. Apart from the commission, and perhaps a small charge for investments, the renewal expenses of an endowment assurance policy were not necessarily heavier than those of a whole-life policy, and he would suggest that a small percentage of the premiums should be taken, say 3 per-cent, and the balance of expenses treated as a constant charge on each policy instead of a percentage upon the sum assured.

Under this mode of treatment the profit from loading under endowment assurances would be somewhat heavier than under the whole-life policies. With regard to interest, the author had referred to the modifications in the contribution method that would become necessary if the office adopted a lower rate of interest; but surely, if the principle were correct, it must hold in all cases. It seemed to him that the principle was not at fault, but the application. The net premium valuation at a very low rate of interest was only an approximation to a true valuation; a true valuation would be a valuation at an approximately true rate of interest, with provision for future bonuses and expenses. The author drew attention to that matter, but he had not proceeded any further with it. It seemed to him (the speaker) that the net premium valuation rate had nothing to do with the interest profit of the policy; he would go even a little further, and say that it seemed to him the difference between the rate assumed when the premiums were calculated and the effective rate of interest was the true profit. With regard to the mortality, he quite agreed with the author that there was not very much profit from that source in these days. The profit assumed in the past was nearly all from suspended mortality.

MR. H. H. AUSTIN said that the author had divided the paper into the three usual headings of interest, mortality and loading. So far as the interest profit was concerned, it seemed rather a pity that he had not shown the results under that heading separately, because that item did not present much difficulty. With most valuation tables it was fairly constant, and also if the bonus arising from interest were found for any age or term to be x, for instance, for a surplus margin of $\frac{1}{2}$ per-cent, then for a surplus margin of 1 per-cent it would be $2x$. For that reason it could be fairly easily be disposed of, and attention could be concentrated on the loading results. With regard to the loading profit, that was the main point to be considered in discussing relative bonuses. With certain assumptions as to premiums and expenses, the author had brought out certain rates of bonus, but when those assumptions were varied, the rates were also varied. He was struck with the fact that the bonuses brought out under endowment assurances appeared to be less than under the whole-life; his impression previously was that the contrary usually held good. He had taken Mr. Lidstone's average premiums, and working on the $O^{[M]}$ data, he found the uniform bonuses corresponding to the author's Table F, for whole-life assurances at ages 25, 35, 45 and 55, came out at 44, 44, 45 and 52, showing that the endowment assurance bonuses were rather higher. That, however, simply resulted from varying the premiums brought into calculation.

Of course, no general rule could be laid down on the subject, since the premiums charged by different offices varied as much as 10 per-cent or more, and a variation of 10 per-cent on an average premium would provide uniform annual bonuses ranging from 8*s*. 6*d*. at age 20 to 18*s*. at age 60; and under endowment

assurances from 12*s.* on a 30-year term, to 18*s.* on a 15-year term policy. He thought those figures showed the great importance of the premiums charged in a consideration of relative rates of bonus. Mr. Allin had referred to the question of the expenses, and that also was a matter that considerably affected the results. In dealing with questions of bonus, it was usual to treat initial expenses as a percentage of the sum assured, but in considering the two classes under discussion, where it was necessary to get as near as possible to the facts, it seemed to him that that was hardly correct. He had taken out the experience of a set of representative offices, and found that the average sums assured under endowment assurances were roughly about one-half those under whole-life assurances, and that was rather an important point to be considered.

The general expenses of dealing with a policy of £500 were practically the same as with a policy of £5,000. Taking the author's assumptions as to £3 per-cent initial expenditure, he (Mr. Austin) assumed that the number of policies in the whole-life and endowment assurance classes were as 4 to 1, and the average sums assured £500 and £250. Allowing 30*s.* per-cent, or half the expenditure to be in proportion to the sum assured, he divided up the balance in proportion to the number of policies, and that brought out an initial expense for whole-life policies of about £2. 15*s.* per-cent, and for endowment assurances about £4 per-cent. The result on the uniform annual bonus would be an increase on the whole-life of 6*d.* to 1*s.* per-cent, but a decrease on the endowment assurances of about 2*s.* per-cent for all terms. Of course, a similar argument would apply to the renewal expenses.

A further question arose in considering the growth of endowment assurances. If, for instance, those same rates of expenditure were assumed, *i.e.*, £2. 15*s.* and £4, and the proportion of endowment assurances rose from one-fourth to two-thirds, the effect on the initial expenditure of the office was to increase it from £3 to about £3. 5*s.* per-cent, and that would seem to show that as endowment assurances became more popular, with smaller sums assured, the general office expenses would tend to increase, other things being equal. In reference to the mortality profit, the author seemed to consider this as arising from the difference between the $O^{[M]}$ and the O^{M} Tables, but he himself was not quite sure that that sufficiently expressed the case. The $O^{[M]}$ Table was an average table made up of the experience of a large number of offices, and there was no doubt that some offices experienced mortality much more favourable and others a mortality considerably less favourable than that shown. He found that differences in the mortality considerably affected the whole-life policies, but it did not have much effect on the endowments. That would seem to follow from Mr. Chatham's paper, in which, although only 80 per-cent of the select mortality was assumed, the endowment assurance bonuses were not increased to any great extent.

Assuming lives entering at ages 28 and 43, and experiencing the mortality of two years older, under whole-life business, the uniform

annual bonus per-cent was affected to the extent of 4*s*. at age 30 and 7*s*. at age 45; while with a difference of five years under the endowment assurances it was affected at age 30 to the extent of from 1*s*. to 2*s*., and at 45 to the extent of 2*s*. to 5*s*., according to the length of term. It would appear, therefore, that a difference of five years on endowment assurances for terms 15, 20, and 30 years had far less effect on the bonus than a difference of two years under the whole-life policies. With reference to the favourable mortality experienced under endowment assurances in the past, that, as had been rightly pointed out, would probably not continue, but it seemed to him a question whether the better lives would not exercise selection in the future and enter under endowment assurances, and so tend adversely to affect the mortality that would be experienced under the whole-life business.

Mr. C. R. V. COUTTS said the last speaker had referred to the fact that in the author's tables he brought out smaller bonuses under endowment assurances than under the whole-life policies. He thought that was chiefly due to the fact that he had based his premiums on the $H^{[M]}$ Table and assumed his mortality to follow the $O^{[M]}$ Table. As a matter of interest he had compared the net premiums under the $O^{[M]}$ and $H^{[M]}$ Tables for a whole-life policy, and for a 20-year endowment assurance; the $H^{[M]}$ were from 7 to 12 per-cent higher than the $O^{[M]}$ net premiums for the whole-life, but in the case of the endowment assurances they were only about 3 per-cent higher; so that there was a margin of about 5 per-cent extra loading on the whole-life premiums on the $O^{[M]}$ basis, which naturally, assuming the mortality to follow the $O^{[M]}$ Tables, would give higher bonuses on the whole-life policy. He did not quite understand why the author based his premiums on the $H^{[M]}$ instead of on the $O^{[M]}$, because it seemed that he had introduced by so doing an unnecessary deviation between the endowment assurance bonuses and the whole-life bonuses.

He did not agree that the mortality profit could be dismissed quite so easily as the author suggested. Of course it varied with the office to a large extent, but he was sure that many offices obtained a profit from mortality on policies that were more than 5, 10 or even 15 years in force. The author also dismissed the question of profit from the surrenders, lapses and non-participating business, on the ground that, as the participating policyholders had not in any way contributed to that profit, therefore no question of equity arose in considering the method to be adopted in distributing it. But in a mutual office the profit had to be distributed amongst the participating policyholders, and therefore it was necessary to consider the method of distributing, and the question of equity arose. If, for example, it were all given to the policies where the lives assured had not attained 50 years, the older policyholders would probably raise a question of equity! The profit from miscellaneous sources must, as in the case of the mortality profit, vary with the offices. In some companies the amount was not so small that it could be dismissed as a negligible quantity. The question of division of the

profit was a very interesting one, and he thought the remarks made by previous speakers really led to a *reductio ad absurdum* of the so-called equitable or contribution method of dividing profits. They showed that the profits allocated to each policy depended largely on the basis adopted to allocate the expenditure, which in itself had been shown on any practicable basis to be inequitable. A policy for £5,000 cost less in general management in the ordinary way than a policy of £500, and yet whether the expenses were allocated on the basis of the premiums or on the basis of the sum assured, the £5,000 policy was not treated fairly. So far as equity was concerned, it seemed to follow that the whole scheme of division of profits on a basis of that kind must fall to the ground.

A similar point arose in connection with the valuation rate of interest. If a net premium valuation were employed, and the rate of interest were changed from 3 to $2\frac{1}{2}$ per-cent, it at once threw a lot of so-called profit from loading into interest-profit, and thus distributed it on quite a different basis. Both those valuations could not be equitable. If equity demanded the division on the 3 per-cent basis, then the $2\frac{1}{2}$ per-cent valuation must be inequitable. He was one who hoped that in the course of time the net premium method of valuation would be treated with rather less respect than at present. It seemed to him to be quite indefensible on scientific grounds. The main argument used in defence was that the method afforded a rough-and-ready way of providing for a simple reversionary bonus, if there was a margin of one-half per-cent in the rate of interest, or for a compound reversionary bonus, if there was a margin of 1 per-cent or rather more. But instead of using a rule-of-thumb method like that, why not employ a valuation based on a rate of interest which approximated to that actually earned by the office, with a provision for the expenses? It might be possible, as a matter of practice, to treat the profit from mortality and the profit from miscellaneous sources as a set-off to the expenditure on commission and expenses of management, and get a net deduction from the future premiums for the expenses in that way, and then value the bonus that it was hoped to give in the future as a liability. That was surely a truer method of ascertaining divisible surplus than the net premium basis.

Mr. R. P. HARDY said that, having regard to the considerable and growing disproportion of the volume of endowment assurances to ordinary assurances, he could well understand that the minds of many people were being considerably exercised, and he ventured to think they would be still more exercised in the future. He had had occasion to look into the question himself, and at that stage his mind was still open, but the general trend of his opinion was that the reversionary bonuses of endowment assurances should be less than those for ordinary whole-term assurances. He thought it was very difficult to find a general test that might be laid down in the text-books as applicable, having regard to the various opinions, and still more varied practice respecting the loadings placed upon the premiums, the almost wonderful effect that a quarter per-cent

in the realized interest would make, and the unexpected results that the more favourable rate of mortality had upon the surplus yielded in endowment assurances. Again, there was the burning question of the expenses and their distribution, to say nothing of the sundry profits, which varied considerably in almost every office. Therefore, he did not see how any general test could be set up, and he thought that such would have to be framed by each actuary dealing with the facts in his own office as they came before him.

He observed that the author said he thought the difference between the rate of mortality in endowment assurances and that in whole-term policies was likely to disappear. He must ask the author not to rely too closely upon that conclusion. Then there was a formula given which professed to represent the profit from mortality. With very great respect for the author, he submitted that that was hardly a full statement of the case, and in that connection he referred to a paper he had read in New York (*Transactions of Fourth International Congress*, vol. I, p. 150 *et seq.*) in which he showed that a favourable mortality in the case of endowment assurances was by no means an unmixed blessing. The tables in that paper were prepared merely for the purposes of illustration, and he did not suggest that the ratio there brought out would necessarily be experienced in practice. Taking an endowment assurance of fifteen years—bearing in mind that he was dealing simply with what were called net premiums, and not with any profit from mortality or even interest—in the first quinquennium there was the considerable surplus of £1,307, in the second quinquennium it fell to £583, and in the third quinquennium it was only £113—not 10 per-cent of what it was at first.

The reason for that was not far to seek. In the case of endowment assurances, if the man did not die, he lived to claim the benefit, and all the profit the office could anticipate was the interest upon moneys suspended and left in the hands of the company until the maturity age was reached, plus the additional premiums the office received—and that profit did not work out at quite as much as some persons might think. But although he was that evening in the unhappy position of not agreeing with the conclusions put forward, the author must not think he was not fully grateful to him for the considerable trouble he had taken in working out a complete view of the subject for consideration. Whether it was possible to agree or not with all the conclusions submitted, the thanks of the Institute were none the less due to the author for endeavouring to hold a torch to lighten what was at present a very dark stretch of country.

Mr. W. P. ELDERTON thought that scant justice was being done to the author in the discussion. It seemed to him the author had set out to give some examples of what might happen in certain events, and he had succeeded in doing so. Doubtless, if other circumstances had occurred, the author would have worked out particulars in another way; but if he had proved nothing else, at any rate he had proved that it was necessary, in order to be equitable,

to analyze the sources of profit and the sources of loss, if expenses could be so called. He agreed with Mr. Coutts that the profit from surrenders and lapses ought to be considered, but he thought it ought to be considered rather on the lines mentioned by Mr. T. G. C. Browne some years ago (*J.I.A.*, xxxii, p. 115), when he was dealing with the distribution of surplus in a discussion that followed the reading of Mr. Lidstone's paper. Mr. Browne then said that he thought the best way to treat it was to consider it as a set-off against the initial expenses. It seemed that that was the most equitable way because if all the expenses were set against the loadings, the whole of the reserve from lapses should appear as profit. If it appeared as profit in that way, since it was said that there was little profit on lapses and surrenders because of the initial and other expenses, the two sources of profit (loading and withdrawals) should be pooled together in considering the relation between loading, expenses and bonuses.

Some observations had been made upon aggregate tables in the paper and discussion. He was afraid he could not agree with Mr. Coutts' remarks on the net premium aggregate table method of valuation. He felt that the net premium method of valuation was capable of a far stronger defence at the present time than ever before. Not only did select tables and other methods involve a much larger amount of work than an aggregate table, but if it was necessary to face the question of the difficulty due to a falling rate of interest, even then it did not necessarily follow that the net premium method of valuation broke down. It might mean that the offices should keep a considerable difference between the valuation rate and the realized rate of interest, but it did not necessarily follow that they need do that by decreasing the valuation rate or by using an artificial net premium. It might be done by artificially increasing the realized rate of interest, by spending the money that would otherwise go to reducing the valuation rate of interest in writing down the marketable and other securities. Obviously if the securities were written down, the nominal amount of the fund was being decreased, which sent up the realized rate of interest.

With regard to the method of dividing the mortality-profit, he did not see in the paper any remark as to how it was done, but he was afraid from some experiments he had made on that subject that he could not quite agree that the whole of the mortality-profit made by the office should go to the policies in the first quinquennium, because, when using an aggregate rate of mortality and working on the attained age, the aggregate rate in the early groups of ages took into account a large quantity of select material, and therefore was in effect something like a select rate. That seemed to him to show that it did not necessarily follow in using an aggregate table that one need consider the particular duration of the policy, provided one dealt entirely with the age.

Mr. H. W. Manly said, with regard to the subject of the paper, that he always felt inclined to take the reverse method of

investigation; that was to say, if he wanted to give the same rate of reversionary bonus to endowment assurances as to whole-life assurances, then it was necessary to make the premiums fit that condition of affairs. Taking select tables and the rate of interest that could be earned, the rate of bonus desired and the actual expenses, premiums can be obtained from exactly the same data for whole-life assurance and endowment assurances, and in this way it would be possible to give the same rate of reversionary bonus. That was subject to a certain amount of special reserve in the valuation, because it would be found that if all the policies were endowment assurances the surplus would give a larger bonus at the first quinquennium than for the second or third quinquennium, so that a certain reserve had to be set aside to make up for the increase in the value of a reversionary bonus as the policy matured. Mr. Coutts had made a remark that mortality profits were to be found on policies more than fifteen years old, but he very much doubted that. The way in which the estimated claims were generally calculated and compared with actual claims was not fair and equitable. If the expected claims were based upon a select table of mortality, it would be found that the expected and the actual often came very close together, and if the mortality amongst the policyholders more than fifteen years in force were compared with the select rates, there would not be much profit found in the mortality. With regard to expenses, there again he rather demurred to Mr. Coutts' statement when he said that a £5,000 policy cost less than a £500. To a certain extent it did—that was to say with regard to office expenses. If all the policies were for £100 it would necessitate having another clerk or two. But having regard to the incidence of commissions, as now allowed, on the first year, and on renewals, he thought the expenses on the £5,000 policy would work out very much about the same ratio as on the £100.

Mr. COUTTS said he did not mean to include commission. He was speaking of expenses as office expenses.

Mr. MANLY said that those were not very material, when distributed over the whole number of assurances. With regard to a net premium valuation, he thought Mr. Coutts had implied that it was unscientific, but there was a great deal to be said for it as a scientific method, although he did not attach that great importance to it that obtained some years ago. He thought it had saved the companies in this country from consuming their reserves or from plunging into insolvency. If anything was going to be substituted for the net premium reserve, he thought the substitute must be something even more stringent, the substitution of a valuation of future bonus at the rate to be declared. So far from wishing to see the net premium barrier broken down, he rather desired to see it made stronger than it was. The suggestion that there was something wrong about it might eventually induce weak and extravagant companies to reduce their reserves by adopting some system of gross premium valuation,

reserving only a percentage for expenses, which might be varied according to circumstances. It had always appeared to him, and the paper somewhat confirmed the view, that the average rate of premium which was charged for endowment assurances would not enable the same reversionary bonus to be given as could be given upon the whole-life policies. It was a question of the premium to be charged.

MR. W. P. PULLEY thought that Mr. Rietschel had not given quite sufficient weight to the effect of favourable mortality upon bonuses, and two illustrations had just occurred to his mind which he thought were good proofs of the favourable effect of mortality. The first related to an office which had a separate temperance section, where it could be plainly seen that the favourable mortality was accompanied by a higher bonus. The second was the case of a company which insured lives from among a favourable class of the community, where a very light mortality was again accompanied by high bonuses.

MR. H. J. BAKER said that there had been some difference of opinion as to the relative bonuses which should he given to whole-life policies and endowment assurances, and it was therefore desirable that, if possible, some general principles should be established for their guidance. It was difficult to ascertain exactly the actual methods adopted in practice, but apparently the majority of offices declaring simple or compound reversionary bonuses gave the same reversionary bonus to endowment assurances and whole-life policies. That uniformity was somewhat remarkable, having regard to the great differences which existed between office and office. For instance, he found that the margin between the rates of interest experienced, and assumed in the valuation, ranged from $\frac{1}{2}$ per-cent to $1\frac{1}{4}$ per-cent in the case of offices giving a simple reversionary bonus, and from $\frac{1}{2}$ per-cent to $1\frac{1}{2}$ per-cent in the case of offices giving a compound reversionary bonus. Among offices which did not adopt either of those methods of distribution, some gave the same cash bonus to whole-life policies and endowment assurances, and others gave the same cash percentages of the actual premiums paid during the valuation period. Others, again, gave a larger reversionary bonus to endowment assurances, and amongst those some gave a bonus which varied according to the original length of the endowment term.

The premiums in Table A formed the basis of the author's calculations and deductions, and, having regard to the practical nature of the investigation, it was important that those premiums should approximate as closely as possible to the rates actually charged by representative offices. In the first place, he did not think it could be considered satisfactory to assume a rate of interest of 4 per-cent in the calculation of premiums for an office that earned only $3\frac{1}{2}$ per-cent. If that was done, part of the loading for expenses and bonuses must be absorbed in providing for the deficiency in the interest. The author, in fact, loaded his premiums very heavily, with the result that they were on the whole distinctly higher than

the average office rates. For instance, taking thirteen offices which declared a uniform compound reversionary bonus, he found that the average whole-life premiums were from 2*s*. 4*d*. to 3*s*. 4*d*. per-cent less than those in Table A, while in no case and at no age was a higher premium payable. The author's endowment assurance premiums were about 1*s*. per-cent above the average for the longer terms, and approximated closely to the average for the shorter terms. Generally speaking, therefore, the author's whole-life premiums were unusually high, as compared with his endowment premiums. That fact had an important influence upon the bonuses which should be given to the two classes. He had accordingly recalculated for age 35 at entry the reversionary bonuses for whole-life policies and 30-year endowment assurances, taking a whole-life premium of £2. 16*s*. 2*d*. per-cent, and £3. 11*s*. 8*d*. per-cent for the 30-year endowment assurance, these being the average rates of the 13 offices. On that basis the total reversionary bonuses at the end of 30 years were approximately as follows:—

	Whole-life Assurance.	30-year Endowment Assurance.
Table B	£346	£376
(Rietschel)	421	406
Table C	254	266
(Rietschel)	322	293
Table E	371	395
(Rietschel)	441	425
Table F	277	285
(Rietschel)	342	312

On the assumptions made in the paper, an interest margin of at least 1 per-cent was required before the same compound reversionary bonus could be given to whole-life policies and endowment assurances, and if the excess interest was only ½ per-cent, a smaller simple reversionary bonus should be given to endowment assurances. Taking the average premiums he had assumed to be payable, with an interest margin of 1 per-cent, a slightly greater compound reversionary bonus, and with an interest margin of ½ per-cent, the same simple reversionary bonus could be allotted to the endowment assurances. Considerable emphasis was laid in the paper on the effect on the bonuses of different modes of allocating the expenses, but the figures he had given showed, he thought, that far greater importance must be attached to the relationship between the premiums actually charged for the different benefits. Premiums charged for endowment assurances by various offices differed to a far greater extent than did those for whole-life assurances, and although two offices might have the same margin as regards interest and identical expense rates and mortality experiences, yet the relationship between the whole-life and endowment assurance premiums might differ to such an extent as materially to affect the bonuses which could be given to the two classes respectively. In other words, to secure an equitable distribution of the surplus, a comparison of the premiums was

necessary, not only between those for whole-life policies and endowment assurances, but also in respect of the length of the endowment term.

The author practically ignored mortality profit or loss, and with the exception of Table D took no account of it in his calculations. His Tables E, F, H and K practically assumed an $O^{[M]}$ valuation and an $O^{[M]}$ mortality experience. He did not agree with the author in attaching so little importance to mortality profit or loss. It should not be forgotten that the $O^{[M]}$ Table was practically an average of the experiences of the contributing offices, and it by no means followed that the deviations from the standard table in the case of an individual office were of so little or no importance. In fact, it was probable that some offices habitually experienced a rate of mortality more favourable than that shown by the $O^{[M]}$ Table, while with other offices the reverse was probably the case. The mortality profit or loss had less effect upon endowment assurances than upon whole-life policies, as was shown in Table D of the paper. If, therefore, the mortality profit or loss were of considerable amount, the relationship between the bonuses actually earned by the two classes might be appreciably affected. The author considered the effect of surrenders on the bonuses to be but small, but that statement was not borne out by the figures given by Mr. Chatham in his paper on "The Analysis of the Profit from Endowment Assurances", published in the *Transactions of the Faculty of Actuaries*, vol. iii, p. 15. On the assumptions made in that paper, Mr. Chatham showed that surrenders had "a very marked effect upon the rate of bonus, increasing it at the end of the first quinquennium by over 40 per-cent", and even at the end of twenty-five years the increase in the bonus was as much as 9 per-cent.

It would be seen from Tables E and F that a fall in the rate of interest had most effect upon the short-term endowment assurances. With an interest margin of 1 per-cent the total bonus at the end of ten years was, for age 35 at entry, £129 for a 10-year endowment, as compared with £114 for a 30-years' term policy; while if the interest margin were only $\frac{1}{2}$ per-cent the figures became £98 and £100 respectively. Having regard to the many and important differences which existed between office and office, and to the magnitude of modern endowment assurance business, he thought it was very important that, before deciding, in an individual case, as to the relative bonuses that should be given to whole-life policies and endowment assurances, a careful investigation should be made giving effect to all the conditions peculiar to the office. Probably the most equitable plan was to keep the endowment assurances in a distinct class for the purpose of bonus distribution. In conclusion, he wished to suggest to the author the advantage of giving in his tables for each age at entry the totals of the reversionary bonuses. He also thought an account of the methods he had adopted in the construction of his tables would be of advantage.

The PRESIDENT said the paper had been a very instructive one, on a subject that was being more and more pressed upon

attention, in consequence of the constantly increasing number of endowment assurances. One of the most important duties of the actuary was to distinguish equitably between the various classes of assured lives. There were many ways of approaching the subject, and he thought the author's way of starting from certain premiums and deriving the results had distinct advantages, and afforded as good an analysis as the plan favoured by Mr. Manly. Mr. Elderton, referring to the question of the valuation basis, remarked that while some companies, in order to meet the fall in the rate of interest, had adopted a lower rate of interest in their valuations, it had not occurred to them that they could maintain the rate of interest by writing down their assets. Such a plan had been forcibly brought before companies lately. It was also a painful thought that in order to raise the rate of interest 4*s*. per-cent in that way, it would be necessary to write about 5 per-cent off the whole of the funds. With regard to the remark of Mr. Coutts as to some other method of valuation taking the place of the net premium method, he hoped soon to see a paper dealing with a question of such great importance. He was conservative enough to hope that some method of valuation very similar to the present net premium method would always be retained. The time had now arrived when some member might usefully take up the study of this particular subject, and deal with other plans that were in practical operation in one or two companies that did not value strictly on the net premium plan. In New York a recent law had sanctioned another method of valuation, called the Select and Ultimate method, and it would be exceedingly interesting to have a paper on that subject.

The cordial thanks of the Institute were then unanimously accorded to the Author.

Mr. RIETSCHEL, in reply, thanked the members for the manner in which they had received the paper. Mr. Adlard had said that some offices would consistently yield a mortality profit, and he quite agreed; an example of a business consistently yielding a mortality profit was afforded by the temperance sections of several companies, but he was treating in his paper of an average office, and he did not think that in such an office the mortality profit was likely to be very great. In order to confirm that he quoted from Mr. Young's work on Insurance, page 210: "The object of administration in this department (mortality), as in others, should be the preservation of regularity of experience, so that one stage of the company's history should not be accidentally benefited to the disadvantage of another era. It may reasonably be assumed, regarding the present constitution of assurance companies' premiums and reserves, that, on the whole, and in the long run, no very appreciable surplus (or loss) can be anticipated from this source." Then, if a consistent profit or loss were shown, it probably ought to be divided in proportion to the difference between the expected and actual death strains, but it would be found necessary to depart from strict theory, because the bonuses resulting from such a process would

form a very irregular series. Again, if the difference between the strains happened to be large for the new policies, it would again be necessary to depart from strict theory, and make some reduction in respect of these policies. Another point was that if there were a consistent profit or loss on mortality, it would imply that the valuation basis was not entirely suitable to the conditions of the office, and therefore it would appear advisable to analyze the experience of the office, in order to ascertain the sufficiency or otherwise of the reserves. Assuming that the office consistently showed a profit from mortality, and the analysis showed that the reserves should be larger than those actually made, the effect of not making true reserves would be to lessen the interest profit by an amount equal to the interest on the difference between the reserves made and those actually required, and also to withdraw a certain amount from the loading profit. The increase in loading profit shown by the true valuation represented what was now regarded as mortality profit. That argument would seem to show that the mortality profit should be added to the loading profit, and he thought that agreed with Mr. Adlard's suggestion.

With regard to the method which he had adopted, he had as far as possible endeavoured to make the paper accord with the practice of offices. On the one hand, he had been bound by the premiums that had been charged, and on the other hand, the valuation basis was fixed regardless of the bonus and other special conditions appertaining to the company. It seemed to him that the scientific method of approaching the bonus question was akin to that suggested by Mr. Manly, namely, to fix the premiums required as far as possible according to the experienced rates of mortality, interest and expenditure, and by means of these to find the "true" surplus loading margins which the office premiums would produce. The next step would be to decide on the form of bonus which the office desired to declare, and that would be entirely decided by competition and business considerations. The relative rates of bonus provided by the "true" surplus margins should then be ascertained. Having decided upon the form of the bonus, the reserves should be based upon experience rates of mortality and interest to provide (a) the bare sum assured and the existing bonuses, and (b) that part of the future bonuses which would have to be earned by the participating policies, less the value of the future office premiums. The existing conditions did not allow of that being carried out. Public opinion and competition determined the valuation basis of the office regardless of conditions peculiar to it, and the object of any contribution method was to find the form the bonuses must take under those conditions, if substantial justice were to be meted out to the various forms of assurance. The paper showed that if an office earned 1 per-cent in excess of its valuation rate the bonuses were compound, and if $\frac{1}{2}$ per-cent were earned the bonuses were simple reversionary.

He observed that his premiums had been called into question. He had not brought forward those premiums as premiums that

should be charged. They were perhaps somewhat high, although he did not think they were outside the range of premiums charged by offices. To a large extent his calculations with regard to the relative rates of bonuses to be allowed to the two classes depended upon the relative premiums that had been charged. In his fourth conclusion he had said: "If endowment assurances and whole-life policies are both to receive the same rate of bonus, the premiums under the former class should be loaded more heavily in proportion than those under the whole-life class." In order to confirm that statement he had calculated premiums on the $H^{[M]}$ $3\frac{1}{2}$ per-cent Table, with a loading for a £1 per-cent simple reversionary bonus, together with a loading of 10 per-cent and 3*s.* for expenses and commission. The effect of this basis would be to increase the endowment assurance premiums in Table A to a much larger extent than the whole-life premiums, and with such premiums an office earning $3\frac{1}{2}$ per-cent could very nearly declare bonuses in the form of a simple reversionary bonus of the same rate for both classes.

Another speaker had referred to the profit on lapses and surrenders. He did not himself think that was likely to yield a very large surplus, and here again Mr. Young seemed to confirm him. Mr. Young, on page 196 of his work, said: "With reference to any possible surplus yielded by the lapse and surrender of policies, the observations contained in Chapter IV would appear conclusively to show that this source is either insignificant or entirely unproductive. Even if a minute fragment did exist in any case, it would merely consist in the former instance of the reserve (reduced by a protective provision for the effect of withdrawals upon the rates of expenditure and mortality), and in the latter event of the difference between the surrender-value granted and the reserve, diminished for the purpose just mentioned." Mr. Allin had suggested that the general expenditure of the office should be calculated on the number of policies, and not on the sum assured. He could not say he had gone very closely into that question, but it seemed to him that it would make the premiums which would have to be charged in respect of the smaller policies almost prohibitive. He thought it was absolutely necessary for the office to base the expenditure, not on the number of policies, but on the sums assured.

On the Relation between the Theories of Compound Interest and Life Contingencies. *By* JOHN MAYHEW ALLEN, F.I.A., *Actuary and Secretary of the General Accident, Fire and Life Assurance Corporation, Limited.*

[Read before the Institute, 25 March 1907.]

INTRODUCTION.

1. IT has often occurred to me that but scant justice has been done to the application of the infinitesimal calculus to the theories of compound interest and life contingencies. This is, perhaps, in some measure due to the popular relegation of the differential and integral calculus to the realms of the so-called "higher mathematics." There are, of course, two aspects of the case to be borne in mind. On the one hand, it is necessary to present the subjects in such a form as will be best suited to the student who is commencing to study them. For this purpose experience shows that a start should be made with particular cases, leaving the generalization until such time as the student shall have obtained a grasp of first principles sufficient to enable him to view the subjects in their general aspect. On the other hand, however, there is no doubt that to the reflective mind there comes a time when the desire is felt to invert the process and deduce the formulæ in their logical sequence from a fundamental general hypothesis.

2. This is an eminently practical age and there is a tendency to look somewhat askance at a theoretical treatise, but there can be no doubt that, apart from the mental satisfaction which it brings, a proper study of the true theory of a subject imparts to the mind that firm knowledge which is essential to confidence in its practical application. What I propose to attempt is to show how the theories of compound interest and life contingencies can be based on the consideration of the forces of interest and mortality respectively. I do not, of course, claim that I shall be expounding any new discovery,—it is, in fact, rather more accurate to state that in mathematical research we do not *discover* so much as *consider*,—but I venture to submit this paper to the Institute in the hope that it may prove of interest in presenting some well-known formulæ in an aspect somewhat different from the usual one, and that it may prove of value if only for the reasons mentioned at the beginning of this paragraph.

Interest and Mortality.

3. A sum of money is observed to increase with the lapse of time; we may therefore regard the sum as exposed to the operation of an incremental force, which we call the "force of interest." A body of lives is observed to decrease with the lapse of time; we may, therefore, regard the body as exposed to the operation of a decremental force, which we call the "force of mortality." In order to deal with these forces we require for each a measure of intensity and a symbolical representation thereof. We also require a term to represent the unit measure of time. The term usually employed is "year", and, as long as we bear in mind that this is here quite an arbitrary measure, we can adopt it as our unit for the sake of convenience of expression. Now, it is not necessary to regard the forces under consideration as of uniform intensity,—in fact, experience teaches us that in the case of mortality the force tends to increase in intensity with the lapse of time,—and we will, therefore, represent the measure of intensity of either force as a function of the time which has elapsed from the initial moment of observation, *i.e.*, the moment of investment of the money in the former case, the moment of birth of the lives in the latter case.

4. If δ_t represent the nominal annual measure of the force of interest per unit exposed, and μ_t the nominal annual measure of the force of mortality per unit exposed, considering in either case the intensity of the force operating at the moment of time which is distant t from the initial moment, then, using $f(t)$ to represent the amount of the money at time t, and l_t to represent the number of the lives at time t, we have:

$f(t)\delta_t dt$ is the infinitesimal instalment added to the money by interest the instant after time t.

$l_t\mu_t dt$ is the infinitesimal number of lives taken away by mortality the instant after time t.

$\int_0^1 f(x+t)\delta_{x+t}dt$ is the total interest earned in the year succeeding time x.

$\int_0^1 l_{x+t}\mu_{x+t}dt$ is the total number of deaths occurring in the year succeeding time x.

$\int_0^{\frac{1}{m}} f(x+t)\delta_{x+t}dt$ is the total interest earned in the $\frac{1}{m}$ of a year succeeding time x.

$\int_0^{\frac{1}{m}} l_{x+t}\mu_{x+t}dt$ is the total number of deaths occurring in the $\frac{1}{m}$ of a year succeeding time x.

Thus we get—

$$\frac{\int_0^1 f(x+t)\delta_{x+t}dt}{f(x)}$$ is the total interest earned in the year succeeding time x per unit exposed at the beginning of that year. This we represent by the symbol i_x and call the "effective rate of interest for the year succeeding time x."

$$\frac{\int_0^1 l_{x+t}\mu_{x+t}dt}{l_x}$$ is the total number of deaths occurring in the year succeeding time x per unit exposed at the beginning of that year. This we represent by the symbol q_x and call the "rate of mortality at age x", though it should, strictly, be called the "effective rate of mortality for the year succeeding age x."

Similar statements hold for the corresponding expressions for $\frac{1}{m}$ of a year, but, if we multiply them by m so as to obtain their nominal annual measures, we get—

$$m.\frac{\int_0^{\frac{1}{m}} f(x+t)\delta_{x+t}dt}{f(x)}$$ is the nominal annual rate of interest, convertible m times a year, for the $\frac{1}{m}$ of a year succeeding time x, for which we can use the symbol $j_x^{(m)}$.

$$m.\frac{\int_0^{\frac{1}{m}} l_{x+t}\mu_{x+t}dt}{l_x}$$ is the nominal annual rate of mortality, convertible m times a year, for the $\frac{1}{m}$ of a year succeeding age x, for which we can use the symbol $r_x^{(m)}$.

5. From the definitions of δ_t and μ_t we see that—

$$\left\{\begin{array}{l} \underset{h=0}{\mathrm{Lt}}\left[f(x+t)\{1+h\delta_{x+t}\}\right]=\underset{h=0}{\mathrm{Lt}}\left[f(x+t+h)\right] \\ \underset{h=0}{\mathrm{Lt}}\left[\ l_{x+t}\{1-h\mu_{x+t}\}\ \right]=\underset{h=0}{\mathrm{Lt}}\left[\ l_{x+t+h}\ \right] \end{array}\right\}$$

whence
$$\left\{\begin{aligned}\delta_{x+t} &= \underset{h=0}{\mathrm{Lt}}\ \frac{f(x+t+h)-f(x+t)}{hf(x+t)}\\ \mu_{x+t} &= \underset{h=0}{\mathrm{Lt}}\ \frac{l_{x+t}-l_{x+t+h}}{hl_{x+t}}\end{aligned}\right\}$$

i.e.,
$$\left\{\begin{aligned}\delta_{x+t} &= \frac{1}{f(x+t)}\frac{df(x+t)}{dt} = \frac{d\log_e f(x+t)}{dt}\\ \mu_{x+t} &= -\frac{1}{l_{x+t}}\frac{dl_{x+t}}{dt} = -\frac{d\log_e l_{x+t}}{dt}\end{aligned}\right\} \quad . \quad . \quad (1)$$

Hence we deduce—

$$\left\{\begin{aligned}\int_0^n \delta_{x+t}dt &= \log_e f(x+n) - \log_e f(x)\\ \int_0^n \mu_{x+t}dt &= \log_e l_x - \log_e l_{x+n}\end{aligned}\right\}$$

or
$$\left\{\begin{aligned}\frac{f(x+n)}{f(x)} &= e^{\int_0^n \delta_{x+t}dt}\\ \frac{l_{x+n}}{l_x} &= e^{-\int_0^n \mu_{x+t}dt}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

6. Equations (2) ought to be capable of derivation from the consideration that—

$$\left\{\begin{aligned}f(x+n) &= f(x)\cdot\underset{h=0}{\mathrm{Lt}}\left[(1+h\delta_x)(1+h\delta_{x+h})\ \ldots\ (1+h\delta_{x+n-h})\right]\\ l_{x+n} &= l_x\cdot\underset{h=0}{\mathrm{Lt}}\left[(1-h\mu_x)(1-h\mu_{x+h})\ \ldots\ (1-h\mu_{x+n-h})\right]\end{aligned}\right\}$$

I do not think it necessary to proceed on these lines just now, but, seeing that to do so involves an interesting general theorem, I have, for the sake of this general interest, dealt with the matter in the Appendix.

7. $\frac{f(x+n)}{f(x)}$ is the ratio in which $f(x)$ is increased by being exposed to interest for n* years after time x, and therefore it is easy to see that $\frac{f(x)}{f(x+n)}$ is the value at time x of a unit receivable at time $x+n$. $\frac{l_{x+n}}{l_x}$ is the ratio in which l_x is diminished by being exposed to mortality for n years after time x, and is represented by ${}_np_x$, the probability that a person aged x will

* n is, of course, not necessarily an integer.

survive n years. Thus the present value of a unit receivable n years hence if a person now aged x be alive then is

$$ {}_n\mathrm{E}_x = \frac{f(x)}{f(x+n)} \cdot \frac{l_{x+n}}{l_x} = e^{-\int_0^n (\delta_{x+t} + \mu_{x+t})dt} \quad . \; . \; . \; . \; . \quad (3) $$

8. At this stage we may examine the results of assuming constant forces of interest and mortality.

If δ_{x+t} be constant and $=\delta_0$, and μ_{x+t} be constant and $=\mu_0$, equations (2) become—

$$ \left\{ \begin{aligned} \frac{f(x+n)}{f(x)} &= e^{\int_0^n \delta_0 dt} = e^{n\delta_0} \\ \frac{l_{x+n}}{l_x} &= e^{-\int_0^n \mu_0 dt} = e^{-n\mu_0} \end{aligned} \right\} \quad . \; . \; . \; . \; . \; . \; . \quad (4) $$

Looking back to the expressions for the effective rates of interest and mortality, we see that—

$$ i_x \text{ becomes } \int_0^1 e^{t\delta_0}\delta_0 dt = e^{\delta_0} - 1, $$

$$ q_x \text{ becomes } \int_0^1 e^{-t\mu_0}\mu_0 dt = 1 - e^{-\mu_0}, $$

showing that the effective rates of interest and mortality both become constant. Representing them respectively by i_o and q_o, we have—

$$ \left\{ \begin{aligned} i_0 &= e^{\delta_0} - 1, \text{ or } \delta_0 = \log_e(1+i_0) \\ q_0 &= 1 - e^{-\mu_0}, \text{ or } \mu_0 = -\log_e(1-q_0) \end{aligned} \right\} \quad . \; . \; . \quad (5) $$

Also it follows from equations (4) and (5) that the amount of a unit in n years becomes $(1+i_0)^n$ and that the present value of a unit due at the end of n years becomes $(1+i_0)^{-n}$, which can also be written v_0^n, where $v_0 = \dfrac{1}{1+i_0}$ and is the value of a unit due at the end of 1 year; whilst the probability that a person of any age will live n years becomes $(1-q_0)^n$, which can also be written p_0^n, where $p_0 = 1 - q_0$ and is the probability that a person of any age will live 1 year.

9. From equation (3) we see that, if the forces of interest and mortality vary in such a manner that $\delta_{x+t} + \mu_{x+t}$ can be expressed as an integrable function of t, the calculation of the numerical values of life benefits can be simplified. The best case to consider is the one where the sum of the forces of interest and mortality is constant. Let us suppose, therefore, that $\delta_{x+t} + \mu_{x+t} = \delta$, a constant, then ${}_n\mathrm{E}_x = e^{-\int_0^n \delta dt} = e^{-n\delta} = v^n$, where

v is calculated at the constant effective rate of interest, i, equivalent to the constant force of interest δ. In such a case it is evident that the values of single-life annuities can be replaced by those of annuities-certain. For instance—

$$\begin{array}{lcl} {}_nE_x & \text{becomes} & v^n \\ a_{\overline{xn}|} & ,, & a_{\overline{n}|} \\ a_x & ,, & a_{\overline{\omega-x}|}. \end{array}$$

10. *Example* 1.—To find, approximately, the value of $a_{35:\overline{10}|}$ at 3 per-cent by the O[M] Table. On reference to the column of μ_x we see that its average value during the 10 years after age 35 is ·00889. The force of interest equivalent to the effective rate, 3 per-cent, is ·02956. We can assume that δ_{x+t} and μ_{x+t} vary during the 10 years in such a manner that their sum is always equal to ·02956 + ·00889, *i.e.*, ·03845, which is actually the average value of $\delta_{x+t}+\mu_{x+t}$ according to the conditions of the problem. Now the value of i (to the nearest $\frac{1}{8}$ per-cent below) equivalent to $\delta = \cdot 03845$ is $i = \cdot 03875$. Looking up $a_{\overline{10}|}$ at $3\frac{7}{8}$ per-cent we find its value (to 3 places of decimals) to be 8·162, which is also the true value (to 3 places of decimals) of $a_{35:\overline{10}|}$.

The reason for taking i to the nearest $\frac{1}{8}$ per-cent *below* is that we assumed a decreasing rate of interest to be replaceable by a constant rate of interest, and must therefore expect the resulting value of $\delta_{x+t}+\mu_{x+t}$ to be too high rather than too low. I do not pretend that the method of calculation illustrated in this example,—*i.e.*, the evaluation of $a_{\overline{xn}|}$ by means of the column μ_x and interest tables,—will be universally applicable. It is evident that the assumptions made postulate a fairly short term and a smooth progression in the values of μ_x. Nor is it to be expected that even under favourable conditions the value obtained will always be true to three places of decimals. We can, however, generally expect a good approximation under favourable conditions. It will be well to take another example.

Example 2.—$a_{40:\overline{10}|}$ at O[M] $3\frac{1}{2}$ %.

δ at $3\frac{1}{2}$ %	= ·03440
Average value of μ_x	= ·01122
	·04562
Equivalent value of i to nearest $\frac{1}{8}$ % below	= ·04625
$a_{\overline{10}|}$ at $4\frac{5}{8}$ %	=7·864
True value of $a_{45:\overline{10}|}$	=7·874

In this example the approximate value is too small by 1 in the second place of decimals.

11. The examples given so far are illustrative rather than practically useful, seeing that the temporary annuity-values in question are given in the volume of O^M Tables. But, in another way, the rationale under consideration may be put to actual practical use. Let us, for the sake of brevity, use the term "rate-certain", for the rate of interest at which the annuity-certain has to be taken. It is evident that, within the limits of the rates of interest ordinarily used, the rate-certain for a given initial age and term exceeds practically by a constant the rate at which the temporary annuity-value is required, because this excess varies at all only by reason of the variation in the difference between corresponding values of i and δ. Therefore, having the values of $a_{x\overline{n}|}$ at a given rate of interest, we can, with the sole aid of tables of values of $a_{\overline{n}|}$, approximate to the values of $a_{x\overline{n}|}$ at other rates of interest.

Example 3.—We saw in Example 1 that the O^M 3 % value of $a_{35:\overline{10}|}$ was the same as the value of $a_{\overline{10}|}$ at $3\frac{7}{8}$ %,—*i.e.*, the rate-certain here exceeds the actual rate by $\frac{7}{8}$ %. Consequently we have as approximations:

$a_{35:\overline{10}|}$ at $3\frac{1}{2}$ % $= a_{\overline{10}|}$ at $4\frac{3}{8}$ % . . . . . $= 7{\cdot}962$

$a_{35:\overline{10}|}$ at $3\frac{1}{4}$ % $= a_{\overline{10}|}$ at $4\frac{1}{8}$ % . . . . . $= 8{\cdot}061$

The true $3\frac{1}{2}$ % value is 7·961.

Example 4.—The office annual premium for a 7 years' term assurance on a life aged 28 next birthday being £1. 3*s*. %, required the values of the single premium for the risk, commuting on the bases of $O^{[NM]}$ $3\frac{1}{2}$ % and 4 % respectively.

In Messrs. Baker & Raisin's Tables, we find the $O^{[NM]}$ 3 % value of $a_{[28]:\overline{6}|}$ to be 5·296. Referring to Oakes' Interest Tables, we find this to be the mean between the $3\frac{3}{4}$ % and the $3\frac{5}{8}$ % values of $a_{\overline{6}|}$. The mean between the $4\frac{1}{4}$ % and the $4\frac{1}{8}$ % values of $a_{\overline{6}|}$ is 5·210, which we adopt as the $O^{[NM]}$ $3\frac{1}{2}$ % value of $a_{[28]:\overline{6}|}$,* whilst the mean between the $4\frac{3}{4}$ % and the $4\frac{5}{8}$ % values of $a_{\overline{6}|}$ is 5·127, which we adopt as the $O^{[NM]}$ 4 % value of $a_{[28]:\overline{6}|}$:

$$1{\cdot}15 \times 6{\cdot}210 = 7{\cdot}142\,;$$

$$1{\cdot}15 \times 6{\cdot}127 = 7{\cdot}046.$$

* This agrees with the true value calculated by the formula $a_{[28]:\overline{6}|} = \dfrac{N_{[28]+1} - N_{35}}{D}$

Thus the required single premium is

£7. 2*s*. 10*d*. % commuting on the O[NM] $3\frac{1}{2}$ % basis,

or £7. 0*s*. 11*d*. % " " " 4 % "

12. We may now investigate the application of the principle of § 9 to assurance-premiums. We have—

$$\bar{A}^1_{x\overline{n}|} = \int_0^n {}_tE_x \mu_{x+t} dt$$

$$= \int_0^n v^t \mu_{x+t} dt$$

where v is calculated at the rate-certain.

If now we assume Makeham's Law of Mortality to hold, we get—

$$\bar{A}^1_{x\overline{n}|} = \int_0^n v^t(A + Bc^{x+t})dt$$

$$= A \cdot \frac{1-v^n}{\delta} + Bc^x \cdot \frac{1-v^n c^n}{\delta - \log_e c},$$

and

$$\bar{A}_{x\overline{n}|} = \bar{A}^1_{x\overline{n}|} + {}_nE_x$$

$$= A \cdot \frac{1-v^n}{\delta} + Bc^x \cdot \frac{1-v^n c^n}{\delta - \log_e c} + v^n$$

These formulæ, it is to be observed, apply only to an aggregate table graduated by Makeham's law, and calculations by them would, besides, be too laborious to be of any practical use. Approximate values of $A^1_{x\overline{n}|}$, $P^1_{x\overline{n}|}$, $A_{x\overline{n}|}$, and $P_{x\overline{n}|}$ can, however, be derived from those of temporary annuities, remembering that—

$$A^1_{x\overline{n}|} = v(1 + a_{x\overline{n-1}|}) - a_{x\overline{n}|},$$

$$P^1_{x\overline{n}|} = v - \frac{a_{x\overline{n}|}}{1 + a_{x\overline{n-1}|}},$$

$$A_{x\overline{n}|} = 1 - d(1 + a_{x\overline{n-1}|}),$$

$$P_{x\overline{n}|} = \frac{1}{1 + a_{x\overline{n-1}|}} - d,$$

the last two being conveniently applied by means of a conversion table. It is, of course, to be borne in mind that v must be taken, or the conversion table entered, at the actual rate, and not at a rate-certain used in calculating the temporary annuity-values.

Example 5.—Having given the table of $O^{[NM]}$ 3 % values of $a_{[x]\overline{n}|}$, to deduce approximately, with the aid of interest tables, the $O^{[NM]}$ $3\frac{1}{2}$ % and $3\frac{3}{4}$ % values of $A^{1}_{[40]:\overline{5}|}$.

$$a_{[40]:\overline{4}|} \text{ at } 3\% = 3{\cdot}642$$
$$= a_{\overline{4}|} \text{ at } 3\tfrac{7}{8}\% \text{ plus } \tfrac{1}{11} \text{ of the diff. for } \tfrac{1}{8}\%$$
$$a_{[40]:\overline{4}|} \text{ at } 3\tfrac{1}{2}\% = a_{\overline{4}|} \text{ at } 4\tfrac{3}{8}\% \text{ plus } \tfrac{1}{11} \text{ diff. for } \tfrac{1}{8}\%$$
$$= 3{\cdot}599.$$
$$a_{[40]:\overline{4}|} \text{ at } 3\tfrac{3}{4}\% = a_{\overline{4}|} \text{ at } 4\tfrac{5}{8}\% \text{ plus } \tfrac{1}{11} \text{ diff. for } \tfrac{1}{8}\%$$
$$= 3{\cdot}578$$
$$a_{[40]:\overline{5}|} \text{ at } 3\% = 4{\cdot}463$$
$$= a_{\overline{5}|} \text{ at } 3\tfrac{7}{8}\% \text{ less } \tfrac{1}{4} \text{ diff. for } \tfrac{1}{8}\%$$
$$a_{[40]:\overline{5}|} \text{ at } 3\tfrac{1}{2}\% = a_{\overline{5}|} \text{ at } 4\tfrac{3}{8}\% \text{ less } \tfrac{1}{4} \text{ diff. for } \tfrac{1}{8}\%$$
$$= 4{\cdot}401$$
$$a_{[40]:\overline{5}|} \text{ at } 3\tfrac{3}{4}\% = a_{\overline{5}|} \text{ at } 4\tfrac{5}{8}\% \text{ less } \tfrac{1}{4} \text{ diff. for } \tfrac{1}{8}\%$$
$$= 4{\cdot}371$$
$$A^{1}_{[40]:\overline{5}|} = v(1 + a_{[40]:\overline{4}|}) - a_{[40]:\overline{5}|}$$
$$\text{at } 3\tfrac{1}{2}\% = {\cdot}9662 \times 4{\cdot}599 - 4{\cdot}401$$
$$= \underline{{\cdot}043}^{*}$$
$$\text{at } 3\tfrac{3}{4}\% = {\cdot}9639 \times 4{\cdot}578 - 4{\cdot}371$$
$$= \underline{{\cdot}042}$$

Example 6. Having given the table of $O^{[M]}$ 3% values of $a_{[x]\overline{n}|}$, to deduce approximately, with the aid of Interest and Conversion Tables, the $O^{[M]}$ $3\frac{1}{2}$% and $3\frac{3}{4}$% values of $P_{[30]:\overline{15}|}$.

$$a_{[30]:\overline{14}|} \text{ at } 3\% = 10{\cdot}788$$
$$= a_{\overline{14}|} \text{ at } 3\tfrac{5}{8}\% \text{ less } \tfrac{4}{9} \text{ diff. for } \tfrac{1}{8}\%.$$
$$a_{[30]:\overline{14}|} \text{ at } 3\tfrac{1}{2}\% = a_{\overline{14}|} \text{ at } 4\tfrac{1}{8}\% \text{ less } \tfrac{4}{9} \text{ diff. for } \tfrac{1}{8}\%$$
$$= 10{\cdot}438.$$
$$a_{[30]:\overline{14}|} \text{ at } 3\tfrac{3}{4}\% = a_{\overline{14}|} \text{ at } 4\tfrac{3}{8}\% \text{ less } \tfrac{4}{9} \text{ diff. for } \tfrac{1}{8}\%$$
$$= 10{\cdot}269.$$

Entering the Conversion Table, we find—

$$P_{[30]:\overline{15}|} \text{ at } 3\tfrac{1}{2}\% = \underline{5{\cdot}361}\%.^{\dagger}$$
$$P_{[30]:\overline{15}|} \text{ at } 3\tfrac{3}{4}\% = \underline{5{\cdot}260}\%.$$

* The true value, to 3 places, is ·043.
† The true value, to three places of decimals, is 5·363 %.

MORTALITY.

13. In §§ 3–8 the effects of forces of interest and mortality were considered simultaneously; it will now be convenient to drop, for the time being, the subject of interest and consider mortality alone.

14. The life table gives a representation of a stationary community supported by a fixed number of annual births. For the sake of completeness it will be as well to give a brief statement of the notation employed.

l_0 denotes the constant number of annual births.

l_x denotes the total number of persons who, in the course of any one year, attain the exact age x. It is to be observed that, whereas hitherto we have regarded the l_x persons as the survivors at time x of l_0 persons all born at some one moment of time, we are now going to regard the l_0 persons as born at intervals throughout a year, requiring therefore to extend our observations over a year in order to count up the l_x survivors who attain exact age x.

d_x denotes the total number of persons registered during any one year as dying aged between x and $x+1$.

L_x denotes the total number of persons aged between x and $x+1$ disclosed by a census supposed to be taken at any moment of time.

T_x denotes the total number of persons aged x and upwards disclosed by such a census.

15. It is generally supposed to be necessary to assume a uniform distribution of deaths (this assumption requires qualification,—a point with which I shall deal shortly), but we need not at present make this assumption. What, however, we must assume, for the purpose of obtaining workable results, is that the l_0 births occurring in any year are evenly distributed over that year. This means that the births are to be treated as occurring by equal infinitesimal instalments (of $l_0 dt$ each) every moment.

16. L_x consists entirely of persons aged $x+t$, where t is allowed to vary, by infinitely small increments, from 0 to 1. The infinitesimal number of persons aged exactly $x+t$ is $l_{x+t}dt$, the survivors, at the moment of the census, of the $l_0 dt$ births which occurred $x+t$ years before.

$$\therefore \qquad L_x = \int_0^1 l_{x+t}dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Similarly

$$T_x = \int_0^{\omega - x} l_{x+t}dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

17.
$$\frac{dL_x}{dx}=\frac{d}{dx}\int_0^1 l_{x+t}dt$$
$$=\int_0^1 \frac{d}{dx}l_{x+t}dt$$
$$=\int_0^1 \frac{d}{dt}l_{x+t}dt$$
$$=l_{x+1}-l_x$$
$$=-d_x$$

Now the mortality-function m_x, to which the name "central death-rate" has been given, is the ratio of d_x to L_x.

$$\therefore \qquad m_x=\frac{d_x}{L_x}=-\frac{1}{L_x}\cdot\frac{dL_x}{dx} \qquad (8)$$

18. I referred in § 15 to the fact that the assumption is generally made that deaths are uniformly distributed. Now the assumption that l_0dt births occur every moment necessitates the consequence that l_0dt deaths occur every moment, in order that T_0, the total population, may be kept stationary; *i.e.*, the assumption that the l_0 annual births are evenly distributed over every year of time necessitates the assumption that the total l_0 annual deaths are also evenly distributed over every year of time. Further,—the l_0dt births occurring every moment cause l_xdt and $l_{x+1}dt$ persons to attain exact ages x and $x+1$ respectively every moment, *i.e.*, the l_x and l_{x+1} attainments of respective ages x and $x+1$ in a year of time are uniformly distributed over that year of time, and therefore, in order that L_x, the population aged between x and $x+1$, may be kept stationary, the d_x deaths of persons aged between x and $x+1$ must be uniformly distributed over every year of time.

The uniform distribution of deaths over every year *of time* is therefore not an independent assumption but a *necessary consequence* of the assumption of uniform distribution of births. What, however, is not a necessary consequence is a uniform distribution of deaths over every year *of age*.

19. The assumption just referred to consists in supposing that

$$\frac{1}{m}d_x=l_x-l_{x+\frac{1}{m}}=l_{x+\frac{1}{m}}-l_{x+\frac{2}{m}}=\ldots=l_{x+1-\frac{1}{m}}-l_{x+1},$$
$$\frac{1}{m}d_{x+\frac{1}{m}}=l_{x+\frac{1}{m}}-l_{x+\frac{2}{m}}=\ldots=l_{x+1-\frac{1}{m}}-l_{x+1}=l_{x+1}-l_{x+1+\frac{1}{m}},$$

and so on; *i.e.*, the assumption is that

$$d_x=d_{x+\frac{1}{m}}=d_{x+\frac{2}{m}}=\ldots=d_{x+1-\frac{1}{m}}=d_{x+1}=\ldots$$

and, in order that the uniform distribution may be perfect, these relations must hold in the limit when $\frac{1}{m}$ is made infinitely small, giving d_x constant for all values of x, and also

$$d_x = \underset{\frac{1}{m}=0}{\mathrm{Lt}} \frac{l_x - l_{x+\frac{1}{m}}}{\frac{1}{m}} = -\frac{dl_x}{dx} = \mu_x l_x$$

i.e., $q_x = \mu_x$.

To summarize—the assumption that deaths are uniformly distributed over every year of age involves—

(*a*) A law of mortality of De Moivre's form.

(*b*) That the "rate of mortality" at any age is equal to the "force of mortality" at that age.

(*c*) That the number of deaths in any time, t, is td_x.

20. Referring to § 16, we can see what form L_x, T_x and m_x take when the assumption just referred to is made. We get—

$$\begin{aligned} T_x - T_{x+n} &= \int_0^n l_{x+t}dt \\ &= \int_0^n (l_x - td_x)dt \\ &= nl_x - \frac{n^2}{2}d_x \\ &= n(l_x - \frac{n}{2}d_x) \\ &= nl_{x+\frac{n}{2}} \quad \ldots \ldots \ldots \quad (9) \end{aligned}$$

Putting $n=1$, we get

$$L_x = l_{x+\frac{1}{2}} \quad \ldots \ldots \ldots \quad (10)$$

Putting $n=\omega-x$, we get

$$T_x = (\omega - x)l_{x+\frac{1}{2}(\omega-x)} \quad \ldots \ldots \ldots \quad (11)$$

Again—

$$\begin{aligned} m_x &= -\frac{1}{L_x}\frac{dL_x}{dx} \\ &= -\frac{1}{l_{x+\frac{1}{2}}} \cdot \frac{dl_{x+\frac{1}{2}}}{dx} \\ &= \mu_{x+\frac{1}{2}} \quad \ldots \ldots \ldots \quad (12) \end{aligned}$$

21. I mentioned in § 15 that the assumption generally made with regard to distribution of deaths requires qualification. Stated fully, it is this—

> The deaths in the year of age x to $x+1$ are assumed to be uniformly distributed over that year of age, for all integral values of x.

The consequences of this assumption are that—

$$\left.\begin{array}{r} l_x - l_{x+t} = td_x,\ t \text{ being } < 1 \\ T_x - T_{x+n} = nl_{x+\frac{n}{2}},\ n \text{ being } < 1 \\ L_x = l_{x+\frac{1}{2}} \\ m_x = \mu_{x+\frac{1}{2}} \\ q_x = \mu_x\text{*} \end{array}\right\} x \text{ being an integer.}$$

22. It is worthy of notice that many of the properties of functions of the Life Table hold good irrespective of any assumption as to distribution of deaths over years of age.

$$_{|n}\mathring{e}_x = \frac{\int_0^n l_{x+t}dt}{l_x} = \frac{T_x - T_{x+n}}{l_x}$$

The central death-rate of the population aged x and upwards is

$$\frac{d_x + d_{x+1} + d_{x+2} + \ldots}{T_x} = \frac{l_x}{\int_0^{\omega-x} l_{x+t}dt} = \frac{1}{\mathring{e}_x}$$

The total lifetime lived after age x by the $l_x - l_{x+n}$ persons who die aged between x and $x+n$ is

$$\begin{aligned} & \int_0^n tl_{x+t}\mu_{x+t}dt \\ & = \int_0^n t\left(-\frac{dl_{x+t}}{dt}\right)dt \\ & = \left[-tl_{x+t} + \int l_{x+t}dt\right]_0^n \\ & = \int_0^n l_{x+t}dt - nl_{x+n} \\ & = T_x - T_{x+n} - nl_{x+n}, \end{aligned}$$

giving $\quad x + \dfrac{T_x - T_{x+n} - nl_{x+n}}{l_x - l_{x+n}}$ as their average age at death.

* For μ_x should be substituted $\mathop{\mathrm{Lt}}\limits_{h=0} \mu_{x+h}$ (*see* Addendum, p. 347).

Whence, putting $x=0$, $n=\omega$, the average age at death of the l_0 persons who die every year is $\mathring{e}_0$.

The T_x members of the population aged x and upwards will, between them, have a total future lifetime of $\int_0^{\omega-x} T_{x+t}dt (=Y_x)$.

The sum of the ages above x of the T_x members of the population aged x and upwards is $\int_0^{\omega-x} t l_{x+t}dt$.

Now
$$\begin{aligned}\frac{dT_{x+n}}{dn} &= \frac{d}{dn}\int_0^{\omega-x-n} l_{x+n+t}dt\\ &= \int_0^{\omega-x-n} \frac{d}{dn} l_{x+n+t}dt\\ &= \int_0^{\omega-x-n} \frac{d}{dt} l_{x+n+t}dt\\ &= -l_{x+n}\end{aligned}$$

Thus
$$\begin{aligned}\int_0^{\omega-x} t l_{x+t}dt &= \int_0^{\omega-x} t\left(-\frac{dT_{x+t}}{dt}\right)dt\\ &= \Big[-tT_{x+t} + \int T_{x+t}dt\Big]_0^{\omega-x}\\ &= \int_0^{\omega-x} T_{x+t}dt\\ &= Y_x\end{aligned}$$

Thus the total future lifetime of the persons constituting the population aged x and upwards is equal to their total past lifetime since age x; or putting $x=0$, the total future lifetime of the members of the whole population is equal to their total past lifetime, and the average age at which the T_0 persons constituting the whole population will die is $\frac{2Y_0}{T_0}$.

To summarize the facts with regard to the whole population:

(*a*) It consists of T_0 members.

(*b*) The average age of the members is $\frac{Y_0}{T_0}$.

(*c*) The average age at which the members will die is $\frac{2Y_0}{T_0}$.

(*d*) It is replenished annually by l_0 births.

(*e*) It is depleted annually by the deaths of l_0 persons of average age $\mathring{e}_0$.

(*f*) These properties depend on the assumption of an even distribution of births, but on no independent assumption as to distribution of deaths.

23. We saw in § 5 that—

$$\int_0^n \mu_{x+t}dt = \log_e l_x - \log_e l_{x+n}.$$

If $n=1$, this becomes—

$$\int_0^1 \mu_{x+t}dt = \log_e l_x - \log_e l_{x+1} = \text{colog}_e . p_x .$$

Let us now consider the utility of the three relations

$$L_x = \int_0^1 l_{x+t}dt$$

$$d_x = \int_0^1 l_{x+t}\mu_{x+t}dt$$

$$\text{colog}_e . p_x = \int_0^1 \mu_{x+t}dt.$$

The first shows that, if we have plotted out the curve for which L_x is represented by the area contained between the curve, the ordinates corresponding to the respective abscissæ x and $x+1$, and the unit base which is the difference between the abscissæ, then the equation of the curve is $y = l_x$.

The second shows that, if we have plotted out the curve for which d_x is represented by a similar area, then the equation of the curve is $y = l_x \mu_x$.*

The third shows that, if we have plotted out the curve for which $\text{colog}_e . p_x$ is represented by a similar area, then the equation of the curve is $y = \mu_x$.

24. In the construction of a mortality-table by Milne's method from statistics of the general population, two curves are obtained, namely :—

(1) The population-curve, whose successive area-sections give the denominators for the successive values of m_x.

(2) The death-curve, whose successive area-sections give the numerators for the successive values of m_x.

There are three notable instances on record of the use of these curves.

* It will be remembered that Messrs. E. M. Moors and W. R. Day made use of this fact in the calculation of the values of μ_x for ages under 12 based on their New South Wales and Victoria Table (*J.I.A.*, vol. xxxvi, p. 175).

(i) In the construction of the Carlisle table, Milne calculated the values of m_x from the relation

$$\frac{\text{Area-section of (2)}}{\text{Area-section of (1)}}=\frac{d_x}{\text{L}_x}=m_x,$$

and used the values of m_x, so found, to pass, without further graduation, to the mortality-table by means of the formula

$$p_x=\frac{2-m_x}{2+m_x}.$$

(ii) In the construction of his Australian Table (*J.I.A.*, vol. xxiv), Mr. A. F. Burridge calculated the values of m_x from the relation

$$\frac{\text{Area-section of (2)}}{\text{Area-section of (1)}}=\frac{d_x}{\text{L}_x}=m_x.$$

The values of m_x, so obtained, he graduated by the graphic method, and he then passed to the mortality table by means of the formula

$$p_x=\frac{2-m_x}{2+m_x}.$$

(iii) In the construction of their New South Wales and Victoria Table (*J.I.A.*, vol. xxxvi), Messrs. E. M. Moors and W. R. Day calculated the values of $\log p_x$ from the relation

$$\log p_x=\log\overline{2\{\text{area-section of (1)}\}-\{\text{area-section of (2)}\}}$$
$$-\log\overline{2\{\text{area-section of (1)}\}+\{\text{area-section of (2)}\}},$$

passing thence to the values of d_x.

The values of d_x, so obtained, they graduated by Woolhouse's formula. They also calculated values of μ_x from the graduated values of d_x and l_x.

25. I have now to suggest a fourth variation, namely:—

Set up (1) the population curve and (2) the death-curve in the usual manner. Calculate the values of μ_x from the relation

$$\frac{\text{Ordinate of (2)}}{\text{Ordinate of (1)}}=\frac{\mu_x l_x}{l_x}=\mu_x.$$

Graduate by the graphic method the values of μ_x so obtained.

·06800
·06400
·02000
·01600
·01200
·00800
·00400
20
25
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65
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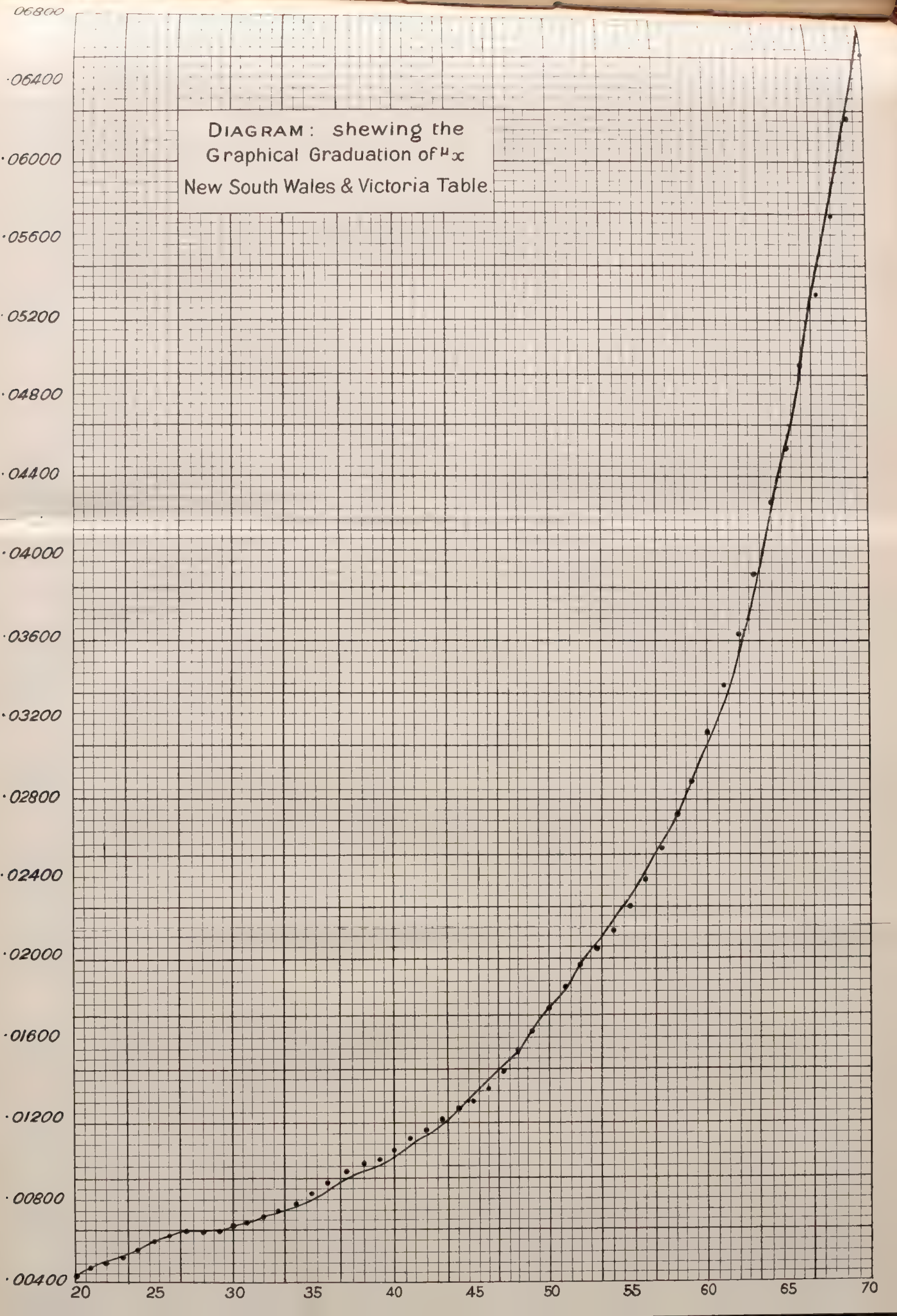
DIAGRAM: shewing the Graphical Graduation of μ_x New South Wales & Victoria Table.
06800
·06400
·06000
·05600
·05200
·04800
·04400
·04000
·03600
·03200
·02800
·02400
·02000
·01600
·01200
·00800
·00400
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70

Having thus obtained a smooth curve of $y = \mu_x$,—

> Read off the lengths of its ordinates, thus obtaining the graduated values of μ_x, and measure its unit area-sections by reading off the lengths of its central ordinates, thus obtaining the graduated values of $\text{colog}_e.p_x$, passing thence to those of $\log_{10}.p_x$ by means of the formula
>
> $$\text{Mantissa of } \log_{10}.p_x = 1 - \text{M}.\text{colog}_e.p_x,$$
>
> where M is the modulus of common logarithms ($= \cdot 43429\ldots$), and the formula can be rapidly applied by means of the arithmometer.

I venture to submit this method as being the most expeditious for passing from Milne's curves to the graduated mortality table including the values of μ_x.

26. In order to illustrate the proposed method, I have applied it to ages 20 to 70 of the male population and death curves of Messrs. Moors and Day's New South Wales and Victoria Table. The authors state (*J.I.A.*, vol. xxxvi, p. 163), that their parallelograms were set back each for half an age in order to facilitate the reading of the central ordinates. The ordinates required for our purpose are therefore those half an age back in each case. These lengths I have read off as accurately as possible by scrutinizing their diagrams with the aid of a lens. The results of these readings are given in Table A. The diagram shows the graphical graduation of μ_x. The ordinates of the smooth curve give the graduated values of μ_x, the comparison of which with the ungraduated values is shown in Table B. The central ordinates of the smooth curve give the graduated values of $\text{colog}_e.p_x$. These, as well as the resulting values of $\log_{10}.p_x$ and of p_x, are given in Table C. Finally, in Table D are shown the graduated values of μ_x and p_x side by side with those obtained by Messrs. Moors and Day. A study of Table D shows that by this paper the values of μ_x are on an average $\cdot 00011$ less, and the values of p_x are on an average $\cdot 00013$ greater than those of Messrs. Moors and Day, the maxima divergences being at age 61, where μ_x is $\cdot 00091$ less and p_x is $\cdot 00091$ greater, and at age 67, where μ_x is $\cdot 00061$ greater and p_x is $\cdot 00053$ less. Considering the difference between the methods employed, I think it can be admitted that the divergence in the results is not great.

Table A.—*Ordinates of Population and Death Curves, and Ungraduated Values of* μ_x.

Age x	Ordinate of Population Curve (1)	Ordinate of Death Curve (2)	Ratio of (2) to (1) Ungraduated μ_x	Age x
20	23,125	103	·00445	20
21	23,750	114	·00480	21
22	24,250	123	·00507	22
23	24,750	133	·00537	23
24	25,000	143	·00572	24
25	25,200	153	·00607	25
26	25,250	162	·00642	26
27	25,000	164	·00656	27
28	24,750	164	·00662	28
29	24,375	162	·00664	29
30	23,125	158	·00683	30
31	21,875	155	·00708	31
32	20,500	153	·00746	32
33	19,500	150	·00769	33
34	18,325	148	·00808	34
35	17,000	146	·00859	35
36	15,750	145	·00921	36
37	14,750	142	·00963	37
38	14,000	139	·00993	38
39	13,125	135	·01028	39
40	12,250	132	·01077	40
41	11,625	131	·01127	41
42	11,125	130	·01169	42
43	10,750	131	·01218	43
44	10,500	132	·01257	44
45	10,375	136	·01311	45
46	10,250	141	·01376	46
47	10,125	147	·01452	47
48	9,875	154	·01560	48
49	9,750	161	·01651	49
50	9,500	167	·01758	50
51	9,250	173	·01870	51
52	9,000	177	·01967	52
53	8,875	182	·02051	53
54	8,750	187	·02137	54
55	8,500	193	·02271	55
56	8,250	199	·02412	56
57	8,000	205	·02563	57
58	7,750	212	·02735	58
59	7,500	217	·02894	59
60	7,125	223	·03130	60
61	6,750	228	·03378	61
62	6,375	231	·03624	62
63	6,000	234	·03899	63
64	5,500	234	·04254	64
65	5,125	233	·04546	65
66	4,625	230	·04973	66
67	4,250	226	·05318	67
68	3,875	222	·05729	68
69	3,500	218	·06229	69
70	3,250	213	·06554	70

TABLE B.—*Comparison of the Graduated Values of μ_x with the Ungraduated Values.*

Age x	μ_x Ungraduated	μ_x Graduated	Variation per 10,000	Accumulated Variation per 10,000	Age x
20	·00445	·00450	+ 5	+ 5	20
21	·00480	·00480	0	+ 5	21
22	·00507	·00507	0	+ 5	22
23	·00537	·00537	0	+ 5	23
24	·00572	·00572	0	+ 5	24
25	·00607	·00607	0	+ 5	25
26	·00642	·00642	0	+ 5	26
27	·00656	·00656	0	+ 5	27
28	·00662	·00662	0	+ 5	28
29	·00664	·00664	0	+ 5	29
30	·00683	·00675	− 8	− 3	30
31	·00708	·00708	0	− 3	31
32	·00746	·00729	− 17	− 20	32
33	·00769	·00759	− 10	− 30	33
34	·00808	·00793	− 15	− 45	34
35	·00859	·00822	− 37	− 82	35
36	·00921	·00862	− 59	− 141	36
37	·00963	·00903	− 60	− 201	37
38	·00993	·00947	− 46	− 247	38
39	·01028	·00993	− 35	− 282	39
40	·01077	·01040	− 37	− 319	40
41	·01127	·01094	− 33	− 352	41
42	·01169	·01142	− 27	− 379	42
43	·01218	·01194	− 24	− 403	43
44	·01257	·01257	0	− 403	44
45	·01311	·01317	+ 6	− 397	45
46	·01376	·01387	+ 11	− 386	46
47	·01452	·01462	+ 10	− 376	47
48	·01560	·01554	− 6	− 382	48
49	·01651	·01651	0	− 382	49
50	·01758	·01741	− 17	− 399	50
51	·01870	·01848	− 22	− 421	51
52	·01967	·01969	+ 2	− 419	52
53	·02051	·02082	+ 31	− 388	53
54	·02137	·02202	+ 65	− 323	54
55	·02271	·02317	+ 46	− 277	55
56	·02412	·02445	+ 33	− 244	56
57	·02563	·02579	+ 16	− 228	57
58	·02735	·02735	0	− 228	58
59	·02894	·02894	0	− 228	59
60	·03130	·03072	− 58	− 286	60
61	·03378	·03282	− 96	− 382	61
62	·03624	·03532	− 92	− 474	62
63	·03899	·03832	− 67	− 541	63
64	·04254	·04182	− 72	− 613	64
65	·04546	·04573	+ 27	− 586	65
66	·04973	·05002	+ 29	− 557	66
67	·05318	·05437	+ 119	− 438	67
68	·05729	·05876	+ 147	− 291	68
69	·06229	·06317	+ 88	− 203	69
70	·06554	·06757	+ 203	0	70

TABLE C.—*Graduated Values of* $colog_e . p_x$, $log_{10} . p_x$ *and* p_x.

Age x	$\text{colog}_e . p_x$	$1 - \text{M} . \text{colog}_e . p_x = \log_{10} . p_x$	p_x	Age x
20	·00465	$\bar{1}$·99798	·99536	20
21	·00492	·99786	·99508	21
22	·00520	·99774	·99481	22
23	·00556	·99759	·99447	23
24	·00590	·99744	·99412	24
25	·00626	·99728	·99376	25
26	·00650	·99718	·99353	26
27	·00661	·99713	·99341	27
28	·00663	·99712	·99339	28
29	·00669	·99709	·99332	29
30	·00690	·99700	·99312	30
31	·00717	·99689	·99286	31
32	·00743	·99677	·99259	32
33	·00774	·99664	·99229	33
34	·00805	·99650	·99197	34
35	·00840	·99635	·99163	35
36	·00880	·99618	·99124	36
37	·00924	·99599	·99081	37
38	·00969	·99579	·99035	38
39	·01015	·99559	·98990	39
40	·01065	·99537	·98940	40
41	·01117	·99515	·98889	41
42	·01168	·99493	·98839	42
43	·01224	·99468	·98783	43
44	·01285	·99442	·98723	44
45	·01350	·99414	·98660	45
46	·01423	·99382	·98587	46
47	·01505	·99346	·98505	47
48	·01600	·99305	·98412	48
49	·01696	·99263	·98317	49
50	·01793	·99221	·98222	50
51	·01906	·99172	·98112	51
52	·02014	·99125	·98005	52
53	·02140	·99071	·97884	53
54	·02258	·99019	·97766	54
55	·02380	·98966	·97647	55
56	·02500	·98914	·97530	56
57	·02653	·98848	·97382	57
58	·02812	·98779	·97228	58
59	·02981	·98705	·97062	59
60	·03173	·98622	·96877	60
61	·03402	·98523	·96656	61
62	·03676	·98404	·96392	62
63	·03990	·98267	·96088	63
64	·04369	·98103	·95726	64
65	·04775	·97926	·95337	65
66	·05211	·97737	·94923	66
67	·05650	·97546	·94506	67
68	·06090	·97355	·94091	68
69	·06530	·97164	·93679	69
70	...	...	...	70

TABLE D.—*Comparison of the Graduated Values of μ_x and p_x by this Paper with those of Messrs. Moors and Day.*

Age x	μ_x		p_x		Age x
	This Paper	Moors and Day	This Paper	Moors and Day	
20	·00450	·00450	·99536	·99534	20
21	·00480	·00482	·99508	·99501	21
22	·00507	·00516	·99481	·99469	22
23	·00537	·00549	·99447	·99437	23
24	·00572	·00580	·99412	·99407	24
25	·00607	·00608	·99376	·99382	25
26	·00642	·00632	·99353	·99361	26
27	·00656	·00650	·99341	·99345	27
28	·00662	·00665	·99339	·99331	28
29	·00664	·00678	·99332	·99317	29
30	·00675	·00694	·99312	·99299	30
31	·00708	·00714	·99286	·99276	31
32	·00729	·00741	·99259	·99246	32
33	·00759	·00773	·99229	·99211	33
34	·00793	·00812	·99197	·99170	34
35	·00822	·00855	·99163	·99127	35
36	·00862	·00901	·99124	·99079	36
37	·00903	·00948	·99081	·99034	37
38	·00947	·00995	·99035	·98988	38
39	·00993	·01039	·98990	·98945	39
40	·01040	·01080	·98940	·98906	40
41	·01094	·01120	·98889	·98867	41
42	·01142	·01159	·98839	·98827	42
43	·01194	·01203	·98783	·98779	43
44	·01257	·01255	·98723	·98723	44
45	·01317	·01318	·98660	·98654	45
46	·01387	·01393	·98587	·98576	46
47	·01462	·01478	·98505	·98489	47
48	·01554	·01568	·98412	·98399	48
49	·01651	·01661	·98317	·98306	49
50	·01741	·01756	·98222	·98213	50
51	·01848	·01851	·98112	·98119	51
52	·01969	·01949	·98005	·98019	52
53	·02082	·02050	·97884	·97911	53
54	·02202	·02169	·97766	·97795	54
55	·02317	·02294	·97647	·97665	55
56	·02445	·02434	·97530	·97521	56
57	·02579	·02589	·97382	·97364	57
58	·02735	·02750	·97228	·97194	58
59	·02894	·02940	·97062	·97005	59
60	·03072	·03146	·96877	·96795	60
61	·03282	·03373	·96656	·96565	61
62	·03532	·03622	·96392	·96310	62
63	·03832	·03904	·96088	·96022	63
64	·04182	·04224	·95726	·95696	64
65	·04573	·04576	·95337	·95350	65
66	·05002	·04958	·94923	·94966	66
67	·05437	·05376	·94506	·94559	67
68	·05876	·05818	·94091	·94134	68
69	·06317	·06271	·93679	·93708	69
70	·06757	·06729	...	...	70

INTEREST.

27. Having for a short time considered the subject of mortality alone, I now propose to devote attention for a brief space to interest apart from mortality.

28. Let us consider a fund which varies by reason of—

(i) Income from interest;

(ii) Income from other sources, which consists of the algebraical excess of trade-income over trade-outgo, and may for brevity be referred to as income from trading.

Let δ_t represent the force of interest-income operating at time t.

,, γ_t represent the force of trading-income operating at time t.

,, f_x represent the total amount of the fund at the end of time x.

,, h_x represent the total interest-income for the year succeeding time x.

,, k_x represent the total trading-income for the year succeeding time x.

29. By reasoning similar to what has been used before, we get the following identities—

$$h_x=\int_0^1 f_{x+t}\delta_{x+t}dt=f_x\cdot\int_0^1 e^{\int_0^t(\delta_{x+t}+\gamma_{x+t})dt}\cdot\delta_{x+t}dt$$

$$k_x=\int_0^1 f_{x+t}\gamma_{x+t}dt=f_x\cdot\int_0^1 e^{\int_0^t(\delta_{x+t}+\gamma_{x+t})dt}\cdot\gamma_{x+t}dt$$

$$h_x+k_x=f_x\cdot\int_0^1 e^{\int_0^t(\delta_{x+t}+\gamma_{x+t})dt}\cdot(\delta_{x+t}+\gamma_{x+t})dt$$

$$=f_x\cdot\left\{e^{\int_0^1(\delta_{x+t}+\gamma_{x+t})dt}-1\right\}$$

$$=f_{x+1}-f_x.$$

This shows, as it should, that the amount of the fund at the end of the year exceeds its amount at the beginning of the year by the sum of the interest and trading incomes for the year.

30. The mean amount of the fund in the $(x+1)$th year is the arithmetic mean of all the amounts which it assumes during the year. Representing this mean amount by F_x, we have

$$F_x=\int_0^1 f_{x+t}dt=f_x\cdot\int_0^1 e^{\int_0^t(\delta_{x+t}+\gamma_{x+t})dt}dt,$$

and the mean force of interest-income for the year is

$$\frac{\int_0^1 f_{x+t}\delta_{x+t}dt}{\mathrm{F}_x}=\frac{h_x}{\mathrm{F}_x},$$

and the mean force of trading-income for the year is

$$\frac{\int_0^1 f_{x+t}\gamma_{x+t}dt}{\mathrm{F}_x}=\frac{k_x}{\mathrm{F}_x}.$$

If, however, we assume the year's *income from all sources* to be evenly distributed over the year, we have—

$$\begin{aligned}\mathrm{F}_x&=\int_0^1 \{f_x+t(h_x+k_x)\}dt\\&=\int_0^1 \{f_x+t(f_{x+1}-f_x)\}dt\\&=f_x+\tfrac{1}{2}(f_{x+1}-f_x)\\&=\tfrac{1}{2}(f_x+f_{x+1}),\end{aligned}$$

and the mean force of interest-income for the year becomes

$$\frac{2h_x}{f_x+f_{x+1}},$$

which agrees, *mutatis mutandis*, with Mr. G. F. Hardy's

$$\frac{2\,\mathrm{I}}{\mathrm{A}+\mathrm{B}},$$

and the mean force of trading-income for the year becomes

$$\frac{2k_x}{f_x+f_{x+1}},$$

which, in Mr. G. F. Hardy's notation, is

$$\frac{2(\mathrm{B}-\mathrm{A}-\mathrm{I})}{\mathrm{A}+\mathrm{B}}.$$

The mean amount of the fund, irrespective of interest-income, in the $(x+1)$th year is the arithmetic mean of all the amounts which it assumes, by operation of trading-income, during the year. Representing this mean amount by $\mathrm{F}_x^{(\gamma)}$, we have—

$$\text{Mean effective rate of interest-income for the year}=\frac{h_x}{\mathrm{F}_x^{(\gamma)}}.$$

If now we assume the year's *trading-income* to be evenly distributed over the year, we have—

$$\begin{aligned} F_x^{(\gamma)} &= \int_0^1 (f_x + tk_x)dt \\ &= \int_0^1 \{f_x + t(f_{x+1} - f_x - h_x)\}dt \\ &= f_x + \tfrac{1}{2}(f_{x+1} - f_x - h_x) \\ &= \tfrac{1}{2}(f_x + f_{x+1} - h_x), \end{aligned}$$

and the mean effective rate of interest-income for the year becomes—

$$\frac{2h_x}{f_x + f_{x+1} - h_x},$$

which agrees, *mutatis mutandis*, with Mr. G. F. Hardy's

$$\frac{2I}{A+B-I}.$$

Similarly, the mean effective rate of trading-income for the year is $\frac{k_x}{F_x^{(\delta)}}$, which, if we assume the year's *interest-income* to be uniformly distributed over the year becomes

$$\frac{2k_x}{f_x + f_{x+1} - k_x},$$

which, in Mr. G. F. Hardy's notation, is

$$\frac{2(B-A-I)}{2A+I}.$$

32. For the sake of completeness we may take the rates of income-from-all-sources, and it is evident that by similar reasoning we get—

$$\text{Mean force of income} = \frac{h_x + k_x}{F_x},$$

which, assuming the income to be evenly distributed, becomes—

$$\frac{2(h_x + k_x)}{f_x + f_{x+1}} \text{ or } \frac{2(B-A)}{A+B};$$

whilst the effective rate of income is—

$$\frac{h_x + k_x}{f_x} \text{ or } \frac{B-A}{A}.$$

Interest, Mortality, and a Third Force.

33. In the early part of this paper the forces of interest and mortality were considered in conjunction. There is, theoretically, no reason why the number of forces should be limited to two. The reasoning will be similar if we introduce a third,—the force of secession. In § 7 we saw that the present value of a unit receivable at the end of n years, if a person aged x lived so long, could be represented by the expression

$$e^{-\int_0^n (\delta_{x+t}+\mu_{x+t})dt}.$$

If, now, we allow a force of secession, σ, to operate, the present value of a unit receivable n years hence, if a person now aged x be then alive and still a member of a certain community, will be given by—

$$_n\mathrm{E}'_x = e^{-\int_0^n (\delta_{x+t}+\mu_{x+t}+\sigma_{x+t})dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

34. In practice δ_{x+t} is constant, μ_{x+t} increases with t, and σ_{x+t} decreases as t increases. If, therefore, we assume the decrease in σ to counteract the increase in μ, we shall have $\delta_{x+t}+\mu_{x+t}+\sigma_{x+t}$ constant. Representing this constant sum by δ, we get—

$$_n\mathrm{E}'_x = e^{-\int_0^n \delta dt} = e^{-n\delta} = v^n \quad . \quad . \quad . \quad . \quad . \quad (14)$$

and
$$a'_{x\overline{n}|} = a_{\overline{n}|} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

the rate-certain at which v and $a_{\overline{n}|}$ are calculated being the effective rate of interest equivalent to the force of interest δ.

35. In view of the fact that the introduction of the rate of secession as a factor into calculations is generally regarded as a dangerous practice, I do not think it worth while to pursue further this suggestion for dealing with secessions; I submit it principally for its theoretical interest. The rationale is, however, not restricted to the case where the third force considered is that of secession, and a brief consideration of another case may therefore not be out of place.

36. Let $(m\mu)_x$ represent the force of marriage at age x, then the present value of a unit receivable n years hence, if a person now aged x and single be alive n years hence without having married in the meantime, will be given by—

$$_n\mathrm{E}''_x = e^{-\int_0^n \{\delta_{x+t}+\mu_{x+t}+(m\mu)_{x+t}\}\, dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and the present value of a unit receivable on the death or marriage, within n years, of a person now aged x and single will be given by—

$$\bar{A}''^{1}_{x\overline{n}|} = \int_0^n {}_t\mathrm{E}''_x\{\mu_{x+t} + (m\mu)_{x+t}\}dt \quad . \quad . \quad . \quad . \quad (17)$$

37. In practice δ_{x+t} is constant, μ_{x+t} increases with t, and $(m\mu)_{x+t}$ (for the ages usually occurring in problems of this kind) decreases as t increases. If, therefore, we assume the decrease in $(m\mu)$ to counteract the increase in μ, we shall have $\mu_{x+t} + (m\mu)_{x+t}$ constant. Representing the constant sum $\mu_{x+t} + (m\mu)_{x+t}$ by β, and the constant sum $\delta_{x+t} + \mu_{x+t} + (m\mu)_{x+t}$ by δ, we get—

$${}_n\mathrm{E}''_x = e^{-\int_0^n \delta dt} = e^{-n\delta} = v^n \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$a''_{x\overline{n}|} = a_{\overline{n}|} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

$$\bar{A}''^{1}_{x\overline{n}} = \int_0^n v^t\beta dt = \beta . \bar{a}_{\overline{n}|} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

the rate-certain at which v, $a_{\overline{n}}$, and $\bar{a}_{\overline{n}}$ are calculated being the effective rate of interest equivalent to the force of interest δ.

38. I have remarked on the danger of introducing rates of secession into calculations, and must therefore not overlook the fact that there is a similar danger in introducing rates of marriage. The risk of death differs from that of secession or marriage in that risks of the latter class depend on the exercise of the human will, whilst the former does not. Rates of secession and marriage based on experience cannot consequently be regarded as very reliable for bases of calculation; the rates actually experienced in connection with transactions where allowance is made for these influences will probably not be the same as those which obtain generally. Allowances made for secessions or marriages must of necessity, therefore, usually be somewhat arbitrary.

39. The subject of marriage and mortality tables has been dealt with (among others) by Dr. Sprague and by Mr. Chatham (*J.I.A.*, vol. xxviii), but the function $\bar{A}''^{1}_{x\overline{n}|}$, mentioned above, concerns a problem somewhat different from those discussed by either of these writers. It sometimes occurs that a person is entitled to the absolute reversion to a fund expectant on the decease or re-marriage of a widow. In such a case it is, of course, on the safe side to neglect the possibility of the falling in of the reversion by reason of re-marriage of the

life tenant when valuing the reversion for the purpose of purchase or loan. The probability of re-marriage is often very small. If, for instance, the life tenant is of even fairly advanced age, this, coupled with the fact that re-marriage involves the forfeiture of her life interest, is sufficient to affect considerably the likelihood of her marrying again. A case may, however, arise where the life tenant is comparatively young and the absolute reversion has to be valued, say, for the purpose of estate duty. In such a case an arbitrary allowance might be made for the marriage risk by using the formula

$$\bar{A}''_x = \bar{A}_x + \bar{A}''^{\,1}_{x\overline{n}|} - \bar{A}^{\,1}_{x\overline{n}|}$$

where n is the term at the expiration of which the probability of re-marriage may be regarded as becoming negligible.

40. By way of illustration let us consider a case where the life tenant is a widow aged 35, and the reversioner is entitled to the absolute reversion expectant on her decease or re-marriage. Valuing by the $O^{[af]}$ Table at 4 per-cent—

$$a_{[35]} = 16\cdot870,\text{ whence } A_{[35]} = \cdot3127,$$

and
$$\bar{A}_{[35]} = 1\cdot02 \times A_{[35]},\text{ approximately,}$$
$$= \cdot3189$$

If no allowance were made for the marriage-risk, the value of the reversion would, on this basis, be ·3189 per unit.

Proceeding now to make the suggested adjustment, we will assume that the probability of re-marriage becomes negligible at age 50, and we have—

$$a_{50} = 13\cdot574,\quad A_{50} = \cdot4395$$

$$_{15|}A_{[35]} = \frac{v^{15}\,.\,l_{50}\,.\,A_{50}}{l_{[35]}} = \frac{\cdot5553 \times 7617 \times \cdot4395}{8938} = \cdot2080$$

$$A^{\,1}_{[35]:\overline{15}|} = A_{[35]} - {}_{15|}A_{[35]} = \cdot3127 - \cdot2080 = \cdot1047$$

$$\bar{A}^{\,1}_{[35]:\overline{15}|} = 1\cdot02 \times A^{\,1}_{[35]:\overline{15}|},\text{ approximately, } = \cdot1068.$$

The force of interest equivalent to effective rate 4 % $= \cdot039$

Assume that $\mu_{[35]+t} + (m\mu)_{[35]+t}$ is constant for the 15 years, and . . . . $= \cdot015$

$\cdot054$

The rate-certain is the effective rate equivalent to the force of interest 5·4 %, *i.e.*, $5\frac{1}{2}$ %.

$$\bar{a}_{\overline{15}|} = a_{\overline{15}|} \cdot \frac{i}{\delta} = 10{\cdot}038 \times \frac{{\cdot}055}{{\cdot}054} = 10{\cdot}224$$

$$\bar{\mathrm{A}}''^{\,1}_{[35]:\overline{15}|} = {\cdot}015 \times 10{\cdot}224 \qquad = \quad {\cdot}1534$$

$$\bar{\mathrm{A}}^{\,1}_{[35]:\overline{15}|} \qquad = \quad {\cdot}1068$$

Adjustment for marriage-risk $= \quad {\cdot}0466$

$$\bar{\mathrm{A}}_{[35]} \qquad = \quad {\cdot}3189$$

$$\bar{\mathrm{A}}''_{[35]} \qquad = \quad {\cdot}3655$$

The adjusted value of the reversion is thus ·3655 per unit, or ·0466 per unit more than its value without the adjustment. The assumption made is that $(m\mu)_{[35]+t}$ is approximately ·012 for $t=0$, diminishes as t increases from 0 to 15, and then vanishes. This assumption is, of course, an arbitrary one, but any assumption with regard to the rate of marriage in such a case must, as I have pointed out, be of an arbitrary nature.

USE OF THE FUNCTION $r_x^{(m)}$.

41. In § 4 reference was made to the nominal annual rate of mortality convertible m times a year, and the symbol $r_x^{(m)}$ was used for it. This function is of theoretical rather than practical use, but, having introduced it into the argument, I think it desirable to give some examples of its applicability, and I have therefore concluded this paper by showing how it may be employed in obtaining formulæ for the relations between $\mathrm{A}_x^{(m)}$ and $a_x^{(m)}$.

42. We saw that—

$$r_x^{(m)} = m \cdot \frac{\int_0^{\frac{1}{m}} l_{x+t}\mu_{x+t}dt}{l_x},$$

and it follows at once by actual integration or from general reasoning that—

$$r_x^{(m)} = m \cdot \frac{l_x - l_{x+\frac{1}{m}}}{l_x}$$

Under the assurance, whose value is represented by $\mathrm{A}_x^{(m)}$, a unit

will be paid $\frac{1}{m}$ of a year after time t (where t increases by intervals of $\frac{1}{m}$ each from 0 to ∞), provided (x) live time t and die in the succeeding interval.

Thus
$$\mathrm{A}_x^{(m)} = \sum_{t=0}^{t=\infty}{}^{(m)} v^{t+\frac{1}{m}} \cdot {}_tp_x \cdot r_{x+t}^{(m)}.$$

Under the annuity-due, whose value is represented by $\mathbf{a}_x^{(m)}$, a payment of $\frac{1}{m}$ will be made at time t (where t increases by intervals of $\frac{1}{m}$ each from 0 to ∞), provided (x) be alive then.

Thus
$$\mathbf{a}_x^{(m)} = \sum_{t=0}^{t=\infty}{}^{(m)} v^t {}_tp_x.$$

43. Let Δ_x be used as the symbol to denote differencing with regard to x over the interval $\Delta x = \frac{1}{m}$; then we can proceed—

$$\frac{\Delta_x {}_tp_x}{\Delta x} = \frac{\Delta_x \dfrac{l_{x+t}}{l_x}}{\Delta x}$$

$$= \frac{l_x \cdot \Delta_x l_{x+t} - l_{x+t} \cdot \Delta_x l_x}{l_x \cdot l_{x+\frac{1}{m}} \cdot \Delta x}$$

$$= -\frac{l_{x+t}}{l_{x+\frac{1}{m}}} \cdot r_{x+t}^{(m)} + \frac{l_{x+t}}{l_{x+\frac{1}{m}}} \cdot r_x^{(m)}$$

i.e.,
$$\frac{\Delta_x {}_tp_x}{\Delta x} = \frac{{}_tp_x}{{}_{\frac{1}{m}}p_x} \cdot \left\{ r_x^{(m)} - r_{x+t}^{(m)} \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Now
$$\frac{\Delta_x \mathbf{a}_x^{(m)}}{\Delta x} = \frac{\Delta_x \sum_{t=0}^{t=\infty}{}^{(m)} v^t {}_tp_x}{\Delta x} = \frac{\sum_{t=0}^{t=\infty}{}^{(m)} v^t \cdot {}_tp_x \cdot \left\{ r_x^{(m)} - r_{x+t}^{(m)} \right\}}{{}_{\frac{1}{m}}p_x}$$

by equation (21).

$$i.e.,\ m\left\{ \mathbf{a}_{x+\frac{1}{m}}^{(m)} - \mathbf{a}_x^{(m)} \right\} = \frac{1}{{}_{\frac{1}{m}}p_x} \left\{ r_x^{(m)} \mathbf{a}_x^{(m)} - \frac{1}{v^{\frac{1}{m}}} \mathrm{A}_x^{(m)} \right\}$$

$$\therefore \quad \mathrm{A}_x^{(m)} = v^{\frac{1}{m}} \left[r_x^{(m)} \mathbf{a}_x^{(m)} + {}_{\frac{1}{m}}p_x \cdot m \left\{ \mathbf{a}_x^{(m)} - \mathbf{a}_{x+\frac{1}{m}}^{(m)} \right\} \right] . \quad (22)$$

Transforming equation (22), we get—

$$\mathrm{A}_x^{(m)}=mv^{\frac{1}{m}}\left[\frac{l_x-l_{x+\frac{1}{m}}}{l_x}\cdot\mathbf{a}_x^{(m)}+\frac{l_{x+\frac{1}{m}}}{l_x}\cdot\left\{\mathbf{a}_x^{(m)}-\mathbf{a}^{(m)}_{x+\frac{1}{m}}\right\}\right]$$

$$=mv^{\frac{1}{m}}\left[\mathbf{a}_x^{(m)}-{}_{\frac{1}{m}}p_x\cdot\mathbf{a}^{(m)}_{x+\frac{1}{m}}\right]$$

$$=m\left[v^{\frac{1}{m}}\mathbf{a}_x^{(m)}-a_x^{(m)}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

$$=m\left[\left\{1-\frac{f_{(m)}}{m}\right\}\cdot\mathbf{a}_x^{(m)}-\left\{\mathbf{a}_x^{(m)}-\frac{1}{m}\right\}\right]$$

$$=1-f_{(m)}\mathbf{a}_x^{(m)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

44. Formulæ (21)—(24) are, some of them, cumbrous-looking, but they are useful on account of their general nature. By taking special values of m we can see of what formulæ they represent the general form.

Putting $m=1$,

No. 21 becomes

$$\Delta_{x\,t}p_x=\frac{{}_tp_x}{p_x}\cdot\{q_x-q_{x+t}\},$$

No. 22 becomes

$$\mathrm{A}_x=v[q_x\mathbf{a}_x+p_x(\mathbf{a}_x-\mathbf{a}_{x+1})],$$

No. 23 becomes

$$\mathrm{A}_x=v\mathbf{a}_x-a_x,$$

No. 24 becomes

$$\mathrm{A}_x=1-d\mathbf{a}_x.$$

Putting $m=\infty$,

No. 21 becomes

$$\frac{d_tp_x}{dx}={}_tp_x(\mu_x-\mu_{x+t}),$$

No. 22 becomes

$$\bar{\mathrm{A}}_x=\mu_x\bar{a}_x-\frac{d\bar{a}_x}{dx},$$

No. 23 becomes indeterminate,

No. 24 becomes

$$\bar{\mathrm{A}}_x=1-\delta\bar{a}_x.$$

Conclusion.

45. In conclusion I can but repeat what I stated at the outset, namely, that I submit this paper, not on the ground that it contains much that is new, but rather in the hope that the mode of presentation of the subjects dealt with may prove of interest. At the same time I think that the paper may attain some useful purpose, if it succeeds in throwing a little more light on the real meaning and utility of the forces of interest and mortality,—functions which, at their first introduction to him, the average student regards as petty obstacles put into the *Text-Books* for his special annoyance, a state of mind which, in some cases, continues long after that first introduction.

APPENDIX.

To find the limiting value of Π, where this symbol is used to denote the product

$$\{1+h\phi(a)\}\{1+h\phi(a+h)\}\{1+h\phi(a+2h)\}\ \ldots\ \{1+h\phi(b-h)\},$$

when h is indefinitely diminished and, consequently, the number of factors is indefinitely increased, $\phi(t)$ being a continuous function of t between the limits a and b.

The number of factors is $\dfrac{b-a}{h}$, which may be denoted by n.

Let $\psi(t)$ be such a function of t that

$$\frac{d\log_e\psi(t)}{dt}=\phi(t).$$

Then we may write—

$$\phi(a)=\frac{\psi(a+h)-\psi(a)}{h.\psi(a)}+a_1,$$

$$\phi(a+h)=\frac{\psi(a+2h)-\psi(a+h)}{h.\psi(a+h)}+a_2,$$

$$\vdots \qquad\qquad \vdots$$

$$\phi(b-h)=\frac{\psi(b)-\psi(b-h)}{h.\psi(b-h)}+a_n,$$

where $a_1, a_2, \ldots a_n$ are small quantities, each of which vanishes in the limit when h is made infinitely small.

Π can now be put into the form—

$$\left\{\frac{\psi(a+h)}{\psi(a)}+ha_1\right\}\left\{\frac{\psi(a+2h)}{\psi(a+h)}+ha_2\right\}\ldots\left\{\frac{\psi(b)}{\psi(b-h)}+ha_n\right\}$$

$$=\frac{\psi(b)}{\psi(a)}\left[\left\{1+\frac{ha_1\psi(a)}{\psi(a+h)}\right\}\left\{1+\frac{ha_2\psi(a+h)}{\psi(a+2h)}\right\}\ldots\left\{1+\frac{ha_n\psi(b-h)}{\psi(b)}\right\}\right]$$

Now suppose

$$\frac{ha_r\psi(a+\overline{r-1}h)}{\psi(a+rh)}$$

to be the greatest, and

$$\frac{ha_s\psi(a+\overline{s-1}h)}{\psi(a+sh)}$$

to be the least of the n terms of the kind, then the value of the expression inside the square brackets is—

$$<\left\{1+\frac{ha_r\psi(a+\overline{r-1}h)}{\psi(a+rh)}\right\}^n \text{ and } >\left\{1+\frac{ha_s\psi(a+\overline{s-1}h)}{\psi(a+sh)}\right\}^n$$

i.e.,

$$<\left\{1+\frac{a_r(b-a)k}{n}\right\}^n \text{ and } >\left\{1+\frac{a_s(b-a)l}{n}\right\}^n$$

where k is, for brevity, put in place of $\dfrac{\psi(a+\overline{r-1}h)}{\psi(a+rh)}$

and l " " " $\dfrac{\psi(a+\overline{s-1}h)}{\psi(a+sh)}$.

Now, when n is indefinitely increased, the limiting value of

$$\left\{1+\frac{a_r(b-a)k}{n}\right\}^n \text{ is } e^{a_r(b-a)k},$$

and the limiting value of

$$\left\{1+\frac{a_s(b-a)l}{n}\right\}^n \text{ is } e^{a_s(b-a)l};$$

but a_r and a_s vanish in the limit, and $(b-a)k$ and $(b-a)l$ are each of them, *ex hypothesi*, finite; hence the limiting value of $e^{a_r(b-a)k}$ is unity, and that of $e^{a_s(b-a)l}$ is unity.

Consequently the limiting value of the expression inside the square brackets is unity, and the limiting value of Π is $\dfrac{\psi(b)}{\psi(a)}$.

But $$\frac{d\log_e\psi(t)}{dt}=\phi(t)$$

$\therefore$ $$\log_e\psi(t)+C=\int\phi(t)dt$$

$\therefore$ $$\log_e\psi(b)-\log_e\psi(a)=\int_a^b\phi(t)dt$$

$\therefore$ $$\frac{\psi(b)}{\psi(a)}=e^{\int_a^b\phi(t)dt}$$

$\therefore$ $$\underset{\substack{h=0\\n=\infty}}{\text{Lt}}\;\Pi=e^{\int_a^b\phi(t)dt}\quad . \quad . \quad . \quad . \quad (A)$$

Turning to § 6 of the paper, we find there two particular cases of this theorem, and we are able at once to write down from equation (A)—

$$f(x+n)=f(x)\,.\,e^{\int_0^n\delta_{x+t}dt},$$

$$l_{x+n}=l_x\,.\,e^{-\int_0^n\mu_{x+t}dt},$$

thus confirming equations (2).

The following corollary of the general theorem is interesting also: The arithmetic mean of the binomial factors which constitute Π is—

$$1+\frac{1}{n}\sum_{t=a}^{t=b-h}\left(\frac{1}{h}\right)\phi(t),$$

but $$e^{\int_a^b\phi(t)dt}=\underset{\substack{h=0\\n=\infty}}{\text{Lt}}\left[\left\{1+\frac{1}{n}\sum_{t=a}^{t=b-h}\left(\frac{1}{h}\right)\phi(t)\right\}^n\right],$$

so that the product of the n binomial factors is, in the limit, equal to the nth power of their arithmetic mean.

Abstract of the Discussion.

Mr. T. P. THOMPSON said that although the author did not himself claim the merits of a discoverer, the theory which he had presented had been relegated so much to the background that he might justly claim that distinction. The fact would not, he thought, be disputed that interest and mortality should, if one desired to be scientifically accurate, be regarded from the point of view presented in the paper before them, namely, that of the forces of interest and mortality; though, if reference were made to the *Text-books* (Part I and II), it would be found that they were not so regarded there. The force of interest was

introduced at the beginning of Part I, but very little more was done with it, until the final chapter of the book. As regarded the force of mortality, that was brought before the student in a few paragraphs in square brackets, at the end of a long chapter in Part II, the brackets denoting that the paragraphs might be omitted at a first reading. He did not think that any student would realize that those sections must be regarded as bringing before him the most important function of the whole book, for on the expression of μ_x in Makeham's modified form was based the graduation of the new tables. As far as interest was concerned, he did not think that any loss was sustained by treating it from the point of view of the yearly rate. The subjects dealt with in Part I, namely, annuities, the rate of interest earned by debentures, and kindred subjects, could be treated quite as well from that point of view as from the point of view of δ. There was one further point to be noticed, namely, that while μ_t was a function which varied with t, δ in practically all cases which they came across was constant: in other words, δ_t remained the same for all values of t. As far as the second part of the *Text-book* was concerned, there was, he thought, a distinct loss in treating the subject from the point of view of q_x, and not from that of μ_x. He should like to mention two or three reasons for holding that view; one the author had put very pithily in § 2 of his paper, when he said there could be no doubt that, apart from the mental satisfaction which it brought, a proper study of the true theory of a subject imparted to the mind that firm knowledge which was essential to confidence in its practical application. He submitted also that to treat it from the point of view of μ_x would make the study of the subject, which he was regarding from a student's standpoint, very much easier than it was at present. A great many of the more intricate problems dealt with in the *Text-book* could not be satisfactorily dealt with except from the point of view of μ_x and the integral calculus, while a great many of the proofs given in the *Text-book* were very much simpler and could be more easily understood if the integral and differential calculus were used, as suggested in the paper. In some cases such proofs were given, which meant to a certain extent a duplicating of the work. It seemed to him to be unnecessarily cumbersome and intricate to a student on his first reading, having studied the greater part of a chapter, to find further proofs of the same formulas at the end. In §§ 14–22 of the paper, the author had given an example, which would be found to be a re-writing of Chapter 5 of the *Text-book*, Part II. Personally, he thought the student would find that way of regarding it much easier than the one previously given. In the concluding sentences of the paper the author had stated that "the paper may attain some useful purpose, if it succeeds in throwing a little more light on the real meaning and utility of the forces of interest and mortality—functions which, at their first introduction to him, the average student regards as petty obstacles", and he wished to point out that a paper of such a character as that which

Mr. Allen had read could be of little utility to the student until he had obtained a thorough knowledge and grasp of the first principles of the integral and differential calculus. If he should not be considered guilty of contempt of Court in speaking of matters *sub judice*, he should like to express the opinion that it would be a good thing to bring those two subjects, the differential and the integral calculus, into Part I of the examinations. This amongst other things would, he thought, obviate a feature to which attention had been drawn on so many occasions, and on which more than one President had commented, namely, the number of failures in Part II of the Examinations.

He would now refer to two or three points of detail. First of all, as to Mr. Allen's proof respecting the assumptions underlying a uniform distribution of deaths. He had always thought that the uniform distribution of deaths meant a uniform distribution for integral ages only, in other words from x to $x+1$ where x was integral. The author had, however, apparently assumed that the uniform distribution held for any year of age whether x was integral or not. He thought that Mr. Allen ought certainly to be congratulated on the new method which he had pointed out of obtaining a graduation of a mortality table where the graphic method could be used. It appeared to him that the author would have obtained a better graduation had he somewhat increased the values of μ_x at the ages 20 to 30, where he had made no variation from the ungraduated data, and slightly reduced the values at ages 30 to 40. It would be noticed that the author only obtained the total deviation equal to zero by making the graduated μ_x very much too high for the last six ages. Throughout the whole table, at any rate after 30, there was a large accumulation of minus error. Possibly there might be reasons for that in the plotting out of the curve which did not appear on the surface. The author had stated that a study of Table D showed that his values of μ_x were on an average ·00013 less than those of Messrs. Moors and Day. If the suggested change were made, he thought that the variation would be very considerably reduced. In the Appendix the author had given the proof of the limiting value of Π where that symbol was used to denote the product of successive values of $1+h\phi$. He would suggest a much simpler and shorter proof, which he thought was sound, and which was suggested by the form of the limit. Starting by taking logarithms, one would get

$$\log \Pi = \Sigma \log [1 + h\phi(a+nh)]$$

Expanding the right-hand side by the formula

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$$

we get $$\log \Pi = \Sigma h\phi(a+nh)$$

$+$ terms involving h^2 and higher powers of h, which vanish when $h=0$.

$$\therefore \quad \underset{h=0}{\mathrm{Lt}} \log \Pi = \underset{h=0}{\mathrm{Lt}} \Sigma h\phi(a+nh)$$

$$= \int_a^b \phi(t)dt, \text{ by definition,}$$

$$\therefore \quad \underset{h=0}{\mathrm{Lt}} \Pi = e^{\int_a^b \phi(t)dt}$$

MR. C. W. KENCHINGTON thought Mr. Allen's paper served a very useful purpose in bringing together a vast amount of matter which had already been in the hands of a good many students and had been used by several tutors, and clearly showed the great benefit of looking at the subjects of interest and mortality from the point of view of the infinitesimal calculus. He would like to support Mr. Thompson in advocating the inclusion of the infinitesimal calculus in Part I of the Institute Examinations. At the present time the subject of finite differences was included in Part I of the syllabus, but such a standard work as Boole on Finite Differences was almost a closed book to the students, because it involved to a large extent a knowledge of the differential calculus. He thought that while the syllabus remained as it was there would always be a tendency for students to attempt to pass the examinations with a minimum of knowledge, and those students who succeeded in so doing found in after years the great disadvantage it had been to them in not extending their research during their days of studentship into those subjects, which were more or less of a theoretical nature. More especially would that be so when they took into account the fact that frequency-curves and correlation were now coming to the fore, and that in days to come actuaries would have to be better mathematicians than they were at the present day.

MR. D. C. FRASER said that he had found it a very useful exercise to read again Mr. King's latest paper (*J.I.A.*, vol. xli, p. 54) in the light of the ideas suggested by Mr. Allen, and he wished to refer specially to three of the functions dealt with in that paper, namely, colog p_x, m_x, and $\mu_{x+\frac{1}{2}}$. Mr. King had remarked that the two latter functions might be considered to be identical for the purposes of his paper, and had commented on the fact that, assuming Makeham's law to hold, the differential coefficients of colog p_x and of $\mu_{x+\frac{1}{2}}$ were the same in form. Examining these functions from Mr. Allen's point of view, it was apparent that the intimate relationship between these three functions was an essential consequence of their definitions. The following exact formulas were given by Mr. Allen:

$$\text{I.} \qquad \operatorname{colog}_e(p_x) = \int_0^1 \mu_{x+t}\,.\,dt$$

$$\text{II.} \qquad m_x = \frac{d_x}{\mathrm{L}_x}$$

where
$$d_x = \int_0^1 l_{x+t}\,.\,\mu_{x+t}\,.\,dt$$

and
$$\mathrm{L}_x = \int_0^1 l_{x+t}\,.\,dt.$$

It would be noticed that the integrals occurring in these formulas were all of the same form, and that they represented areas erected on a unit length of the axis of x. They were accustomed to use $l_{x+\frac{1}{2}}$ as a good approximation to the value of L_x; and if this were justifiable, it followed that similar approximations could be used for the other integral expressions, so that they were led at once to the following approximate relations:

From I. $\quad \text{colog}_e(p_x) = \mu_{x+\frac{1}{2}}$

From II. $\quad m_x = \dfrac{l_{x+\frac{1}{2}} \cdot \mu_{x+\frac{1}{2}}}{l_{x+\frac{1}{2}}} = \mu_{x+\frac{1}{2}}.$

And therefore, approximately,

$$m_x = \mu_{x+\frac{1}{2}} = \text{colog}_e(p_x).$$

These relations were not new; but the point he wished to emphasize was that the numerical values of the functions were so close that for purposes of numerical calculation they could be treated as interchangeable throughout the greater part of the mortality table. If $\text{colog}_e(p_x) = m_x$, then $\text{colog}_{10}(p_x) = \text{M}.m_x$, and taking as an illustration the "original" values of $\text{colog}_{10} p_x$ and m_x given in Tables 5 and 6 of Mr. King's paper, he found that the excess of $\text{colog}_{10}(p_x)$ over $\text{M}.m_x$ was as follows:

At age 20	·000001
30	–·000001
40	·000002
50	–·000001
60	·000002
70	·000010
80	·000119
90	·001630

These figures indicate that up to age 80 the approximation might be used with safety. After age 80, the approximation was not so good, but this was of less importance, from the fact that most mortality tables had to be treated in a special manner after that age. The relation $\text{M}.m_x = \text{colog}_{10}(p_x)$ was much more convenient for calculation than the familiar formula $p_x = \dfrac{2 - m_x}{2 + m_x}$; and when applied to statistics of population led more rapidly and directly to the columns $\log l_x$ and l_x. Mr. Allen had on p. 320 suggested a fourth variation to the processes previously in use for deriving a Mortality Table by Milne's method from statistics of the general population. A fifth variation might be suggested. Whatever process be adopted, the columns d_x and L_x must necessarily be formed for all integral values of x, in order to obtain a check upon the drawing of the curves. Then the following operations would give the Mortality Table with the least possible labour:

$$m_x = \frac{d_x}{L_x}$$

$$\text{colog}_{10}(p_x) = \text{M} \times m_x$$

$$\log_{10}(p_x) = 1 - \text{M} \times m_x.$$

The necessary graduation could be performed either on the rough values of m_x or on those of $\text{colog}_{10}(p_x)$.

[Mr. Fraser's attention having been directed to the fact that the changes in sign, shown in the above figures representing the values of $\text{colog}_{10}(p_x) - \text{M}m_x$, are abnormal, he has asked us to append the following supplementary remarks and comparative figures.—ED. *J.I.A.*]

[The values of $\text{colog}_{10}(p_x) - \text{M}.m_x$ given above were derived from the values of "original $\text{colog}\, p_x$" and "original m_x" as stated by Mr. King for the $\text{O}^{\text{M}(5)}$ Table (see *J.I.A.*, vol. xli, pp. 78–9). It is not clear what formula was employed by Mr. King for the calculation of m_x. The values according to the usual approximation, $m_x = 2\dfrac{1-p}{1+p}$, are given below, along with the values of $\mu_{x+\frac{1}{2}}$, and the values of $\text{colog}_{10}(p_x) - \text{M}.m_x$ and $\text{colog}_{10}(p_x) - \text{M}.\mu_{x+\frac{1}{2}}$ are added.

x	m_x	$\mu_{x+\frac{1}{2}}$	$\text{colog}_{10}(p_x) - \text{M}.m_x$	$\text{colog}_{10}(p_x) - \text{M}.\mu_{x+\frac{1}{2}}$
20	·0065432	·0065430	·0000000	·0000001
30	·0074951	·0074946	·0000000	·0000002
40	·0098317	·0098304	·0000000	·0000006
50	·0155672	·0155643	·0000001	·0000014
60	·0296450	·0296393	·0000009	·0000034
70	·0641868	·0641892	·0000094	·0000084
80	·1487714	·1489992	·0001196	·0000207
90	·3535470	·3571830	·0016299	·0000509

I have used the values of $\text{colog}_{10}(p_x)$ given on p. 153 of "Principles and Methods."]

Mr. Thompson had, he thought, in his criticism of one point, not appreciated Mr. Allen's argument. In § 19 the assumption was made of a uniform distribution of deaths over *every year of age*. It was clear that Mr. Allen intended these words to be taken in the widest sense they would bear, and when they were used without qualification there was nothing to prevent a year of age being reckoned from the attainment of a fractional age. Mr. Allen had in fact used for purposes of illustration two years of age, one commencing at age x and the other at age $x + \dfrac{1}{m}$, and had shown that the assumption of the uniform distribution of deaths over *every year of age* involved a law of mortality of De Moivre's form, and among other consequences the identity of q_x and μ_x. The above assumption reduced the curve representing l_x to a straight line. In § 21 the assumption was then made that the deaths in the year of age x to $x+1$ were uniformly distributed over that year of age *for all integral values of* x, thus reducing the curve representing l_x to a polygon. He feared that in asserting that under these conditions $q_x = \mu_x$ Mr. Allen had employed fallacious mathematical reasoning.

Under the conditions indicated, l_x had finite second differences, and by a well-known approximate formula

$$\frac{dl_x}{dx} = \Delta l_x - \tfrac{1}{2}\Delta^2 l_x.$$

Therefore $$-\frac{1}{l_x}\cdot\frac{dl_x}{dx} = -\frac{\Delta l_x}{l_x} + \tfrac{1}{2}\frac{\Delta^2 l_x}{l_x}.$$

That is $$\mu_x = q_x + \tfrac{1}{2}\frac{\Delta^2 l_x}{l_x},$$

and μ_x consequently differed from q_x by a finite quantity. If $q_x = \mu_x$,

then $$\frac{dl_x}{dx} = \Delta l_x;$$

and this could not be true unless l_x were represented by a continuous straight line, or in other words, unless the Mortality Table followed De Moivre's Law. Presumably, the reasoning employed by Mr. Allen was as follows: Using the same argument as in § 19, and considering the first infinitesimal decrement of l_{x+t} which succeeded l_x, since the deaths d_x were evenly distributed over the year of age x to $x+1$,

$$l_x - l_{x+\frac{1}{m}} = \frac{1}{m}d_x.$$

Therefore, $$d_x = \frac{l_x - l_{x+\frac{1}{m}}}{\frac{1}{m}}.$$

When m was indefinitely increased the right side became $-\dfrac{dl_x}{dx}$.

Therefore, $$d_x = -\frac{dl_x}{dx},$$

and hence, $$q_x = \mu_x.$$

But the same argument might of course be applied to the last infinitesimal decrement of l_{x+t} preceding l_x. Since the deaths d_{x-1} were equally distributed over the year of age $x-1$ to x,

$$l_{x-\frac{1}{m}} - l_x = \frac{1}{m}d_{x-1}.$$

Therefore, $$d_{x-1} = \frac{l_{x-\frac{1}{m}} - l_x}{\frac{1}{m}}.$$

When m was indefinitely increased the right side became $-\dfrac{dl_x}{dx}$.

Therefore, $$d_{x-1} = -\frac{dl_x}{dx}.$$

But it was shown above that

$$d_x = -\frac{dl_x}{dx}.$$

Therefore, $$d_{x-1} = d_x.$$

That is to say, the table followed De Moivre's Law, which contradicted the assumption of § 21.

The fallacy lay in assuming that in the limit, when m was indefinitely increased,

$$\frac{l_x - l_{x+\frac{1}{m}}}{\frac{1}{m}} = -\frac{dl_x}{dx},$$

whether the curve which represented l_x was continuous at the point x or not. Under the conditions of § 21 the curve was not continuous at the point x, which was an angular point of a polygon, and since the use of the Differential Calculus implied continuity *in both directions* at the point, the differential coefficient of l_{x+t} at that point could not properly be taken, and if taken led to fallacious results.

MR. S. G. WARNER said that the paper was of a type to which the Institute was not very frequently accustomed of late, but which would always hold a necessary and honoured place in its proceedings. It was what might be called a philosophical paper, which proceeded on first principles to deal, as completely as possible, with one important aspect of their science. The paper had one dominating merit, namely, that while it was, from its very nature, severely theoretical on the whole, one great aim of the author had been to show how the comparatively difficult and technical processes he exhibited had their bearing on practical work.

A very useful section, in that light, was that in which the author dealt with possible approximations to annuity-values, based on the assumption which he showed to be sufficiently sound over comparatively short periods, that the joint forces of mortality and interest made up a constant. Another very interesting point was with regard to the introduction, along with those, of a possible third force. That was a thing which seemed so much to be invited from the mathematical aspect of the question, and to follow so admirably upon the luminous reasoning which had led up to it, that one could not help feeling rather sorry that it did not better fit in with practice. But, as the author himself had to admit, it was rather difficult to select good instances of the judicious introduction of a third force. The author attempted it with the force of secession, and had immediately to warn them that that was a dangerous thing to handle. He then proceeded to deal with the rate of marriage, which at first sight seemed a little more hopeful; but, there again, the author was compelled to enter a *caveat* and remind them that that was a thing which, unlike the force of mortality, was dependent upon human will, and must, therefore, always be very cautiously dealt with—a very sound piece of criticism. He was afraid even that the instance the author gave of the re-marriage rate in the valuation of a reversion for probate purposes, while it was novel and ingenious, and admirably worked out, might in fact prove rather impracticable.

The demonstrations given were, he thought, worthy of the highest praise, because there was no doubt that one of the chief difficulties which students of actuarial science felt was in connection with the methods of the infinitesimal calculus. The notation of that calculus, and the fundamental ideas of it, were exceedingly difficult to apprehend when the mind was no longer so elastic as it was in the first stages of most men's study; and a good many of them had painful reason to know that the absorption of the amount, perhaps not very great, of that kind of theoretical teaching which they found necessary after they were engaged in practical work, was exceedingly difficult. It seemed to differ not in degree so much as in kind from the rest of the mathematical work they had to do, by reason of the difficulty they felt in getting a thoroughly firm hold of the primal assumptions; and from that point of view it seemed to him that the processes of the author in his paper, proceeding in every case carefully and minutely from first principles, and giving almost what might be called verbal explanations of those recondite processes, were of high value. Their science was essentially a doctrine of series, and as soon as that fact was grasped, and the further obvious fact that the divisions into yearly periods were purely arbitrary ones, they were at once—if they wished to reason thoroughly—thrown upon the infinitesimal calculus and its methods. Another thing which was valuable in the paper was the insistence on the close and logical connection between the forces of interest and mortality. He remembered long ago hearing Mr. King say to some students, when he had finished that part of his course of teaching which dealt exclusively with interest, that although they were saying good-bye to the subject in a sense, they were not really doing so, because throughout the whole of their actuarial training they would, in fact, be working at it. One was reminded of such words as those in reading a paper like Mr. Allen's. Those two forces, one of increment and one of decrement, were essentially of the same nature, and they could never too carefully or constantly remember that fact.

The PRESIDENT said that the members of the Institute on the present occasion owed special thanks to Mr. Allen, who had not only prepared a paper whose merits had been admitted by all speakers, but who had taken the trouble to come for the purpose of reading it all the way from Perth. He ventured to hope that the fact of the author living so far from London would not prevent him appearing frequently at the meetings of the Institute, and that they would have further papers from him. He was much struck with the fact that the author began with certain fundamental principles, and used the infinitesimal calculus to arrive at his results, and in that connection he noticed a remark from Mr. Thompson as to Part II of the *Text-Book*, where demonstrations involving the use of the differential or integral calculus were so arranged as to lead students to pass them over at a first reading. He might in that connection say that the question of placing the elements of the

integral and differential calculus in the examination of Part I was now under consideration by the Council. By that means those who had passed Part I would probably find it much easier to pass on to the work of Part II. He asked the members to accord a hearty vote of thanks to the author for his paper.

The motion was carried with acclamation.

Mr. J. M. ALLEN, in reply, said that he would endeavour to deal with some of the points raised. Taking first Mr. Thompson's point with regard to § 19, he noticed that Mr. Fraser had taken up what he intended to mention, namely, that the assumption dealt with there was, that the deaths were uniformly distributed over every year of age. By that he meant that he was for the time being considering every year, no matter when it started, whether from x or from $x+\frac{1}{m}$. With regard to Mr. Fraser's criticism, all he could say at the moment was that he was open to conviction that there was a false step; and he would be happy to look into the matter.

With reference to Mr. Thompson's remarks on the graphic graduation, he did not propose to start a discussion as to the *pros* and *cons* of such a graduation. They all knew that one of the principal features of the method was that no two graduators would produce the same curve. He had tried many graduations personally in the present case, the object in view, as Mr. Thompson had said, being to reduce the ultimate accumulation of error to zero. He found it very difficult to do so in any other way except by following the original data in the early years, and as he noticed particularly that those original data did follow very much their preconceived notions of the flow of the force of mortality in the general population at those particular ages, he ultimately determined to adhere to them as far as age 29. He should be very interested to see the graduation which Mr. Thompson had made, going in an upward direction at the earlier ages. He did not propose the example which he had given as a mortality table that was to be used practically, but having propounded a certain method of construction he thought it well to give a practical illustration, and he hoped, if only it served that purpose, it might be of some value.

With regard to the appendix, he should be very interested to study more fully Mr. Thompson's proof. He might say that he had considered it desirable to obtain some proof of the theorem dealt with there, as it occurred in the course of the paper, and thought it would be found that the argument he had used in this particular theorem was very similar and parallel to the proof of the original theorem of the integral calculus, which regarded the definite integral as the limiting value of the sum of a series. Mr. Kenchington had made a few remarks with regard to the question of the paper, from an educational point of view. He had not been aware of what the President had told the members that evening, that the desirability of including the differential and integral calculus

in Part I was under discussion: but if he might express a personal view on the question, he was very glad to hear it. Although he had no longer anything to do with training students for examinations, he had had some little experience in that connection, and he thought it would be of value if at any rate the elements of the calculus could be cleared off before the students attacked Part II.

Mr. Fraser had suggested another method of obtaining a mortality curve. What he should like to say particularly with regard to his own method was, that he regarded it as being useful, for the reason that one obtained the values of μ_x instead of having to make an independent calculation of them, as was usually the case, and also that one avoided calculating the function m_x which was never required at all, except as an intermediate step. He quite agreed with Mr. Warner that one could not deal satisfactorily with the question of a third force. He tried two illustrations, but had come to the conclusion that they could not be regarded as more than somewhat arbitrary examples. Nevertheless, he thought it was at any rate interesting to show how some use could be made of that particular treatment, and that was the reason why he had included the example with regard to the force of marriage.

[Mr. Allen has sent us the following additional note for publication.—Ed. *J.I.A.*]

I have considered the point raised by Mr. Fraser in connection with § 21.

Under the assumption there referred to the curve representing l_x is, as Mr. Fraser points out, represented by a polygon whose angular points are at the successive integral values of x, and l_x is consequently a discontinuous function. At any of the angular points two tangents can be drawn to the curve, one applicable to each of the two sections that join at the point, and it is, therefore, desirable to avoid any reference to a differential coefficient precisely at such a point. If, however, for

$$q_x = \mu_x$$

we substitute

$$q_x = \operatorname*{Lt}_{h=0} \mu_{x+h}$$

the objection disappears.

It is to be borne in mind that in § 21 is considered a discontinuous curve of l_x used to represent approximately the actual curve of l_x according to the mortality table, and that the μ of the relation

$$q_x = \operatorname*{Lt}_{h=0} \mu_{x+h}$$

is that obtained from the approximate curve and not the μ of the mortality table.

Before closing these remarks I should like to draw attention to the fact that although Mr. Fraser's objection to $q_x = \mu_x$ is well founded on account of the *reductio ad absurdum* to which it leads, I do not think that he is justified in bringing the formula

$$\frac{dl_x}{dx} = \Delta l_x - \tfrac{1}{2}\Delta^2 l_x + \ldots$$

to bear on the argument, for the following reason :

By MacLaurin's theorem

$$u_{x+1} = u_x + \frac{du_x}{dx} + \frac{1}{\underline{|2}}\frac{d^2u_x}{dx^2} + \ldots$$

i.e., $$(1+\Delta)u_x = e^{\frac{d}{dx}}u_x$$

whence $$1+\Delta = e^{\frac{d}{dx}}$$

or $$\frac{d}{dx} = \log_e(1+\Delta)$$

and $$\frac{du_x}{dx} = \Delta u_x - \tfrac{1}{2}\Delta^2 u_x + \ldots$$

Now a condition for the admissibility of MacLaurin's theorem is that u_x be a continuous function of x, which, under the conditions of § 21, l_x is not. Consequently the formula

$$\frac{dl_x}{dx} = \Delta l_x - \tfrac{1}{2}\Delta^2 l_x + \ldots$$

is inadmissible.

On the Rationale of Formulæ for Graduation by Summation.

By GEORGE J. LIDSTONE, F.I.A., *Actuary and Secretary of The Equitable Life Assurance Society.*

1. SO much has been written on the algebra of formulæ for graduating an irregular series by means of successive summations, and on the theoretical errors introduced by this process, that there is possibly some danger of the underlying principles, on which all such formulæ are founded, being overlooked or insufficiently understood. In the present note an attempt is made to deal with this matter from first principles, with the object of rendering clear how such formulæ work and why it is that they produce, with greater or less success according to the nature of the particular formula, a smooth succession of values from rough and irregular data.

2. The main principle underlying all graduation formulæ or methods is that the irregularities shewn by an observed series of rates of mortality, or of sickness, withdrawal, &c., arranged according to successive ages, durations, &c., arise from paucity of data and other accidental causes, and are not true characteristics of the observed series; so that if the data were strictly accurate and of infinite extent, we should expect the observed rates to form a smooth and continuous series. In practice, of course, the data can never reach these ideal conditions, but the more nearly these conditions are approached, the smaller will be the irregularities shewn by the observations. The object of graduation is to obtain as close an approximation as possible to the smooth series which would be produced naturally if perfect and unlimited data were available.*

3. The following remarks by James Sorley (see *J.I.A.*, xxii, 311–2) on this part of the subject are so clear and instructive that it will be useful to quote them at length. He says:

"The adjustment of mortality tables framed from the records "of actual observations, may be held to be undertaken mainly for "two reasons:

"(1) To correct errors in the records themselves—mis-"statements, either wilful or clerical—and which may "be called Personal Errors;

"(2) To smooth down such irregularities, arising from "paucity of numbers, as we have good reason to "believe should not be expected to recur were a "similar class of lives again similarly exposed. "These, for the want of a better name, may be "called Natural Errors.

. . . .

"Now the process of graduation is simply the alteration of "the expression for the law of mortality of the community "observed, from an irregular rectilinear form to a curve—a "line, it may be, presenting frequent flexures, but no angles. "For this alteration two good reasons may be given: First, the "principle of continuity, founded upon the reign of Law existing "throughout all nature—expressed in the maxim, *Natura non* "*agit per saltum*—teaching the inherent improbability that "sudden fluctuations up and down in the death-rate as we

* For the present purpose a somewhat narrow view has to be taken of the proper function of graduation; for a much broader view, with which the present writer is in entire sympathy, *see* remarks by G. F. Hardy, *J.I.A.*, xxxiii, 491.

"pass from age to age, are likely to recur at corresponding "ages, were a similar body of lives again observed. Second, "Experience, derived from inspection of other observations in "which we find that like fluctuations do not exist, or possibly "that the deviations from the average or curve line are in opposite "directions; and which has also clearly established the fact, that "as numbers are increased irregularities are in large measure "diminished. Hence Experience, applied, of course, with "judgment, is a most important factor, enabling us, for instance, "to avoid eliminating as accidental deviations any well-established "physical fact. . . ."

4. Again, Sprague says (*J.I.A.*, xxvi, 93):

"If we obtained another large set of observations on a similar "body of lives, we should no doubt find that they would exhibit "similar irregularities, but not at the same ages; and if we could "get observations on a sufficiently large number of lives, these "irregularities would either wholly disappear or be reduced to "insignificant proportions. Now the object of graduation is to "remove all irregularities of this kind, and to obtain the smooth "curve that would be yielded by our observations if they were "sufficiently numerous."

5. The ungraduated observations may therefore be conveniently regarded as subdivided into two parts—(1) the underlying true, smooth curve, and (2) superimposed, or as it were *embroidered*, thereon the positive or negative errors of observation. Now if the unadjusted numbers be subdivided into two parts, and these be separately graduated by any formula of graduation by summation, the sum of the two graduated portions will be identically the same as the result of a single graduation of the whole by the same formula. It will, therefore, be convenient, and tend to a clear understanding of the subject, to consider separately the effect of the graduation formula on each of the parts above referred to; and this we may do although the exact line of division is unknown.

6. In the first place, then, a graduation formula must be so constructed as to reproduce, without sensible or systematic alteration, the smooth series which is the first and principal portion of the observations; for otherwise the smoothing effect of the graduation would be purchased at the expense of a distortion or systematic deformation of the main curve. The required condition would be secured if the smooth series could throughout be represented with sufficient accuracy by a uniform

analytical law (as, *e.g.*, Makeham's); but in many cases this may not be the case. Generally, however, it may safely be assumed that successive *small* sections, each consisting of, say, 15 to 20 terms, can be expressed—with approximate accuracy sufficient for practical purposes—by a comparatively simple analytical expression, which will be the same in form for each successive section, though each section may involve a different set of constants. Although formulæ for graduation by summation might easily be based on other forms, it has been found in practice convenient and sufficient to adopt the form

$$u_x = u_0 + x\Delta u_0 + \frac{x(x-1)}{2}\Delta^2 u_0 + \frac{x.(x-1)(x-2)}{6}\Delta^3 u_0 + \ldots$$

that is, a given order of differences (usually, but not necessarily, the third) is assumed to be constant.

7. The assumption that the smooth series can be represented, over successive small sections, by the law referred to in the last paragraph—or by any other analytical law—leads to simple relations between the various terms, and by means of these relations we may deduce an unlimited number of expressions for the central value of each section in terms of the remaining values. Any one of such expressions will accurately reproduce the original smooth series provided that series conforms to the assumed law, and we are free to select some particular expression, partly for its convenience in practical application, and partly for its power of smoothing the second portion of the observed results, namely, the superimposed errors (*see* par. 5). The class of expressions obtained by means of successive summations of the ungraduated values is found to answer both these requirements, and hence graduation formulæ on this basis have been widely adopted. These formulæ, by a series of summations, the nature of which is too well known to need discussion here, lead to an expression of the following form as the equivalent of u_0 (where u represents a term of the smooth series), namely,

$$\text{(A)} \qquad au_0 + b(u_{-1} + u_1) + c(u_{-2} + u_2) + \ldots$$

where $a, b, c \ldots$ are numerical coefficients. If the smooth series followed rigidly the assumed law, the substitution of expression (A) for u_0 would introduce no error at all; but as the law is in general only approximate, a slight theoretical error is introduced. The size of this small error—that is, the extent to which the substitution of (A) for u_0 leads to distortion of the true smooth series (through that series not exactly following the assumed law)

—has been investigated by Sprague (xxix, 232-6), G. F. Hardy (xxxii, 371-8), Todhunter (xxxii, 382-5), and, quite recently and very exhaustively, by King (xli, 54-80); and it is unnecessary to say more than that their investigations show that for most practical purposes the error is negligible with all the well-known formulæ.

8. The investigations referred to in paragraph 7 demonstrate how a graduation formula of the class under discussion operates on the smooth series of values which it is the object of graduation to disentangle from the errors which are superimposed thereon; and they show that (as stated in paragraph 7) an unlimited number of formulæ can be constructed which will reproduce this smooth series with sufficient exactness for all practical purposes. We have now to consider how the graduation formulæ act upon the errors themselves. If u represent a term of the smooth series and ϵ the superimposed error, the observed value will be $u+\epsilon$, where ϵ may be positive or negative. We have seen that the graduation substitutes expression (A) for u_0, and similarly it will substitute the following expression for ϵ_0, namely:

$$\text{(B)} \qquad a\epsilon_0+b(\epsilon_{-1}+\epsilon_{+1})+c(\epsilon_{-2}+\epsilon_2)+\ldots$$

9. In order to understand how and why the graduation formula effects this smoothing process, and why one produces smoother results than another, we must consider the values taken by the coefficients $a, b, c \ldots$; that is, the form taken by a series of summations.

Taking as an example three successive summations in fives, it is easily seen that the results opposite the central term u_0 will be as follows:

1st Summation. $\quad u_{-2}+u_{-1}+u_0+u_1+u_2.$

2nd Summation.

$$u_{-4}+2u_{-3}+3u_{-2}+4u_{-1}+5u_0+4u_1+3u_2+2u_3+u_4$$

3rd Summation.

$$u_{-6}+3u_{-5}+6u_{-4}+10u_{-3}+15u_{-2}+18u_{-1}+19u_0$$
$$+18u_1+15u_2+10u_3+6u_4+3u_5+u_6$$

Taking, as another example, a summation in fours, followed by one in fives, followed by another in sixes, we shall have (writing coefficients only for brevity):

1st Summation. 1, 1, 1, 1.

2nd Summation. 1, 2, 3, 4, 4, 3, 2, 1.

3rd Summation.

1, 3, 6, 10, 14, 17, 18, 17, 14, 10, 6, 3, 1.

10. In each case it is seen that the coefficient of the central term or terms (distinguished by different type) is the greatest, the coefficients gradually decrease as we get farther away from the centre, and terms equidistant from the centre have equal coefficients. These relations may be shown to be perfectly general whatever the number and extent of the summations. Now all graduation formulæ of the class under discussion express the central graduated value, say u'_0, in terms of the successive summations of a simple function of the ungraduated u's. For example, G. F. Hardy's Friendly Society Formula may be written:

$$\frac{1}{120}\cdot[4][5][6][-u_{-2}+u_{-1}+u_0+u_1-u_2],$$

and Mr. Spencer's 21-term formula is

$$\frac{1}{350}[5][5][7][-u_{-3}+u_{-1}+2u_0+u_1-u_3]$$

and so on. If these formulæ be expanded so as to show the coefficient of each ungraduated u involved in the graduated value u', the coefficients are found to follow very much the same laws as those previously referred to for simple summations; but with this difference—that some of the terminal values must always have negative coefficients, for otherwise the graduated value would involve an error equal to $\Delta^2 u$ multiplied by a constant coefficient (see Sprague, *J.I.A.*, xxvi, 111). The following Table shows the coefficients of several well-known formulæ:

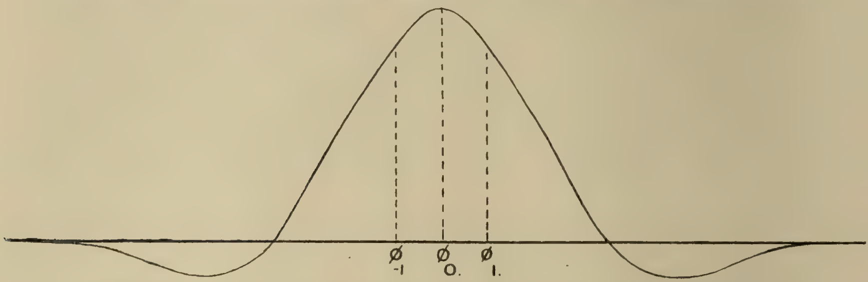
TABLE I.

Showing the values of ϕ_t, where $u'_0=\Sigma_{-t}^{+t}u_t\times\phi_t$, by various well-known Graduation Formulæ.

Distance from Central Term t	Spencer 21-term Formula	Karup	G. F. Hardy Friendly Society Formula	Higham*	Woolhouse
0	·172	·200	·200	·200	·200
± 1	·163	·182	·183	·192	·192
± 2	·135	·139	·142	·144	·168
± 3	·095	·085	·083	·080	·056
± 4	·052	·034	·033	·024	·024
± 5	·017	·000	·000	·000	·000
± 6	−·005	−·013	−·017	−·016	−·016
± 7	−·015	−·014	−·017	−·016	−·024
± 8	−·015	−·010	−·008	−·008	·000
± 9	−·009	−·003	·000	·000	⋮
±10	−·003	·000	⋮	⋮	⋮
±11	·000	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮

* $\frac{1}{125}[5][5][5]\{-u_{-2}+u_{-1}+u_0+u_1-u_2\}$

11. These coefficients (except those for Karup's formula, which is remarkably similar to Hardy's) are shown graphically in Diagram A.* It will be seen that in each case the coefficients tend to lie on a symmetrical curve of the following typical shape:



If ϵ' represent a graduated error and ϵ an ungraduated error, the general expression (B) may be put into the form

$$\epsilon'_0 = \ldots \phi_{-t}\epsilon_{-t} + \phi_{-t+1}\epsilon_{-(t-1)} + \ldots + \phi_0\epsilon_0 + \phi_1\epsilon_1 + \ldots + \phi_t\epsilon_t \ldots$$

where $\phi_0 = a$, $\phi_{-1} = \phi_{+1} = b$, and so on. Now, if the coefficients closely follow the above type of curve—so that the central coefficient ϕ_0 is the greatest, the ϕ's diminish very gradually and smoothly as t (the distance from the central term) increases, and when t becomes considerable the coefficient ϕ vanishes or becomes very small—the successive differential coefficients will be absolutely regular; and the successive finite differences will also be regular if the interval of differencing be small in relation to the whole extent of the curve.

12. Now consider the following scheme showing the values of successive graduated errors:

$$\begin{array}{rllllll}
 & \ldots\ \epsilon_{x-t} & \epsilon_{x-t+1} & & \epsilon_0 & \epsilon_1 & \epsilon_2\ \ldots \\
 & \quad\times & \quad\times & & \times & \times & \times \\
\epsilon'_x = & \ldots\ \phi_{-t} & +\phi_{-t+1} + & \ldots & +\phi_0 & +\phi_1 & +\phi_2 + \ldots \\
\epsilon'_{x+1} = & \ldots\ \phi_{-t-1} & +\phi_{-t} & + \ldots & +\phi_{-1} & +\phi_0 & +\phi_1 + \ldots \\
 & \cdot\ \cdot & \cdot\ \cdot & \cdot\ \cdot & \cdot & \cdot & \cdot
\end{array}$$

Here it is seen that the change from ϵ'_x to $\epsilon'_{x+1} \ldots$ is the sum of a number of partial changes: *e.g.*, $\epsilon_{x-t} \times \phi_{-t}$ changes to $\epsilon_{x-t} \times \phi_{-t-1} \ldots$; $\epsilon_{x-t+1} \times \phi_{-t+1}$ changes to $\epsilon_{x-t+1} \times \phi_{-t} \ldots$; and so on. Each one of these partial changes will be regular and gradual because it depends only on the change in ϕ,

* In order to save space, only one-half of the curve is shown in each case, viz., the left-hand half (corresponding to the central and preceding terms) for Spencer's and G. F. Hardy's formulæ, and the right-hand half (corresponding to the central and following terms) for Woolhouse's and Higham's formulæ. In each case the curve is in fact continued symmetrically on the other side of the maximum ordinate.

which we have already seen to be regular. Therefore, if the coefficients, ϕ, change with great regularity (as they will do if they follow such a curve as is above referred to) the graduated errors will also change with great regularity *whatever the values of the ungraduated errors, i.e., however irregular these may be;* and the more closely the coefficients follow such a curve, the smoother will be the progression of the graduated errors.

13. Returning to Diagram A, and arranging the formulæ according to the degree of likeness to the type (commencing with the smallest degree), we have (1) Woolhouse, (2) Higham, (3) G. F. Hardy, (4) Spencer; and this is precisely the order in which they would appear if arranged according to the smoothness of graduation produced—*i.e.*, according to graduating power. In fact, a formula which is to produce a smooth graduation must itself be smoothly graduated in respect of its coefficients. To quote Karup: "That formula will work most smoothly in which the "coefficients . . . form a curve which, . . . in its ascending and "descending part very nearly resembles a straight line and runs "as evenly and smoothly as possible through those points where "a turn is unavoidable."

14. Diagram A shows in a very clear and striking way the inferiority of Woolhouse's formula, the coefficients of which follow a very irregular curve. Higham's formula is a very great advance on Woolhouse's, and G. F. Hardy's Friendly Society Formula is slightly smoother still. But the curve representing Mr. Spencer's coefficients shows a really remarkable likeness to the typical curve, and will obviously lead to an extremely smooth graduation. The improvement in the sweep of the coefficients is especially remarkable at the two most important points, namely, (1) at the apex where the curve makes a very easy ascent to and descent from the maximum point; and (2) at the tails of the curve, where the coefficients run so smoothly into the base-line that the curve may almost be said to become asymptotic.

15. We may now apply two well-known formulæ to graduate series consisting of utterly irregular values, and an examination of the results, showing the work that a graduation formula *does*, will greatly assist in clearly understanding *how* that work is done. In Table II, page 360, are given two series of digits ranging from 0 to 9, being in fact the last recorded figures in a section of a logarithm table and a table of q_x respectively: these numbers are, of course, completely irregular in their distribution —more so, in fact, than the errors in any observed series of rates

of mortality, &c., could be, because in their case small deviations from the mean are more frequent than large ones, whereas the values in Table II are as likely to differ by the maximum deviation as by the minimum. The graduation of these figures will therefore prove a severe test of smoothness and graduating power. Two graduations of each series have been made, namely, one by Mr. G. F. Hardy's Friendly Society Formula (represented in his notation by $\frac{1}{120}[4][5][6]\{2[3]-[5]\}u_0$ and the other by Mr. Spencer's 21-term formula, represented in the same notation by $\frac{1}{350}[5][5][7]\{-u_{-3}+u_{-1}+2u_0+u_1-u_3\}$. The graduated values are given in the Table, and in Diagrams B and C they are also represented graphically, in which form the graduation is more easily appreciated.*

16. It will be seen that in each case the graduation is remarkably successful. Starting with two series of digits ranging from 0 to 9, and occurring with absolute irregularity, we obtain from Series I graduated values ranging only between 4·86 and 3·91 by Spencer's formula, and between 5·00 and 3·75 by G. F. Hardy's; while in Series II the graduated values range between 4·38 and 3·06 by Spencer's formula, and between 4·62 and 2·90 by G. F. Hardy's. Thus the range is very greatly reduced; but—what is much more important for the purpose in hand—the roughness is immensely diminished. The graphic representation of the graduated values shows that these run with almost complete smoothness and regularity: whereas in the rough data the roughness is present to a marked degree in the units themselves, in the graduated values the roughness is hardly observable in the second decimal place.

17. Thus the graduation formulæ are completely successful in replacing a chaotic series of values—which may be taken to represent the unknown errors that are superimposed on the underlying regular series which we seek—by a smooth and well-graduated series: and from the way in which the formulæ are derived, it follows that they will closely reproduce, without sensible alteration, the underlying regular series itself: thus, since the sum of two smooth series must itself be a smooth series,

* In these diagrams, the straight line represents the average value of the unadjusted values. The graduated values tend in general to group themselves round the straight line; but in the particular case represented in Diagram B, the graduated values are seen to lie almost entirely above the straight line. This is because the average value of those ungraduated values for which graduated values are obtained, happens in this case to be appreciably greater than the general average of the *whole* ungraduated series, including the values at the beginning and end of the series, where *no* graduated values are obtainable.

our total graduated values must themselves progress smoothly—that is, the graduation is efficient from the point of view of smoothness. It is also evident that the errors are not only smoothened but also much reduced, *i.e.*, the final graduated results not only run more regularly but also are, on the average, much nearer the truth than the ungraduated values, as is shown mathematically by Mr. G. F. Hardy, *J.I.A.*, xxxii, 376–8.

18. It must not, however, be assumed that the graduated values, however smooth, necessarily represent more than an approximation to the smooth curve which would be yielded by unlimited data: the errors, though reduced and smoothened, are not altogether eliminated. For example, take the series given in Table II, and Diagrams B & C. The underlying "smooth curves" to which these correspond would be straight lines representing the mean values of the tabulated digits: instead we find curves with a number of waves, sometimes small and shallow ("surface ripples"), and at other times longer and deeper. These waves, as we shall see later, are due to the accidental grouping of the irregularities in the original series, and they are a characteristic feature of the method of graduation now under discussion, although in many instances the waves are so small (except at extreme ages) as to be hardly noticeable for practical purposes.

19. The origin and cause of the wave-like form of the graduated errors, referred to in the preceding paragraph, may now be explained. We have seen (par. 10-11) that the smooth progression in the graduated errors is obtained by spreading out each isolated error in a curve such that adjacent values each take a part of the error; and this process, while it has been seen to produce a smooth curve, yet gives full weight to the actual errors, though this weight is spread over a number of terms instead of being concentrated on one. If the errors are so grouped that positive and negative errors alternate with considerable regularity, the smoothed curve will shew only "surface ripples"; but a succession of positive or of negative errors will necessarily lead to a more marked elevation or depression of the curve. Take, for example, Series I, in Table II. Diagram B shows a considerable "hump" at and near the 16th and 17th values, and on referring to the ungraduated figures it is seen that there is at and near those points a combination of large values. Again, Diagram C, relating to Series II, shows an even greater depression at and around the 24th and 25th values, where there is a sequence of three very low

ungraduated values 1, 1, 1. These features of the graduated curve are present in almost equal degree in the adjustment by Spencer's Formula and by G. F. Hardy's, and it is evident that they would present themselves in any graduation based on similar principles, though they will be least marked with formulæ having the lowest central coefficients.

20. This tendency to convert a short sequence of errors into a feature of the graduated curve—instead of eliminating them, as a perfect graduation method would do—must be regarded as the principle inherent defect in graduation formulæ of this character. If the experience be a large one, so that the errors, or roughnesses, in the observations are not very considerable, the waves thus introduced will not be of great importance because the elevations and depressions will be small relatively to the general height of the curve; but in a small experience they will be both theoretically and practically more serious, and some more powerful method, which will take into account the weights of the observations, may be desirable. At the youngest ages in any experience, and especially in a small one, the deaths will generally be small and the average deviation from the true value of q_x will bear a very considerable proportion to that true value, and will be many times greater than any reasonable value of Δq_x. Here a sequence of large negative deviations in q (arising from quite a small displacement of deaths) may produce a well-marked minimum in the graduated values; but, in many cases the evidence would (if standing alone) be quite insufficient to justify the retention of this minimum as a real feature of the true curve, such as might be expected to repeat itself in another similar experience.

21. In order to obtain a simple example of the graduation of the errors (considered separately from the underlying smooth curve) we took, in Table II, a series of irregular values which grouped themselves round a straight line representing the average value, so that the average value was the same throughout the series. It is, of course, otherwise when we are dealing with rates of mortality. If θ_x represent the deaths, E_x the exposures, and q_x the true rate of mortality, the average deviation of θ_x from $q_x E_x$ will be proportional to $\sqrt{q_x p_x E_x}$, and the average deviation of $\frac{\theta_x}{E_x}$, the observed rate of mortality, will be proportional to $\sqrt{q_x p_x E_x} \div E_x$, *i.e.*, to $\sqrt{q_x p_x \frac{1}{E_x}}$. This quantity will usually diminish with the age up to a certain point, and afterwards increase; that is, it will form a kind of distorted U-shaped curve,

in place of the straight line. It would seem, however, that the surface ripples and waves which have already been discussed, would group themselves round this U-shaped curve in very much the same way that they were found to do round the straight line in the simple examples taken. Especially will this seem to be probable when it is considered that a very large proportion of each individual error is distributed over a comparatively small number of terms, for which the change in the U curve would be comparatively small. (For example, in Hardy's formula 85 per-cent of each error is distributed over five terms, of which the given term is the centre, and in Spencer's formula 77 per-cent). It is thought, therefore, that the general conclusions arrived at may be regarded as sufficiently correct; but it would be of great interest to see the result of a graduation of a set of representative errors based on the theoretical probable errors taken age by age.

22. To sum up:

(1) A formula for graduation by summation proceeds on the assumption that successive small sections of the true, smooth curve can be represented with sufficient accuracy by a regular function, usually one of the nth degree: that is, one having its nth finite difference constant, where n is usually (though not necessarily) taken equal to three. [Pars. 6, 7.]

(2) By means of this function a formula is found by which any given value is expressed in terms of neighbouring values; and this formula, as applied to the smooth series, will reproduce its terms with practical accuracy. [Pars. 6, 7.]

(3) The formula is so chosen that the coefficients of successive terms form a more or less smooth curve approximating to the type discussed in Par. 11, with the result that, however irregular the individual errors, they are smoothed into a very regular curve; and the more nearly the coefficient-curve approaches the type, the smoother will be the graduated errors. [Pars. 12-14.]

(4) Although the graduated errors will form a *smooth* curve, ripples and waves (of greater or less importance according as the data are small or extensive) will be present as the result of the grouping of individual errors, and the formulæ may thus turn a sequence of errors into a feature of the graduated curve instead of eliminating them. [Pars. 19–20.]

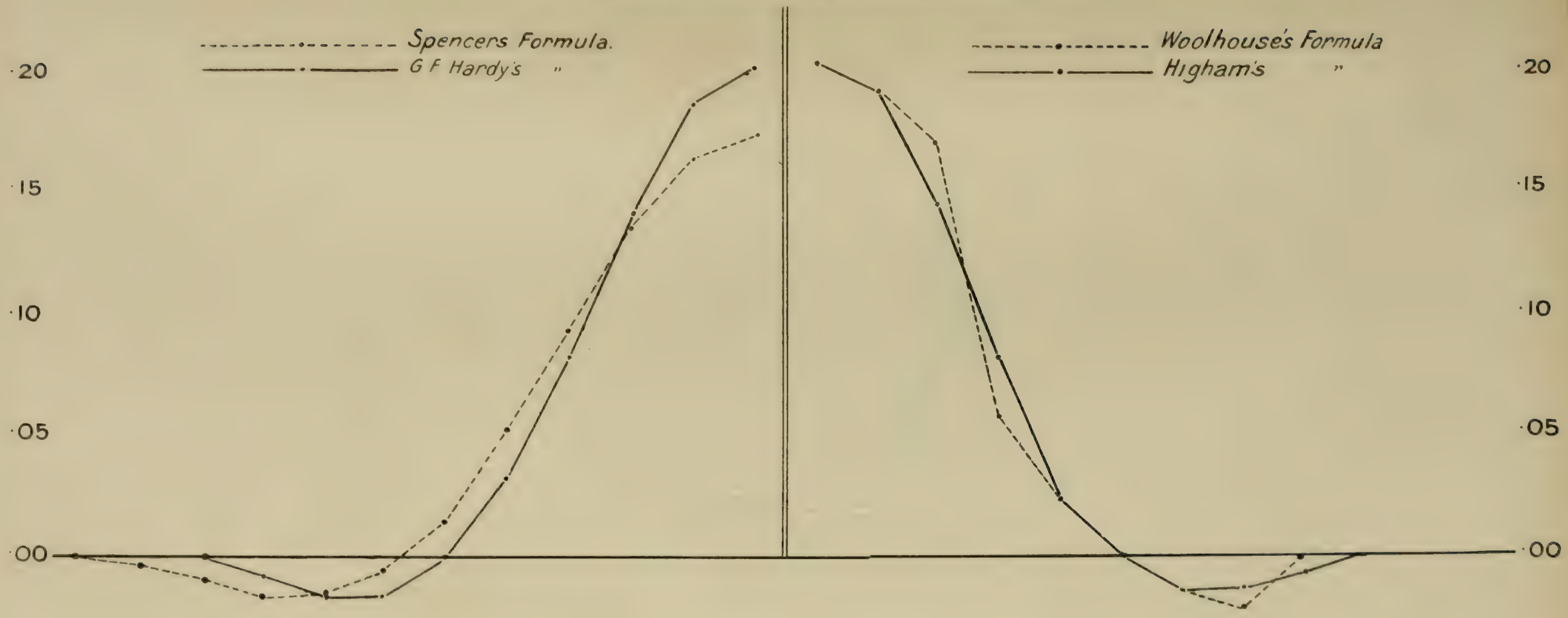
Table II.

Showing the Graduation of two extremely irregular Series, (1) by G. F. Hardy's Friendly Society Formula, (2) by Spencer's 21-term Formula.

Series I			Series II		
Ungraduated	Graduated		Ungraduated	Graduated	
	Hardy's Formula	Spencer's Formula		Hardy's Formula	Spencer's Formula
0	...	...	9	...	...
9	...	...	0	...	...
7	...	...	2	...	...
5	...	...	4	...	...
3	...	...	0	...	...
1	...	...	8	...	...
8	...	...	0	...	...
6	...	...	5	...	...
3	3·87	...	3	3·96	...
0	3·75	...	3	4·18	...
7	3·80	3·91	7	4·31	4·22
4	4·00	4·08	4	4·31	4·25
1	4·27	4·31	5	4·23	4·22
7	4·60	4·54	4	4·11	4·14
3	4·84	4·71	0	4·03	4·08
9	5·00	4·83	5	3·93	4·03
5	4·93	4·86	9	3·94	4·02
1	4·77	4·80	1	4·06	4·05
7	4·60	4·63	1	4·11	4·05
2	4·42	4·46	8	4·07	3·96
7	4·30	4·31	2	3·94	3·79
2	4·18	4·26	4	3·66	3·55
7	4·22	4·26	7	3·28	3·30
2	4·28	4·33	1	2·98	3·12
6	4·43	4·39	1	2·90	3·06
1	4·50	4·46	1	2·95	3·14
5	4·53	4·45	5	3·23	3·34
9	4·53	4·42	5	3·67	3·59
3	4·41	4·42	2	4·05	3·85
6	4·33	4·46	8	4·23	4·04
0	4·33	4·52	2	4·26	4·19
3	4·51	4·57	4	4·22	4·23
6	4·64	4·57	5	4·13	4·27
9	4·73	4·52	5	4·16	4·30
2	4·62	4·44	0	4·29	4·38
5	4·35	4·31	7	4·51	...
7	4·07	4·21	3	4·62	...
0	3·91	4·17	9	...	...
2	3·95	4·17	5	...	...
4	4·13	4·17	0	...	...
5	4·36	...	4	...	...
7	4·35	...	7	...	...
8	...	...	2	...	...
0	...	...	4	...	...
1	...		7	...	...
2	...	...	...	...	...
2	...	...	...	...	...
3	...	...	...	...	...
3	...	...		...	...
4	...	...	...	...	...

Diagram A

Shewing graphically the values of the coefficients in four of the principal graduation formulae.



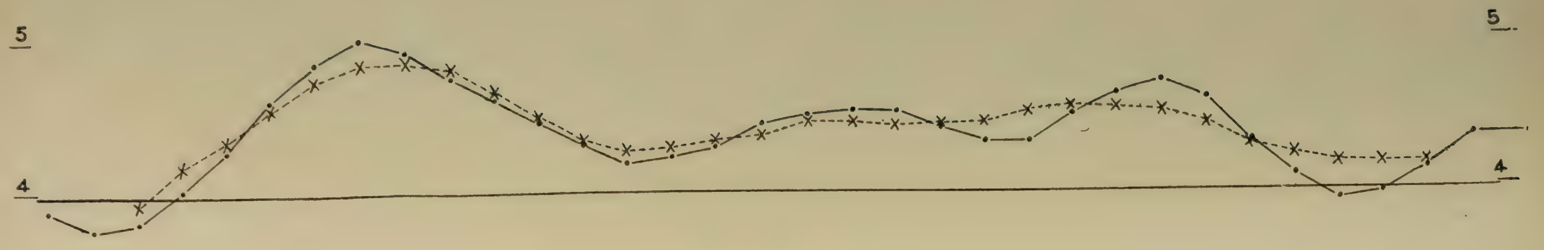


Diagram. B.

Shewing graphically the graduation of Series I. by G. F. Hardy's Friendly Society Formula and by Spencer's 21-term Formula. The Ungraduated Values are not shewn, as they would require a diagram of twice the size to exhibit the Extreme Values, viz, 0 and 9. The Straight Line represents the Mean Value, about which the graduated Values group themselves.

•——•——•——• *G. F. Hardy's Formula.*
x·····x·····x·····x *Spencer's "*

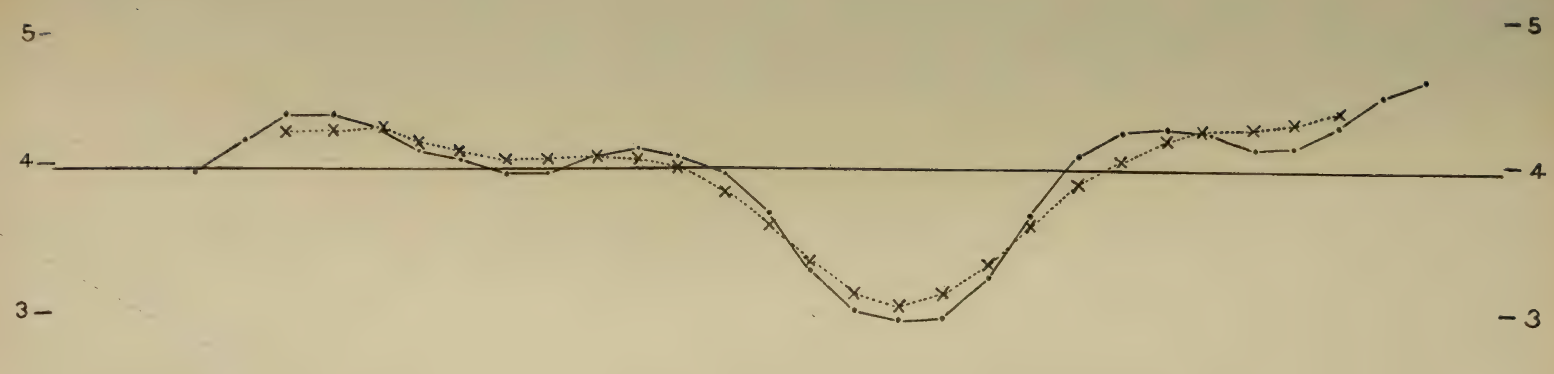


Diagram C.

Shewing graphically the graduation of Series II. by G. F. Hardy's Friendly Society Formula and by Spencer's 21-term Formula. The Ungraduated Values are not shewn, as they would require a diagram of twice the size to Exhibit the Extreme Values, viz. 0 and 9. The Straight Line represents the Mean Value, about which the graduated Values group themselves.

•—•—•—• G. F. Hardy's Formula
×····×····×····× Spencer's "

Some Illustrations of the Employment of Summation Formulas in the Graduation of Mortality Tables. By JOHN SPENCER, F.I.A., *Assistant Actuary of the English and Scottish Law Life Assurance Association.*

I.

A NEW GRADUATION OF THE GOVERNMENT FEMALE ANNUITANTS' (1883) ULTIMATE TABLE AND AN EXAMINATION OF DR. SPRAGUE'S GRADUATION OF THAT TABLE BY THE GRAPHIC METHOD.

MR. GEORGE KING'S recent investigation of the magnitude of the error introduced into certain functions of a mortality table following Makeham's law when these are re-graduated by summation formulas (*J.I.A.*, xli, 54), has again attracted attention to the subject of the graduation of mortality tables, and has recalled the lively controversy which took place some years ago between Dr. Sprague and the late Mr. Woolhouse over the merits and demerits of the methods championed by those distinguished authorities. Not the least interesting contribution to the discussion which followed the reading of Mr. King's paper was furnished by Dr. Sprague himself, who wrote in defence of certain criticisms expressed by him with regard to methods of summation generally and Mr. Woolhouse's formula in particular. Dr. Sprague remarked that he had "objected to Mr. Woolhouse's "formula of graduation, on two grounds, which it has in common "with all other summation formulas; that is to say, (1) because "it does not remove the irregularities of the original series of "facts, but only reduces them, and (2) because it distorts the "law of the facts to a greater or less extent." He went on to say that the results of his re-graduation of the H^M (*Text-Book*) Table by Woolhouse's formula had led him to the conclusion that the second objection might be disregarded in practice, but that there still remained objection (1), which he considered by far the more important. As to the main source of Dr. Sprague's objections to Woolhouse's method, many Actuaries are, I think, at the present time, in agreement. The opinion is widely accepted that smoothness of graduation is the most important consideration, and from this point of view there is no doubt whatever that the results yielded by the application of Woolhouse's formula leave much to be desired.

It seems clear from the general tenour of Dr. Sprague's remarks on Mr. King's paper that he still considers all summation

formulas alike as open to condemnation on the ground that they do not lead to sufficient smoothness in the graduated results. It does not, however, follow that the state of things which existed when Dr. Sprague's famous paper on the graphic method (*J.I.A.*, xxvi, 77) was written, applies at the present time. Other formulas have in the meantime been deduced for the express purpose of overcoming the objections which had been urged against the earlier formulas, and it would not be proper to condemn these new formulas without careful examination of their capabilities. Of the more recent formulas which I believe possess greater graduating power than Woolhouse's, and yield much smoother results, I may more particularly mention Mr. G. F. Hardy's Friendly Society Formula (*J.I.A.*, xxvii, 277), Dr. Karup's 19-term formula (*Transactions of the Second International Congress of Actuaries*, p. 92, formula 5), and the 21-term formula which was deduced by the writer and employed in the graduation of the rates of mortality exhibited by the Manchester Unity Experience 1893–1897. With regard to the last of these I expressed the opinion some years ago (*J.I.A.*, xxxviii, 338) that the formula was more powerful than any published up to that time, and that the results yielded by it compared very favourably with those brought out by employing the graphic method. Further consideration of the subject, and the application of the formula in one or two cases which Dr. Sprague had selected for illustrative purposes in expounding his views on the graphic method, has not altered my view as to the suitability of the formula for employment in the graduation of q in mortality tables of the aggregate or ultimate type, and notwithstanding that much has been written recently on various aspects of the subject of graduation I think it may possibly be useful to show what the formula in question is capable of when applied to graduate a series of unadjusted data presenting irregularities much greater in extent than those observable in the O^M Table, which I have already graduated by the 21-term formula. For this purpose one could not have a better set of facts than that employed by Dr. Sprague in dealing with the graphic method, namely, the Government Female Annuitants' (1883) Ultimate Table. As is well known, the data in the case of that table, up to about age 50, were very scanty, and the unadjusted rates of mortality were so irregular as to call into play a great deal of labour and skill in the application of the graphic method. I have accordingly graduated the table in question by the

21-term formula, the function operated on being the rate of mortality, as in the case of the Manchester Unity Tables and the O^M Table, to which the formula has also been applied.

It may be remembered that the 21-term formula is obtained by applying the operations $\frac{[7][5]^2}{350}$ to the series

$$\{-u_{-3}+u_{-1}+2u_0+u_1-u_3\}.$$

This series may be written in the alternative form

$$([1]+[3]+[5]-[7])u_0,$$

whence, following Mr. G. F. Hardy's plan of expressing the sum of a given number of values in terms of the middle value and its central second differences, we have—

$$\begin{aligned} [1] &= u_0 \\ [3] &= 3u_0+b_0 \\ [5] &= 5u_0+5b_0 \\ [1]+[3]+[5] &= 9u_0+6b_0 \\ [7] &= 7u+14b_0 \\ [1]+[3]+[5]-[7] &= 2u_0-8b_0 \end{aligned}$$

assuming 3rd differences to be constant.

Since $\frac{[7][5]^2}{350}u_0=\frac{175}{350}\left(u_0+\frac{48+24+24}{24}b_0\right)=\frac{1}{2}(u_0+4b_0)$ on the same assumption, it is clear that the employment of these operations will produce a formula which graduates correctly as far as third differences.

It will be found that the actual second differences, which, deducted from $2u_0$ make up the series summed, are

$$\{b_{-2}+2b_{-1}+2b_0+2b_1+b_2\},$$

the assumption that these are equivalent to $8b_0$ contributing towards the error of the formula in the fourth and higher orders of differences.

The resulting formula may be written—

$$u'_0=\frac{1}{350}\{60u_0+57u_{\pm1}+47u_{\pm2}+33u_{\pm3}+18u_{\pm4}+6u_{\pm5} \\ -2u_{\pm6}-5u_{\pm7}-5u_{\pm8}-3u_{\pm9}-u_{\pm10}\}$$

where $u_{\pm1}$ is written for $(u_{-1}+u_{+1})$, and so on.

The working process is apparent from the way in which the formula is arrived at, but an actual example of this is shown in my former paper (*J.I.A.*, xxxviii, 339). I may, however, explain that in applying the formula on the present occasion I have, in order to ensure accuracy, deferred division by 350 until completion of all the summations.

TABLE 1.

Government Female Annuitants' (1883) Ultimate Table.
Unadjusted Rates of Mortality, and Graduated Rates as brought out by Sprague's Graphic Method and Spencer's 21-term formula.

Age	Unadjusted	Sprague	Spencer	Age	Unadjusted	Sprague	Spencer
20	·00000	·0178	·01596	60	·02445	·0219	·02200
1	·00000	·0176	·01595	1	·01921	·0232	·02334
2	·00000	·0173	·01594	2	·02445	·0249	·02501
3	·00000	·0169	·01587	3	·02692	·0270	·02699
4	·00000	·0162	·01570	4	·02884	·0294	·02924
25	·03704	·0151	·01537	65	·03395	·0321	·03169
6	·03030	·0143	·01482	6	·03230	·0350	·03426
7	·01176	·0135	·01406	7	·03933	·0380	·03697
8	·01010	·0128	·01313	8	·04306	·0410	·03992
9	·02308	·0121	·01218	9	·04129	·0440	·04327
30	·01227	·0116	·01140	70	·04730	·0470	·04719
1	·00000	·0110	·01086	1	·04836	·0500	·05177
2	·00897	·0105	·01053	2	·05297	·0560	·05706
3	·01210	·0103	·01047	3	·06553	·0630	·06300
4	·01071	·0103	·01060	4	·07011	·0700	·06955
35	·01567	·0106	·01089	75	·08206	·0770	·07654
6	·01389	·0109	·01119	6	·08446	·0840	·08393
7	·00721	·0112	·01150	7	·08832	·0910	·09160
8	·01268	·0118	·01174	8	·09905	·0990	·09977
9	·00350	·0120	·01198	9	·10816	·1080	·10851
40	·01991	·0122	·01212	80	·11771	·1180	·11806
1	·01228	·0121	·01226	1	·13472	·1290	·12845
2	·01441	·0119	·01231	2	·13395	·1400	·14011
3	·00956	·0116	·01225	3	·15668	·1520	·15294
4	·01094	·0114	·01198	4	·15495	·1660	·16707
45	·01358	·0113	·01162	85	·18826	·1830	·18216
6	·01483	·0113	·01129	6	·20047	·2000	·19832
7	·00824	·0114	·01120	7	·22447	·2170	·21506
8	·00795	·0115	·01139	8	·20629	·2340	·23246
9	·01328	·0121	·01193	9	·26396	·2500	·24968
50	·01019	·0130	·01278	90	·28120	·2650	·26641
1	·01550	·0140	·01382	1	·29412	·2830	·28240
2	·01611	·0155	·01494	2	·27170	·3000	·29760
3	·01753	·0163	·01605	3	·33333	·3200	·31416
4	·01772	·0170	·01707	4	·32231	·3390	·33444
55	·01548	·0177	·01795	95	·39744	·3600	·36103
6	·02022	·0184	·01871	6	·25532	·3820	·39676
7	·01923	·0191	·01940	7	·52941	·4100	·44474
8	·01842	·0199	·02012	8	·56250	·4390	·50839
9	·02329	·0208	·02095	9	·57143	·4660	·59144
...	...	...	...	100	·00000	·5000	·69800
...	...	...	...	1	·66667	·5450	·83256
...	...	...	...	2	·00000	·6050	1·00000

Table 1 shows the unadjusted rates of mortality with the corresponding graduated rates according to Dr. Sprague's and the new graduations.

In deducing the rates at the beginning of the table the observations for ages 19 to 26 were grouped, and the average ungraduated rate of mortality (·01587) was inserted at all ages prior to 27. In this way, as Dr. Sprague says, in making a similar, though not identical, grouping in adjusting the facts graphically, "we roughly assign to the observations at each age a weight proportional to the observed numbers", and it is possible to utilize the formula so as to obtain graduated values back to age 20. No other grouping of any kind was made except above age 90. Here, in order to ensure smoothness of progression in the rates, the irregularities observable in the rough facts about age 95 were lessened by the adoption of an average rate of ·35176 for ages 94 and 95, and of ·40206 for ages 96 to 98. Further, I transposed the rate of mortality at age 101, namely ·66667, to age 100, so as to avoid the zero rate given by the ungraduated data at the latter age. After these trifling adjustments had been made the formula was applied, and the resulting values for ages 20 to 90 are given in Table 1. The values of q for ages 91 to 102 were obtained by a different process, which will be explained later.

We may now proceed to compare the two sets of results as regards (1) smoothness, and (2) fidelity to the original facts, as evidenced by the extent to which the graduated rates will reproduce the actual mortality when multiplied at each age by the number exposed to risk. The smoothness of the curve may be well observed by examining the third differences of q_x, and since in a perfectly-graduated table these differences would be quite inappreciable in extent, their sum over a given section of the table may conveniently be looked upon as a measure of smoothness in comparing two sets of graduated rates. Following Dr. Sprague's plan, it is proposed to deal with the table in sections.

Table 2 gives for both the graphic and the new graduations the third differences of q_x, with the expected and actual deaths :

Table 2.

Government Female Annuitants' (1883) Ultimate Table.
Third Differences of q_x; and divergences of Expected from Actual Deaths.
Ages 20–49.

Age (x)	$\Delta^3 q_x \times 10^5$		Actual Deaths	Sprague			Spencer		
	Sprague	Spencer		Expected Deaths	Deviation	Accumulated Deviation	Expected Deaths	Deviation	Accumulated Deviation
20	0	−6	...						
1	−20	−4	...						
2	−10	−6	...	2·1	2·1	2·1	2·1	2·1	2·1
3	70	−6	...						
4	−30	1	...						
25	10	4	2	·8	−1·2	·9	·8	−1·2	·9
6	−10	15	2	1·0	−1·0	−·1	1·0	−1·0	−·1
7	20	19	1	1·2	·2	·1	1·2	·2	·1
8	−30	7	1	1·3	·3	·4	1·3	·3	·4
9	20	−3	3	1·6	−1·4	−1·0	1·6	−1·4	−1·0
30	20	6	2	1·9	−·1	−1·1	1·9	−·1	−1·1
1	−10	−8	...	2·2	2·2	1·1	2·1	2·1	1·0
2	10	−3	2	2·4	·4	1·5	2·3	·3	1·3
3	−30	−15	3	2·5	−·5	1·0	2·6	−·4	·9
4	0	0	3	2·9	−·1	·9	3·0	0	·9
35	30	−8	5	3·4	−1·6	−·7	3·5	−1·5	−·6
6	−70	7	5	3·9	−1·1	−1·8	4·0	−1·0	−1·6
7	40	−10	3	4·7	1·7	−·1	4·8	1·8	·2
8	−30	10	6	5·5	−·5	−·6	5·6	−·4	−·2
9	20	−9	2	6·8	4·8	4·2	6·8	4·8	4·6
40	0	−2	13	7·9	−5·1	−·9	7·9	−5·1	−·5
1	20	−10	9	8·9	−·1	−1·0	9·0	0	−·5
2	0	12	12	9·8	−2·2	−3·2	10·3	−1·7	−2·2
3	0	12	9	10·9	1·9	−1·3	11·5	2·5	·3
4	0	21	12	12·5	·5	−·8	13·1	1·1	1·4
45	−10	4	17	14·2	−2·8	−3·6	14·5	−2·5	−1·1
6	50	7	21	16·0	−5·0	−8·6	16·0	−5·0	−6·1
7	−20	−4	13	18·0	5·0	−3·6	17·7	4·7	−1·4
8	−20	−12	14	20·3	6·3	2·7	20·1	6·1	4·7
9	40	−11	26	23·7	−2·3	·4	23·4	−2·6	2·1
Totals	±·00640	±·00242	186	186·4	±50·4	±43·7	188·1	±49·9	±37·3

In arriving at the third differences of the new rates I have taken the values of q_x as graduated, to five decimal places. Dr. Sprague gives his values to four decimal places only, and it might suggest itself to some that in order to afford a proper comparison the new rates should be cut down to four places. The adoption of this plan would, however, deprive one to some extent of the greater smoothness which the extra figure renders possible. In applying a summation formula, there is no difficulty in obtaining results to five decimal places, but, on the other hand, one of the objections to the graphic method lies in the

difficulty experienced in reading off accurately, from the graduated curve, values to even three significant figures; and if an attempt were made to work out the results to another decimal place the labour would be enormously increased, if not indeed prohibitive. I accordingly have no hesitation in adopting the plan indicated above.

There can, I think, be no doubt that over this section of the table, where the data are least numerous, and where therefore the summation method is commonly supposed to lead to the least satisfactory results, the new rates are distinctly smoother than Dr. Sprague's. This fact is well brought out in examining the third differences of q_x. There are 19 changes of sign in Dr. Sprague's values, as compared with 10 in mine, the former showing 18 instances where the difference is ·00020 and upwards, as against 1 only in mine; while the sums of the differences, irrespective of sign, vary considerably, the value according to the new method being only about 40 per-cent of that shown by Dr. Sprague's figures.

As regards the expected deaths, the graphic rates bring out a closer result in the aggregate. A final conclusion as to the closeness of the graduated rates to the rough facts cannot, however, be arrived at by looking at this feature over a section of the table, as a comparison of this nature might be materially affected by including another age or two in the group under examination. The closeness of the correspondence between the graduated and ungraduated rates is, I think, best seen by comparing the magnitude of the deviations and accumulated deviations irrespective of sign, and if this point of view be adopted it will be seen that up to age 49 the new rates show somewhat better results than those according to the graphic method.

A few words may be said as to the general characteristics of this section of the curve. The method of graduation adopted at the beginning of the table has the effect in each case of starting the curve with a high value. Dr. Sprague's curve shows a minimum at ages 33–34, a maximum at age 40, and a second minimum at ages 45–46. My curve has the same general features, though the maximum and second minimum points occur later, namely, at ages 42 and 47 respectively.

It may be remembered that before graduating the data graphically Dr. Sprague made two preliminary sets of groupings of the observations for ages 19–51, with regard to which he remarked, "The laws indicated by these two groupings are

"essentially different, and we have to consider which of them is "to be preferred. According to the former of them the rate "of mortality must be considered as approximately constant "from ages 19–49. . . . According to the latter, the rate of "mortality is considerable at the youngest ages, falls to a "minimum between the ages of 30 and 35, and thereafter "continually increases. . . . If we had no further "information beyond what is contained in the table, it would be "difficult to say which of the two groupings should be preferred, "and the safer plan would be to take the former and make the rate "of mortality approximately constant from 19 up to 49" (*J.I.A.*, xxvi, pp. 87, 88). In his recent paper Mr. King quoted the above sentences from Dr. Sprague in commenting on one of the drawbacks attending the application of the graphic method, namely, the difficulty of determining the law followed by the rough data, owing to the fact that an alteration in the groupings often leads to quite different conclusions. Mr. King's criticism is, I think, quite pertinent, but I wish to call attention to one point arising from it on which, in this particular case, misapprehension is likely to occur. Mr. King says that Dr. Sprague eventually discarded the first grouping, "and adopted the second, which "presented a high rate of mortality at the commencement, a "minimum between ages 30 and 35, and thereafter an "uninterrupted increase, and his reason for doing so was that he "found in other statistics an indication of the same minimum. "Now Dr. Sprague himself showed that if the series with "which he was dealing be graduated by Woolhouse's formula, "or by another of the summation formulas, the minimum "between ages 30 and 35 is clearly brought out, as also "a second well-marked minimum between ages 45 and 50, "which is not disclosed by any of Dr. Sprague's groupings" (vol. xli, pp. 72 and 73). To this, Dr. Sprague replied (vol. xli, p. 94): "Mr. King says with regard to the figures given by me "that the graduations by the summation formulas used by "Mr. Higham bring out not only the minimum mortality "between ages 30 and 35, but also a second well-marked "minimum between ages 45 and 50, which is not disclosed by "any of my groupings. I cannot allow this to pass without "notice, but all that can be said is that in my opinion the latter "minimum is not well marked, but is one of the minor "irregularities which it is the business of a good graduation to "eliminate." It is quite true that the grouping provisionally

adopted by Dr. Sprague shows an uninterrupted increase in the rate of mortality after age 35. It might, however, not unnaturally be assumed, especially in view of Dr. Sprague's reply, that no place had been found for the second minimum in the finally-graduated graphic curve, but as a matter of fact this feature has been retained, as will be seen by referring to Table 1. Dr. Sprague's reasons for retaining the second minimum are set out on pages 104 and 105 of his paper, and the form of curve finally adopted is shown in his Diagram 8.

The new rates bring out the second minimum prominently, and, it is also noticeable that the serrated line between ages 32 and 43 brought out by Woolhouse's formula, which Dr. Sprague had criticized, is entirely got rid of in the new adjustment, being replaced by a smoothly progressing curve. The graphic curve at this point will not bear very minute examination (*see* the differences in Table 2).

TABLE 3.

Government Female Annuitants' (1883) *Ultimate Table.*

Third Differences of q_x; and divergences of Expected from Actual Deaths.

AGES 50–69.

Age (x)	$\Delta^3 q_x \times 10^5$		Actual Deaths	SPRAGUE			SPENCER		
	Sprague	Spencer		Expected Deaths	Deviation	Accumulated Deviation	Expected Deaths	Deviation	Accumulated Deviation
50	−120	−9	22	28·1	6·1	6·1	27·6	5·6	5·6
1	60	−8	37	33·4	− 3·6	2·5	33·0	− 4·0	1·6
2	10	−5	43	41·4	− 1·6	·9	39·9	− 3·1	− 1·5
3	0	2	51	47.4	− 3·6	− 2·7	46·7	− 4·3	− 5·8
4	0	5	59	56·6	− 2·4	− 5·1	56·8	− 2·2	− 8·0
55	10	10	57	65·2	8·2	3·1	66·1	9·1	1·1
6	0	8	83	75·5	− 7·5	− 4·4	76·8	− 6·2	− 5·1
7	10	11	86	85·4	− ·6	− 5·0	86·8	·8	− 4·3
8	0	7	90	97·2	7·2	2·2	98·3	8·3	4·0
9	20	4	123	109·8	−13·2	−11·0	110·6	−12·4	− 8·4
60	0	−2	138	123·6	−14·4	−25·4	124·2	−13·8	−22·2
1	− 10	−4	116	140·1	24·1	− 1·3	140·9	24·9	2·7
2	0	−7	157	159·9	2·9	1·6	160·6	3·6	6·3
3	− 10	−8	182	182·6	·6	2·2	182·5	·5	6·8
4	− 10	2	209	213·1	4·1	6·3	211·9	2·9	9·7
65	− 10	10	258	243·9	−14·1	− 7·8	240·8	−17·2	− 7·5
6	0	16	254	275·2	21·2	13·4	269·4	15·4	7·9
7	0	17	317	306·3	−10·7	2·7	298·0	−19·0	−11·1
8	0	9	353	336·1	−16·9	−14·2	327·2	−25·8	−36·9
9	300	5	343	365·5	22·5	+ 8·3	359·4	16·4	−20·5
Totals	±·00570	±·00149	2,978	2986·3	±185·5	±126·2	2957·5	±195·5	±177·0

We now proceed to the next section of the table. Table 3 deals with ages 50 to 69, giving, as before, the third differences of q_x and the expected and actual deaths.

Here, as before, I venture to think that the new rates are smoother than those of Dr. Sprague, notwithstanding the fact that his rates for ages 53 to 57 form a straight line, as also do the values for ages 66 to 71. A noticeable feature of the graphic rates is the pronounced irregularity which is caused by the values for ages 50 to 53. This abrupt break in the curve was probably left in accidentally, and may easily be dispensed with without unduly departing from the facts. The new rates show no such irregularity, and over this section the differences progress much more evenly than those shown by the graphic method.

Turning now to the comparison of expected and actual deaths, it will be seen that on the whole the graphic rates reproduce the actual deaths somewhat more closely than the new rates. Up to about age 61 there is nothing to choose between the two, but after that age, and especially above age 66, the advantage is slightly on the side of the graphic rates. The greater closeness of Dr. Sprague's values to the unadjusted rates above age 66 is probably due to the fact that the graphic rates increase by first differences, instead of exhibiting a positive second difference, which I venture to think is warranted by the observations, and which is to be found in other similar standard tables at the same period of life. This holding-back of the increase in the graphic rates, which even Woolhouse's, the least powerful of the better-known formulas, avoids, gives rise to another abrupt break in the smoothness of the graphic curve at about age 70, to which I will refer in dealing with the next section.

Table 4 deals with the graduated data for ages 70 to 90. The third differences of the new values proceed smoothly up to about age 80, though they are less regular afterwards owing to greater fluctuations in the unadjusted rates above age 90, caused on the one hand by rapid diminution in the number under observation, and on the other by the increase in the rates themselves. The magnitude of the differences is, however, quite small, and their sum is appreciably less than that of the corresponding differences of the graphic values, notwithstanding the presence of nine zero values in the latter. There is no change of sign in the second differences of my rates until age 87 is reached, and these progress steadily and avoid the jumps shown at various points by the corresponding graphic differences.

TABLE 4.

Government Female Annuitants' (1883) Ultimate Table.
Third Differences of q_x; and divergences of Expected from Actual Deaths.
AGES 70–90.

Age (x)	$\Delta^3 q_x \times 10^5$		Actual Deaths	Sprague			Spencer		
	Sprague	Spencer		Expected Deaths	Deviation	Accumulated Deviation	Expected Deaths	Deviation	Accumulated Deviation
70	−200	− 6	396	393·5	− 2·5	− 2·5	395·1	− ·9	− ·9
1	−100	− 4	401	414·6	13·6	11·1	429·3	28·3	27·4
2	0	−17	432	456·7	24·7	35 8	465·4	33·4	60·8
3	0	− 4	518	498·0	−20·0	15·8	498·0	−20·0	40·8
4	0	−12	532	531·1	− ·9	14·9	527·8	− 4·2	36·6
75	100	22	592	555·5	−36·5	−21·6	552·2	−39·8	− 3·2
6	0	7	567	563·9	− 3·1	−24·7	563·4	− 3·6	− 6·8
7	0	24	548	564·7	16·7	− 8·0	568·4	20·4	13·6
8	0	3	565	564·7	− ·3	− 8·3	569·1	4·1	17·7
9	−100	43	562	561·1	− ·9	− 9·2	563·8	1·8	19·5
80	100	−10	549	550·4	1·4	− 7·8	550·6	1·6	21·1
1	100	13	558	534·3	−23·7	−31·5	532·0	−26·0	− 4·9
2	100	−34	479	500·6	21·6	− 9·9	501·0	22 0	17·1
3	−300	11	481	466·6	−14·4	−24·3	469·5	−11·5	5·6
4	0	−49	399	427·5	28·5	4·2	430·2	31·2	36·8
85	0	8	404	392·7	−11·3	− 7·1	390·9	−13·1	23·7
6	−100	−84	342	341·2	− ·8	− 7·9	338·3	− 3·7	20·0
7	0	−31	299	289·0	−10·0	−17·9	286·5	−12·5	7·5
8	400	−25	210	238·2	28·2	10·3	236·6	26·6	34·1
9	−400	− 5	208	197·0	−11·0	− ·7	196·8	−11·2	22·9
90	400	215	160	150·7	− 9·3	−10·0	151·6	− 8·4	14·5
Totals	±·01200	±·00382	9,202	9,192·0	±279·4	±283·5	9216·5	±324·3	±435·5
	to age 87								

Dr. Sprague's reliance in the present instance upon values of q_x to two or three significant figures, a course necessitated perhaps by the difficulty of drawing the curve and reading off the graduated values satisfactorily, may be referred to at this point. It will be seen that his rates for ages 66 to 78 are given to two significant figures only. While, in its effect on monetary values, this plan is perhaps not open to objection, I may point out that a variation in q_x of ·0005 would account for a difference of about 4 in the expected deaths at most of these ages, and I venture to think that the extent of the observations in the table under notice demands another decimal place at least, especially as, from another point of view, it is desirable to avoid the break in the flow of the curve which is caused by a change in the first difference of q_x. The following comparison of the rates of mortality for ages 69 to 73 enables me to

throw into relief the break in Dr. Sprague's curve, to which I have already referred, at ages 71–72, and to show the effect of the cutting-down process on the magnitude of the third differences. To facilitate comparison the O^{af} rates for the same ages are also shown.

Government Female Annuitants' (1883) Ultimate Table.

Age	Sprague q_x	Sprague Δ	Sprague $Δ^2$	Sprague $Δ^3$	Spencer q_x	Spencer Δ	Spencer $Δ^2$	Spencer $Δ^3$
69	·04400				·04327			
		·00300				·00392		
70	4700		·00000		4719		·00066	
		300		+·00300		458		+·00005
71	5000		300		5177		71	
		600		−·00200		529		−·00006
72	5600		100		5706		65	
		700				594		
73	6300				6300			

O^{af} *Ultimate Rates.*

Age	q_x	Δ	$Δ^2$	$Δ^3$
69	·03991			
		·00367		
70	4358		·00048	
		415		−·00010
71	4773		38	
		453		+·00014
72	5226		52	
		505		
73	5731			

The expected deaths brought out for ages 70 to 90 by the graphic rates are, it will be seen, on the whole closer to the observed numbers than is the case with the new values. The difference is, however, not great, and a considerable proportion of this arises about age 72, being, I think, due to the greater smoothness of my rates at that point. The fact that the sum of the accumulated deviations according to the new graduation is relatively large, and that these deviations are mostly positive in sign, arises from the circumstance that the observations have been dealt with in sections, the values in the 70–90 group being immediately preceded by a negative deviation which, had it been carried forward, would have materially modified the figures.

While, as already explained, my values of q_x up to age 90 are those resulting from the application of the formula, a different process was employed in deducing the graduated values of q_x for ages 91 to 102. In his valuable contribution to the discussion on Mr. King's paper, Mr. Lidstone, discussing the

application of summation formulas, suggested a method of dealing with the observations for the final ages based on the assumption that Makeham's law applies. The principal objects in view being to form a satisfactory junction with the main curve, and to reproduce the actual deaths as closely as possible, Mr. Lidstone suggested that the three Makeham constants A, B, and c, should be determined from three equations obtained by making (1) the graduated term at the junction of the two curves agree by the summation and the Makeham method, (2) the differential coefficient or the flow of the curve at that point agree, and (3) the expected deaths for the final section coincide with those observed. Mr. Lidstone did not state which function he would operate upon, nor did he explain by what steps he would proceed to bring the expected deaths into agreement with the actual. In dealing with the present table I have adopted the principle of Mr. Lidstone's suggested method, the actual process being based on the assumption that the rate of mortality might be expressed in the form

$$q_x = a + bx + fx^2 + dc^x,$$

c being assumed to be known, and, in this case, taken as $\log_{10}^{-1} \cdot 04$.

The necessary four equations were obtained by taking

(1) $q_{90} = \cdot 26641$, as found by using the formula.

(2) $q_{91} = \cdot 28240$. (This value was determined by prolonging the original graduation to age 91.)

(3) $q_{102} = 1 \cdot 0$, and

(4) $\Sigma \mathrm{E}q$, the expected deaths $= 368 \cdot 9$, *i.e.*, $3 \cdot 9$ more than the actual deaths for this section, so taken in order to get rid of the aggregate deviation up to age 90 inclusive.

No doubt it would be possible to terminate the curve with equally satisfactory results by applying Makeham's law. The method adopted, however, has the merit of being simple and direct, it avoids the difficulty of finding the constant c, and brings the curve to a termination at the point desired.

The plan here followed of transferring to the final section the deviation in the expected deaths brought out over the main body of the table, is not strictly defensible on theoretical grounds. In the particular instance, however, the deviation is so small that no practical objection can be urged against the course adopted.

The resulting values form a very satisfactory junction with the main curve, and, I think, reflect the observed facts with considerable fidelity. Table 5 compares the values for the final ages with those given by Dr. Sprague.

TABLE 5.

Government Female Annuitants' (1883) Ultimate Table.
Third Differences of q_x; and divergences of Expected from Actual Deaths.

AGES 91–102.

Age (x)	$\Delta^3 q_x \times 10^5$		Actual Deaths	SPRAGUE			SPENCER		
	Sprague	Spencer		Expected Deaths	Deviation	Accumulated Deviation	Expected Deaths	Deviation	Accumulated Deviation
91	−400	236	115	110·7	−4·3	−4·3	110·4	−4·6	−4·6
2	300	259	72	79·5	7·5	3·2	78·9	6·9	2·3
3	−100	283	63	60·5	−2·5	·7	59·4	−3·6	−1·3
4	500	311	39	41·0	2·0	2·7	40·5	1·5	·2
95	−500	342	31	28·1	−2·9	− ·2	28·2	−2·8	−2·6
6	−300	373	12	18·0	6·0	5·8	18·6	6·6	4·0
7	900	411	18	13·9	−4·1	1·7	15·1	−2·9	1·1
8	400	449	9	7·0	−2·0	− ·3	8·1	− ·9	·2
9	400	488	4	3·3	− ·7	−1·0	4·1	·1	·3
100	...	...	...	1·5	1·5	·5	2·1	2·1	2·4
1	...	...	2	1·6	− ·4	·1	2·5	·5	2·9
2	...	...	...	·6	·6	·7	1·0	1·0	3·9
	±·03800	+·03152	365	365·7	±34·5	±21·2	368·9	±33·5	±25·8

The expected and actual deaths in respect of the whole table are summarized in Table 6, which shows clearly how little difference there is between the two tables as regards closeness of the graduated to the ungraduated facts.

To sum up, I have, I submit, shown that throughout the table the new graduation is appreciably smoother than Dr. Sprague's. In this connection it will be remembered that neither Dr. Sprague nor Mr. Woolhouse regarded the values resulting from the application of their processes as unalterable, but considered it quite legitimate to amend them if such a course were found necessary. For example, Mr. Woolhouse says, "It is possible, however, that "at one or two exceptional places there may yet remain some slight "traces of irregularity, but they are sure to be quite isolated, "and so trivial as to be readily amended by mere inspection" (xxix, 239). Again, Dr. Sprague remarks, "It is right that I "should mention that a slight correction has been applied to these

"figures. There is always a liability to error in estimating the "tenths of an interval, and small errors may also arise from "inequalities in the ruling of the paper, or from the curve being "drawn unsteadily. In order to remove these errors I difference "the quantities, and when I find the series of differences presents "irregularities I remove these by inspection" (xxvi, 98). I do not in the least object to the process of "re-touching" being applied if it should be found necessary in order to remove irregularities, or to bring the values into closer conformity with the observed facts, but I wish to lay stress on the fact that the values in Table 1, brought out by my formula for ages 20 to 90, are given precisely as they emerged from the processes of summation, not a single figure having been altered.

TABLE 6.

Government Female Annuitants' (1883) *Ultimate Table.*
Comparison of Expected and Actual Deaths. Whole Table.

Age Group	Actual Deaths	SPRAGUE			SPENCER		
		Expected Deaths	Deviation	Accumulated Deviation	Expected Deaths	Deviation	Accumulated Deviation
20– 30	11	9·9	− 1·1	− 1·1	9·9	− 1·1	− 1·1
31– 40	42	42·2	·2	− ·9	42·6	·6	− ·5
41– 45	59	56·3	− 2·7	− 3·6	58·4	− ·6	− 1·1
46– 50	96	106·1	10·1	6·5	104·8	8·8	7·7
51– 55	247	244·0	− 3·0	3·5	242·5	− 4·5	3·2
56– 60	520	491·5	−28·5	−25·0	496·7	−23·3	−20·1
61– 65	922	939·6	17·6	− 7·4	936·7	14·7	− 5·4
66– 70	1,663	1676·6	13·6	6·2	1649·1	−13·9	−19·3
71– 75	2,475	2455·9	−19·1	−12·9	2472·7	− 2·3	−21·6
76– 80	2,791	2804·8	13·8	·9	2815·3	24·3	2·7
81– 85	2,321	2321·7	·7	1·6	2323·6	2·6	5·3
86– 90	1,219	1216·1	− 2·9	− 1·3	1209·8	− 9·2	− 3·9
91–102	365	365·7	·7	− ·6	368·9	3·9	0
Totals	12,731	12730·4	...	...	12731·0	...	...

The closeness of the expected to the actual deaths in my graduation compares favourably with Dr. Sprague's until about age 65. After that age the balance of advantage is somewhat on the side of the graphic rates, owing in part to a reason already referred to. The difference is very slight, however, and must be regarded as remarkably so when the different characteristics of the two processes are considered. The great advantage of methods of summation is found in the facility with which they may be applied without calling for the exercise of any degree of skilled

labour, and in this respect the method contrasts very favourably with the various processes which the application of the graphic method involves—the experimental arrangements of the data in the preliminary groupings, the drawing of the various sections of the curve, the reading-off of the graduated values, with the rectification and careful final adjustment of the values.

I do not suggest that my graduated table is the best that could possibly be produced, but I claim to have established my main point, namely, that the 21-term formula at any rate is not open to the objection that Dr. Sprague has urged against some others on the ground of their incapacity to yield sufficiently smooth results. Presumably, Dr. Sprague regards his table as one from which all irregularities which ought properly to be got rid of have been removed, and if this be conceded it is impossible to take exception in this respect to my own graduation, having regard not only to the general similarity in shape of the two curves, but also to the fact that the mechanical graduation passes with greater smoothness through those points where changes in the curvature take place.

Before leaving this section of my paper, I may be permitted to express the view that the utility of mechanical methods of graduation has not yet been fully recognized, and that their possibilities are far from being exhausted. It is true that graduation by summation is not always feasible, and in some cases even when the method can be satisfactorily applied it may, in order to subserve practical ends, be preferable to rely upon some alternative method of graduation based on the assumption that the series to be graduated follows some mathematical law. At the same time, if the facts are sufficiently well distributed to admit of the application of any but a purely analytical method of graduation, a suitable summation formula will, with rapidity and precision, bring to light the smooth curve representing the law of the facts, with but a tithe of the labour attending the application of the graphic method, and with a fidelity throughout the table which is not to be looked for when a mathematical law of mortality is assumed to prevail.

It has recently been suggested as a suitable method of procedure in adjusting mortality tables such as that dealt with above, that a preliminary graduation should be made by a summation formula, the results being then graduated graphically in order to obtain greater smoothness. I venture to suggest that in almost every case in which a preliminary graduation by a

formula of comparatively short range, such as Woolhouse's, would be of any service as a basis for a graphic adjustment, the employment in the first instance of the 21-term formula would lead to much more satisfactory results, as regards smoothness, than could be looked for if a graphic process were applied, and that the labour which would be entailed by a second graduation according to the graphic method would therefore be wholly unnecessary. In evidence of this I may again refer to the results produced in the Government Female Annuitants' Table up to age 50 by the application of my formula. Here, where one would have least expected it, in view of the violent irregularities in the original values, order has been evolved out of chaos, a result which will, I think, surprise not only those who regard the graphic method as possessing a monopoly of smoothing power, but others who, though familiar with Woolhouse's time-honoured method and its capabilities, have had no opportunity of examining the work which may be performed with the aid of a much more powerful formula.

II.

Completion of the Graduation of the O^M Table by the 21-term formula; with some remarks on Graduation by reference to a standard table.

On a former occasion I applied my 21-term formula to graduate q_x of the O^M Table, and the adjusted rates up to age 79 were published in the *Journal* (vol. xxxviii, p. 342), the main object I then had in view being to compare the smoothness of my graduated results with that of the official graduation. The O^M Table was chosen for the reason that it afforded an instance of a table in the graduation of which the cardinal point aimed at was to fit the observations with a curve of the greatest possible accuracy and smoothness, the graduator not being fettered with the task of moulding the data according to a specified pattern, as is the case when Makeham's formula is to be applied. It is obviously not to be expected that any summation formula could, throughout the table, yield results quite as satisfactory as regards smoothness as those based on a mathematical formula such as Makeham's; and the same reason that induced me on that occasion to avoid selecting as a standard of comparison a Makeham curve, led me, as I then pointed out, to compare the two sets of values of q_x up to age 79 only, since at that point the graduated O^M Table practically merges into the $O^{M(5)}$.

In his recent paper Mr. King referred appreciatively to my O^M graduation, but commented on the fact that no values were given beyond age 79, and seemed a little doubtful whether the formula would have given equally satisfactory results had the graduation been prolonged to the end of life. On reading Mr. King's paper I had welcomed his statement that "applied "to q_x . . . . the formula would probably prove sufficiently "satisfactory to the end"; but in replying on the discussion Mr. King seemed to modify this view, though I cannot resist the feeling that he was thinking of a graduation not of q_x, which was the function I operated upon, but of the l_x column, to which I have never applied my formula except for experimental purposes. In any case, I think it may be well, as the question has been touched upon, to complete the O^M graduation, and I accordingly give in Table 7 the graduated values of q for ages 80 to 102.

TABLE 7.

O^M *Table. Graduation of q_x by Spencer's 21-term formula.*

AGES 80–102.

Age	q_x	Δq_x	Age	q_x	Δq_x
80	·14008	·01182	92	·31414	·02086
81	·15190	·01242	93	·33500	·02423
82	·16432	·01269	94	·35923	·03003
83	·17701	·01272	95	·38926	·03859
84	·18973	·01283	96	·42785	·05035
85	·20256	·01320	97	·47820	·06469
86	·21576	·01386	98	·54289	·08026
87	·22962	·01469	99	·62315	·09441
88	·24431	·01575	100	·71756	·10599
89	·26006	·01694	101	·82355	·17645
90	·27700	·01806	102	1·00000	...
91	·29506	·01908			

In arriving at the rates for the concluding ages I followed a plan similar to that adopted when the Manchester Unity Tables were dealt with, hypothetical values of q_x, consisting of the series of c, c^2, c^3, to c^{10}, where $c=\log_{10}{}^{-1}\cdot039$, having been utilized at ages 102 to 112 for the purpose of arriving at the rates of mortality for the last ten ages. The adjusted rates progress with considerable smoothness, and if the facts are to be allowed to speak for themselves, I do not think that any material improvement in this respect could be hoped for. It remains to be seen how far, as regards fidelity to the observed values, the rates fulfil the requirements of a good graduation.

Table 8 shows the expected and actual deaths, with the deviations, not only for the particular section, but also in respect of the remainder of the table, which I reproduce from my former paper.

TABLE 8.

O^M *Table. Graduation by Spencer's 21-term formula. Comparison of Expected and Actual Deaths.*

WHOLE TABLE.

Age Group	Expected Deaths	Actual Deaths	Deviation	Accumulated Deviation
10– 14	10	10	0	0
15– 19	98	97	+ 1	+ 1
20– 24	785	806	− 21	−20
25– 29	2,642	2,615	+ 27	+ 7
30– 34	5,196	5,202	− 6	+ 1
35– 39	7,589	7,557	+ 32	+33
40– 44	9,623	9,731	−108	−75
45– 49	11,607	11,526	+ 81	+ 6
50– 54	13,707	13,670	+ 37	+43
55– 59	15,542	15,594	− 52	− 9
60– 64	17,044	17,093	− 49	−58
65– 69	17,719	17,677	+ 42	−16
70– 74	16,117	16,150	− 33	−49
75– 79	12,241	12,197	+ 44	− 5
80– 84	7,275	7,317	− 42	−47
85– 89	2,862	2,865	− 3	−50
90– 94	695	692	+ 3	−47
95–102	93	88	+ 5	−42
	140,845	140,887	±586	±509

Looking for a moment at ages 80 and upwards, it is noteworthy that the deviation of −37 brought out over this section is in the opposite direction to that shown by the figures in Mr. King's Table 3 (vol. xli, p. 76), or, in other words, if we applied a correction on the lines indicated by Mr. King's results, we should arrive at a still greater deficiency in the expected deaths at these particular ages. On the whole, however, the graduated table very accurately represents the observations at the old ages, the expected deaths above age 74 being 23,166, against 23,159 actual, while, above age 84, the corresponding figures are 3,650 against 3,645. I must therefore take exception to the suggestion that, applied to q_x, my formula does not produce such satisfactory results at old ages as in other parts of the table.

Over the whole table we notice an under-statement of the expected deaths of 42, which, on a total of over 140,000 deaths

—·03 per cent—must be regarded as a very satisfactory result. The total deviation in the case of Mr. Hardy's graduated table is +83, which arises entirely from the graduated values for age 90 and upwards, a divergence which need not occasion surprise when it is borne in mind that no use was made of the rough facts for these particular ages in deducing the graduated table. The deviations at ages 90 to 102 in Table 8 is +8 only, and, while very little weight need be attached to the circumstance, there can be no doubt that my graduated values at these ages fit the observed data much more closely than do the official values.

Notwithstanding the smallness of the divergence between actual and expected deaths, as shown in Table 8, and its unimportance from a practical point of view—I estimate roughly that it would only slightly affect the third decimal place of the annuity-values at certain ages—it is clear that it does for the most part arise from the theoretical error of the formula, and, had I originally contemplated the publication of the whole table, I should have applied the principle of making a final adjustment so as to get rid of the deviation brought out, as I think that, in general, such a correction ought to be made. A simple and sufficiently accurate way of giving effect to this final rectification, and one, moreover, that would not in any way have affected the smoothness of the curve, would have been to increase each value of q by the constant quantity $\frac{42}{\Sigma E_x}$ which works out in the present case at about a unit in the fifth decimal place of q.

It may in some cases be desired to trace more accurately to its source any deviation resulting from the application of a given formula, and in this event the method of graduation by reference to a standard table which Mr. Lidstone has suggested, will, I think, often be found of great value, its efficacy being, of course, greatest when it is possible to select as a standard a perfectly smooth table exhibiting similar characteristics to those shown by the data under examination. We have in the graduated $O^{M(5)}$ Table an obvious and a very suitable standard by which to test the O^M graduation, and I have employed this, in applying Mr. Lidstone's method, with the view of discovering the situation of the deviation resulting from my graduation of the O^M Table. It will be remembered that Mr. Lidstone suggested that the differences between the unadjusted values and the corresponding values of the standard table should be graduated, the resulting quantities being added to the standard values. Since, as will

be seen from Mr. King's tables, the corrective quantities we are dealing with will, in the case of most formulas, when the function q is graduated, be extremely small, affecting the fifth decimal place of q at a small number of ages only, it will usually be desirable in following out this method to see that the standard values are accurately calculated to, at any rate, five decimal places. In the present instance, the graduation of q having already been completed directly in the usual way, another method of procedure was decided upon. The $O^{M(5)}$ values of q, accurately deduced from the values of colog p given in the "Account of Principles and Methods" (p. 153), were re-graduated by my formula, and the deviations of the re-graduated from the original values were tabulated. As the effect of graduation by reference to a given standard table will, provided the same formula be employed, be to produce precisely the same series of corrective quantities in the case of every ungraduated table, it is clearly possible to tabulate these quantities once and for all, and so obtain the means of rectifying any graduation that may have been performed in the usual way by operating on q_x. This alternative process will lead to results identical with those which would be reached by proceeding in the manner explained by Mr. Lidstone, as will be seen from the following very simple demonstration, which may be helpful to those students who do not find the proposition obvious at first sight.

If u be the ungraduated value, and u^s the standard value at any age; u' and $u^{s'}$ the corresponding graduated values, and u^r the corrected value, the series to be graduated on the lines suggested by Mr. Lidstone, will consist of values of $u-u^s$, the difference between the unadjusted and standard values. The graduated difference will, of course, be $u'-u^{s'}$, and we shall have—

$$\begin{aligned} u^r &= u^s + (u' - u^{s'}) \\ &= u' + (u^s - u^{s'}), \end{aligned}$$

which is in the form indicated by the alternative method of correction, the quantity $u^s - u^{s'}$ being quite independent of the unadjusted table which is the subject of graduation.

As the 21-term formula is, I believe, the most powerful hitherto employed in practice, and as it may sometimes be desired, not only to take advantage of the greater smoothness to which it leads, but at the same time to get rid as far as practicable of the minute error which it generally involves, I show

in Table 9, for ages 20 to 80, the amount by which the adjusted values of q, as ordinarily obtained, should be increased when the $O^{M(5)}$ Table is taken as a standard, in order to give effect to the principle of Mr. Lidstone's method.

TABLE 9.

Graduation by reference to the $O^{M(5)}$ Table.

Corrections to be applied to q_x when graduated by Spencer's 21-term formula; being the differences between the $O^{M(5)}$ Values of q_x based on the Makeham constants, and those arrived at by graduating the Makeham values by the 21-term formula.

Age	Correction	Age	Correction	Age	Correction
20	+·000001	40	+·000004	60	+·000016
1	1	1	4	1	17
2	1	2	5	2	18
3	1	3	5	3	19
4	1	4	6	4	20
25	1	45	6	65	21
6	1	6	6	6	23
7	1	7	7	7	24
8	2	8	7	8	25
9	2	9	7	9	26
30	2	50	8	70	27
1	2	1	9	1	29
2	2	2	9	2	29
3	2	3	·000010	3	28
4	2	4	11	4	28
35	2	55	11	75	26
6	2	6	12	6	24
7	2	7	13	7	21
8	3	8	14	8	18
9	3	9	15	9	13
				80	7

The values are seen to be exceedingly small, the correction only slightly affecting the fifth decimal place of q from about age 50 upwards. It is not worth while to alter the O^M values of q given above and in my former paper, but by multiplying each of the quantities in Table 9 by the appropriate "Exposed to Risk", we are able to trace the effect of the final adjustment on the expected deaths. The quantities referred to will be found to account for an increase of 58 deaths, so that the deficiency of 42 formerly observed is replaced by an excess of 16 deaths, about ·01 per-cent only of the number actually recorded. Had the corrections been worked out above age 81 we should, as clearly shown in Mr. King's paper, have arrived at a series of deductive quantities, increasing numerically to the end of life. I have not,

however, thought it practically useful to tabulate these, for two reasons. In the first place, having regard to the relatively small numbers exposed to risk at the concluding ages in most tables, it is clear that the corrections above age 81 would have little or no effect on the expected deaths—in a large table like the O^M they would have accounted for less than 2 deaths—and, secondly, it is not clear that the Makeham hypothesis faithfully represents the mortality at the final ages in most published tables, and if it does not these corrections will not necessarily be applicable. The assumption that Makeham's law holds at extreme old ages is, of course, amply justified by the undoubted practical advantages to which it leads, but regarded as a piece of graduation pure and simple it can in my view rarely be said that at these ages a Makeham curve represents the observed facts with very great fidelity. It is earlier in the table—say from age 40 to 75—that Makeham's law is, as a rule, most closely followed, and it is here that, owing to the distribution of the "Exposed to Risk", the very slight deviation in the expected deaths to which most summation formulas give rise when applied to q_x will usually be situated.

Another method of correcting the values resulting from the first graduation would be to apply the formula a second time, afterwards adding to each value in the first graduated series the difference between that quantity and the corresponding value in the second series. In the expression given above, u' would take the place of u^s as the standard, $u^{s\prime}$ would become u'', the value resulting from the second graduation, and we should have

$$\begin{aligned} u^r &= u' + (u' - u'') \\ &= 2u' - u''. \end{aligned}$$

This is the method suggested by Mr. J. A. Higham when he remarks—" I graduate first the data, then the differences between "the data and my results, and when the two are combined the "work has been done correctly to the seventh order of differences" (*J.I.A.*, xxxi, 320). As compared with the effect of graduating by reference to a Makeham standard, recourse to this alternative method, which is the same thing as graduation by reference to the first adjustment, might be expected to lead to still greater accuracy in the sense of closer conformity to the observed facts, but from this point of view the only perfectly accurate values are the ungraduated data themselves, and a little consideration will

show that in applying a correction on these lines we should be undoing the work performed by the graduation formula, the smoothness of the results being to a certain extent impaired. Mr. Higham showed that by following the method indicated and graduating the values of $(l_x - l'_x)$ which Dr. Sprague had brought out by applying Woolhouse's formula to re-graduate the H^{M} (*Text-Book*) Table (*J.I.A.*, xxix, 235), the corrections arrived at corresponded closely, though they were not quite identical, with those necessary to cure the distortion originally produced, and he further pointed out that by repeating the process the work could be carried to the utmost degree of exactness. That would clearly have been the case in the particular example taken, a perfectly smooth series founded on Makeham's law, but, in practice, where one is, of course, concerned with the application of formulas of summation to graduate irregular data, different considerations would arise as each successive correction would, while affording greater accuracy, involve further departure from the first graduated series, with corresponding sacrifice of smoothness, so that eventually we should be brought back to irregularities not less pronounced than those exhibited by the original values.

In the notation employed above, the second corrected value would be

$$\begin{aligned} u^{r_2} &= u^r + (u' - u^{r\prime}) \\ &= 2u' - u'' + u' - 2u'' + u''' \\ &= 3u' - 3u'' + u''' \end{aligned}$$

Similarly the third correction would be

$$u^{r_3} = 4u' - 6u'' + 4u''' - u'''', \text{ and so on.}$$

It will follow from the above remarks that if orderly progression in the finally adjusted values be regarded as of paramount importance, it will be preferable, in applying the principle of graduation by reference to another table, to select as a standard a perfectly smooth table such as is afforded when Makeham's law applies, since any impairment of the curve as regards smoothness would thus be avoided, as will be seen from the values given in Table 9.

It is evidently an adjustment by reference to the first graduation that Mr. King has in mind when he says: "If, "however, anyone so desired it was easy to eliminate practically

" all the error. When a table had been graduated by one " of the formulas one had merely to difference centrally four " times and multiply the fourth central difference by the proper " factor, which is 5·4 for Woolhouse's formula, and add that to " the original result, and the table was obtained graduated " correctly to the sixth differential coefficient" (*J.I.A.*, xli, 96). I may, however, point out that the effect of applying the suggested fourth difference correction merely, instead of taking into account higher orders of difference also, which is what is done automatically when Mr. Higham's first correction is applied, would be to introduce great irregularities into the graduated results, and in consequence one would to a considerable extent be deprived of the advantages of graduation.

I show in Table 10, for ages 50-69, of the Government Female Annuitants' Ultimate Table, the result of applying some of the corrections here dealt with, which may serve to illustrate the above remarks.

TABLE 10.

Government Female Annuitants (1883) *Ultimate Table.*
Graduations of q by Spencer's 21-*term Formula.*

AGES 50–69.

x	(1) First Graduation		(2) Second Graduation		(3) Graduation by reference to $O^{M(5)}$ Table		(4) Graduation by reference to First Graduation		5) First Graduation amended as suggested by Mr. King	
	q_x	Δq_x	q_x	Δq_x	q_x	Δq_x	q_x	Δq_x	q_x	Δq_x
50	·01278	·00104	·01297	·00085	·01278	·00105	·01259	·00123	·01291	·00116
1	1382	112	1382	95	1383	112	1382	129	1407	100
2	1494	111	1477	98	1495	111	1511	124	1507	136
3	1605	102	1575	95	1606	102	1635	109	1643	152
4	1707	88	1670	90	1708	88	1744	86	1795	38
55	1795	76	1760	84	1796	76	1830	68	1833	101
6	1871	69	1844	82	1872	69	1898	56	1934	− 19
7	1940	72	1926	86	1941	72	1954	58	1915	135
8	2012	83	2012	97	2013	83	2012	69	2050	− 5
9	2095	105	2109	114	2096	105	2081	96	2045	117
60	2200	134	2223	135	2201	135	2177	133	2162	96
1	2334	167	2358	159	2336	166	2310	175	2258	218
2	2501	198	2517	183	2502	199	2485	213	2476	185
3	2699	225	2700	207	2701	225	2698	243	2661	250
4	2924	245	2907	230	2926	245	2941	260	2911	384
65	3169	257	3137	255	3171	258	3201	259	3295	232
6	3426	271	3392	282	3429	270	3460	260	3527	246
7	3697	295	3674	317	3699	295	3720	273	3773	232
8	3992	335	3991	358	3994	336	3993	312	4005	221
9	4327	...	4349	...	4330	...	4305	...	4226	...

It is interesting to note the improvement as regards smoothness yielded by the second graduation. The sum at any age of the values of either q_x or Δq_x in columns 2 and 4 is, of course, equal to twice the corresponding value in column 1. The values of q_x in column 5 are obtained, in the way indicated by Mr. King, from those in column 1, the fourth central differences of the latter having been multiplied by 12·6 the appropriate coefficient, and the results added to the corresponding column 1 values. The results bear out what was said above as to the question of smoothness, which is the point upon which stress is here laid. Had Woolhouse's formula, which is mentioned in Mr. King's remarks, been taken, the values in column 5 would have been much more unsatisfactory, as although the coefficient of $-\Delta^4$ in that formula is only 5·4, as against 12·6 in the 21-term formula, the average magnitude of the differences of q_x resulting from the Woolhouse graduation is many times greater, and these larger differences multiplied by 5·4 would be grafted on a much less smooth series of graduated values. Still greater irregularities would have been arrived at by dealing with the earlier ages in the particular table, but in order to avoid exaggerating the effect of applying the suggested fourth-difference adjustment I have purposely dealt with a section of the table where the fourth differences of q are small relatively to the values of q themselves.

III.

An investigation of the smoothing power of certain formulas; and of the variations in q_x to which they give rise.

In the excellent discussion which followed the reading of Mr. King's paper several speakers expressed the opinion that the most important question in considering the merits of a graduation formula is that of the smoothness which it brings out in the adjusted data. I venture to offer a few notes, which it is hoped may be found helpful to those interested in this subject.

Apart from actual trial, which must always be the best test in comparing the smoothing power of different formulas, methods of comparison have been suggested dependent upon the progression of the coefficients which enter into the graduation formula. Mr. G. F. Hardy says (*J.I.A.*, xxxii, p. 376): "It is convenient to

" have some test (other than that of actual trial) of the smoothness " of the curve that will be brought out by any given formula, " though any test must of course be more or less arbitrary, " and can only be relative. It has been assumed throughout that " fourth differences are zero, and in general, if all accidental " irregularities were effectually removed, the third (or even the " second) differences of such functions as q_x or d_x should be small. " If we assume that each of the ungraduated values upon which " our graduated results are based is affected by a similar probable " error, we shall then be able to express the probable error in " the graduated values, and in the successive orders of " differences of both the ungraduated and graduated curves." On the assumptions made Mr. Hardy showed that in the case of Woolhouse's formula the accidental errors in the original values and their third differences would be respectively reduced to $\frac{53}{125}$ths and $\frac{1}{15}$th of their original amount, and he also gave figures relating to some other formulas. In his paper read before the London Congress, Dr. Karup says, (*Transactions of the Second International Congress of Actuaries*, pp. 93 and 929) " that " formula will work most smoothly in which the coefficients " . . . form a curve which is very little distorted, and which " in its ascending and descending part very nearly resembles a " straight line, and runs as evenly and smoothly as possible " through those points where a turn is unavoidable, because as " the calculation proceeds each single observation receives in " succession all the coefficients as factor, and thus an irregularity " is less likely to be reproduced in the adjusted numbers the " more gradually it enters into the calculation and the more " evenly it is distributed over a considerable space." Applying these principles Dr. Karup went on to show that the 19-term formula deduced by him fulfilled the required conditions more closely than Higham's simpler 17-term formula, both of these showing a very great improvement over Woolhouse's formula. I propose now to touch upon some characteristics of various well-known formulas, to show how these emerge from the tests described above, and subsequently to investigate how far the conclusions to which we are led are confirmed when we come to deal with graduated results arrived at by applying the formulas in question.

TABLE 11.

Some well-known Graduation Formulas.

	Coefficents						
	(1) J. Spencer (*a*)	(2) Woolhouse	(3) J. A. Higham (*a*)	(4) J. A. Higham (*b*)	(5) G. F. Hardy	(6) Karup	(7) J. Spencer (*b*)
u_0	·2312	·200	·2106	·200	·2000	·2000	·1714
$u_{\pm 1}$	·2094	·192	·1920	·192	·1833	·1824	·1629
$u_{\pm 2}$	·1438	·168	·1419	·144	·1416	·1392	·1343
$u_{\pm 3}$	·0656	·056	·0794	·080	·0833	·0848	·0943
$u_{\pm 4}$	·0094	·024	·0240	·024	·0333	·0336	·0514
$u_{\pm 5}$	−·0156	·000	−·0053	·000	·0000	·0000	·0171
$u_{\pm 6}$	−·0188	−·016	−·0160	−·016	−·0166	−·0128	−·0057
$u_{\pm 7}$	−·0094	−·024	−·0144	−·016	−·0166	−·0144	−·0143
$u_{\pm 8}$	...	...	−·0069	−·008	−·0083	−·0096	−·0143
$u_{\pm 9}$	...	...	...	...	...	−·0032	−·0085
$u_{\pm 10}$	...	...	...	...	...	...	−·0029

Table 11 gives the formulas which will be here noticed. Formula (1) was employed in the graduation of the bulk of the sickness rates brought out by the Manchester Unity Experience 1893–1897 (*vide J.I.A.*, xxxviii, p. 335). No. (2) is Woolhouse's well-known formula. No. (3) is Mr. J. A. Higham's first 17-term formula resulting from the processes

$$\frac{1}{1875}\{12[5]^4-25[5]^2[9]\}u_0,$$

and recommended by him on account of its smoothing power. No. (4) is Mr. J. A. Higham's simpler and better-known formula represented by the operations

$$\frac{[5]^3}{125}(-u_{-2}+u_{-1}+u_0+u_1-u_2).$$

No. (5) is Mr. G. F. Hardy's Friendly Society formula. No. (6) is the 19-term formula suggested by Dr. Karup (*Transactions of the Second International Congress of Actuaries,* p. 92, formula 5), namely,

$$u'_0=\frac{[5]^3}{625}\{-2u_{-3}+3u_{-1}+3u_0+3u_1-2u_3\}$$

No. (7) is the 21-term formula more fully referred to in the first section of this paper, and needs no further introduction.

In passing, the following shorter method of applying formula (1) may be noticed:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Ungraduated Rate ÷ 10 $= u$	Sum in 3's	u_{-2} + u_{+2}	(2) − (3)	$\frac{1}{3}u$	(4) + (5)	Sum in 5's	Sum in 4's	Sum in 4's $= 10\frac{2}{3}u$	log(9)	(10) − log $10\frac{2}{3}$ $= \log q$

Mr. J. A. Higham's smoother formula (No. 3 of Table 11) is more complex than his other 17-term formula, and is not nearly so easy to apply. The following alternative method of utilizing the formula may be compared with the process indicated in Mr. Higham's paper (*J.I.A.*, xxv, 19).

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Ungraduated Rate $= u$	Sum in 3's	Sum of u in 5's	Sum of u in 9's	(4) × $\frac{1}{12}$	u_{-4} + u_{+4}	(1) + (2) + (3)	(5) + (6)	(7) − (8)	Sum in 5's	Sum in 5's $= 156\frac{1}{4}u$	log (11)	log (11) − log $156\frac{1}{4}$ $= \log q$

The above formulas are classified in Table 12, in which are given certain data which may be useful in comparing the formulas and in studying their peculiar qualities. (*See* Table 12, next page.)

The values in column 7 are intended to facilitate a comparison on the lines suggested by Dr. Karup. Obviously the smoother the coefficient curve the smaller will be the irregularities in the third differences, and the sum of these, irrespective of sign, may be employed as a simple and convenient measure of the extent of these irregularities. Column 8 gives what I have termed the smoothing coefficients, or the ratios in which, on the assumptions made, errors in the third differences of the graduated values are reduced by the application of the formula. These quantities have been calculated in the manner explained by Mr. Hardy, namely, by tabulating the third differences of the coefficients of each formula, and taking the ratio of the square root of the sum of the squares of these quantities to $\sqrt{20}$, which is the square root of the sum of the squares of 1, −3, 3, −1, the third differences of unity, the coefficient of the ungraduated value (*J.I.A.*, xxxii, p. 376). As has been pointed out by Mr. Todhunter (*J.I.A.*, xli, p. 92), it does not appear to follow from Mr. Hardy's remarks that he regards the probable error theory

TABLE 12.

Classification of the Formulas in Table 11

No.	(1) Author	(2) Summations	(3) Values summed, expressed in terms of u_0 and its Second Differences	(4) Number of terms in formula	(5) Fourth Difference Error. Coefficient of $-\frac{d^4}{dx^4}u_0$	(6) Sum of Coefficients of 5 central terms	(7) Sum of Third Differences, irrespective of sign, of Coefficients entering into formu'a	(8) Smoothing Coefficients
1	J. Spencer (*a*) .	$\frac{[5][4]^2}{320}$	$4u_0-3(b_{\ 1}+b_0+b_1)$	15	3 86	·938	·250	$\frac{1}{60}$
2	Woolhouse . .	$\frac{[5]^3}{125}$	u_0-3b_0	15	5·40	·920	·832	$\frac{1}{15}$
3	J. A. Higham (*a*)	*See* p. 388		17	6·00	·878	·207	$\frac{1}{90}$
4	Do. (*b*)	$\frac{[5]^3}{125}$	$u_0-(b_{-1}+b_0+b_1)$	17	6·40	·872	·320	$\frac{1}{56}$
5	G. F. Hardy . .	$\frac{[4][5][6]}{120}$	$u_0-(b_{-1}+b_0+b_1)$	17	$\left\{\begin{array}{c}6{\cdot}50\\ +\frac{1}{12}\frac{d^2}{dx^2}u_0\end{array}\right\}$	·850	·167	$\frac{1}{95}$
6	Karup	$\frac{[5]^3}{625}$	$5u_0-(2b_{-2}+4b_{-1}+3b_0+4b_1+2b_2)$	19	7·80	·843	·166	$\frac{1}{105}$
7	J. Spencer (*b*) .	$\frac{[7][5]^2}{350}$	$2u_0-(b_{-2}+2b_{-1}+2b_0+2b_1+b_2)$	21	12·60	·766	·115	$\frac{1}{160}$

as rigidly applicable to mortality tables, and the assumption which has been made in deducing these ratios, namely, that each ungraduated value is affected by a similar probable error, may possibly be open to criticism. It is to be observed, however, that Mr. Hardy suggests the test described by him as a relative one only, and from this point of view it seems to me important to avoid the assumption that, because the probable error theory may not strictly apply, the method proposed is of no value, as any exceptional distribution of errors which gave rise in the case of a particular formula to a modification of the "reduction of probable error" in the differences of, say, q, might possibly lead to a more or less proportionate change in the corresponding ratios or smoothing coefficients appertaining to other formulas. It is true that the question is to some extent affected by differences of range in the various formulas, but these are not I think likely to affect the results appreciably, except possibly at the ends of the table, where the observed numbers are as a rule very small, and where in consequence the average deviation of the unadjusted from the true values may be relatively large. It may therefore be that, if we had the means in any given case of accurately determining the probable errors affecting the ungraduated data, the ratios of the quantities in column 8 of Table 12 to each other would not be very sensibly modified, and, as this is the only way in which it is necessary for us to use the values, an investigation of the subject from this special point of view is, I think, essential in considering the validity of the suggested test.

In studying the formulas in Table 11, and the figures given in Table 12, it is important to observe the distinction between the capacity of a formula to remove or reduce the larger irregularities in the rough data—the "waves" as distinct from the "ripples"—thus producing a curve free from minor points of inflexion, and its power to bring out a curve which, while giving expression to the "waves", passes smoothly through the points where changes of curvature take place. Generally speaking, the former quality varies with the range of the formula, and is accompanied by loss of theoretical error. Column 6 of Table 12, which shows the sum of the coefficients of the five central terms of each formula, affords a convenient means of observing the extent to which the flattening-out of the curve is carried by the different formulas. The ability of a formula to produce a series of values whose third differences are relatively small depends on the shapeliness of the coefficient curve. This quality, which may

exist to a considerable extent in a formula embracing a comparatively small number of terms, is reflected in columns 7 and 8, which roughly confirm each other as regards the relative smoothness of the formulas here dealt with. The figures given throw into relief the pronounced inferiority of Woolhouse's formula. A comparison of this with formula (1), which extends to the same number of terms, suggests that it may be possible to obtain smoothness without sacrificing conformity to the observed facts. The fourth difference error of the latter is less than three-fourths of that shown by Woolhouse's formula, which would lead us to expect a closer correspondence to the rough facts, while its smoothing coefficient would appear to indicate much steadier progression in the third differences of the graduated values.

TABLE 13.

Government Female Annuitants' (1883) *Ultimate Table.*

*Ungraduated Values of q_x and Graduated Values according to various Formulas.**

Age	Ungraduated Value	(1) Spencer (*a*)	(2) Woolhouse	(3) Higham (*a*)	(4) Higham (*b*)	(5) G. F. Hardy	(6) Karup	(7) Spencer (*b*)
35	·01567	·01161	·01124	·01132	·01133	·01112	·01112	·01070
6	1389	1166	1146	1146	1136	1138	1141	1115
7	721	1146	1135	1150	1152	1154	1148	1150
8	1268	1127	1179	1153	1155	1167	1160	1174
9	350	1149	1144	1169	1178	1174	1179	1198
40	1991	1201	1220	1202	1198	1203	1206	1212
1	1228	1258	1223	1242	1243	1234	1236	1226
2	1441	1280	1283	1264	1256	1258	1255	1231
3	956	1269	1245	1260	1260	1251	1248	1225
4	1094	1228	1245	1221	1218	1220	1214	1198
45	1358	1171	1152	1167	1167	1167	1165	1162
6	1483	1114	1112	1115	1114	1116	1121	1129
7	824	1077	1094	1088	1095	1094	1096	1120
8	795	1082	1102	1096	1102	1107	1110	1139
9	1328	1136	1137	1157	1157	1170	1169	1193
50	1019	1253	1264	1261	1265	1268	1267	1278
1	1550	1399	1410	1393	1395	1391	1388	1382
2	1611	1541	1512	1519	1515	1510	1513	1494
3	1753	1652	1624	1634	1628	1626	1626	1605
4	1772	1734	1744	1729	1731	1727	1721	1707
55	1548	1798	1812	1803	1805	1806	1801	1795

We shall be able to compare the formulas and their characteristics most effectually by applying them to the actual

* In deducing the graduated values of q_x given in Table 13, the unadjusted rates of mortality were employed throughout, without grouping of any kind. On this account, the values of q_{35} and q_{36}, according to formula 7, differ from the corresponding rates of Table 1.

work of graduation, and accordingly I give in Table 13 the results obtained by graduating q_x for ages 35 to 55 in the case of the Government Female Annuitants' (1883) Ultimate Table. This section of the table has been selected for the reason that the observed facts are not only extremely irregular, but exhibit peculiarities which render it difficult, in an examination of the rough data, to determine what is the true law to which they conform. In this connection reference may be made to Dr. Sprague's very exhaustive scrutiny of the same data in his paper on the graphic method (*J.I.A.*, xxvi, pp. 85–90).

It will be seen that all the formulas agree in placing a minimum point at age 47, and, with one exception, a maximum point at age 42. When, however, we come to examine the values for ages 35 to 42, we observe striking differences between the various curves as regards smoothness. The Woolhouse graduation shows a very jagged line at these ages, already referred to, with maxima at ages 36 and 38, and the corresponding minima at ages 37 and 39. Formula (1) gives a distinct improvement on this, with a maximum at age 36, a minimum at age 38, and on the whole a much more wavy line. The other formulas all get rid of these subsidiary points of inflexion, the curves however, varying considerably in character. The steadiness with which the first differences progress becomes more marked when we arrive at the values in the last three columns, formula (7) in this respect showing a distinct superiority to the others. The more potent effect of the longer range formulas on the larger irregularities existing in the rough facts may be well observed by examining the maximum and minimum points at ages 42 and 47 respectively. At the former age the values vary from ·01283 in the case of the Woolhouse graduation to ·01231 according to formula (7), and if we compare in the case of each formula the sum of the values for ages 40 to 44, namely,

(1)	(2)	(3)	(4)	(5)	(6)	(7)
·06236	·06216	·06189	·06175	·06166	·06159	·06092

we obtain an idea of the greater extent to which the more powerful formulas compress the curve at this point. It will be seen that the values vary just as one would have assumed from an examination of the figures given in column 6 of Table 12, the rates brought out by the shorter range formulas being made up of a greater proportion of the values at and about the

maximum points. A similar effect is noticed when we consider the minimum point at age 47. Here the rate of mortality varies from ·01077 in the case of formula (1) to ·01120 in that of formula (7), while the sums of the values for ages 45 to 49 are respectively

(1)	(2)	(3)	(4)	(5)	(6)	(7)
·05580	·05597	·05623	·05635	·05654	·05661	·05743.

The relatively small central coefficients in the longer range formulas is evidenced here by an increase in the values at and adjacent to the minimum point, and we again obtain a result conforming strictly to what column 6 of Table 12 would have led us to expect.

Following the plan adopted in a preceding section we may more easily compare the smoothness of the curves by examining the third differences of the graduated values, and accordingly these are shown in Table 14.

TABLE 14.

Government Female Annuitants' (1883) *Ultimate Table.*

Third Differences of Ungraduated Values of q and of the Graduated Values given in Table 13.

Age	Ungraduated Values	(1) Spencer (*a*)	(2) Woolhouse	(3) J. A. Higham (*a*)	(4) J. A. Higham (*b*)	(5) G. F. Hardy	(6) Karup	(7) Spencer (*b*)
35	+ ·01705	+ ·00026	+ ·00088	+ ·00009	− ·00026	+ ·00007	+ ·00027	− ·00001
6	− ·02680	+ 40	− 134	+ 14	+ 33	− 3	+ 2	+ 11
7	+ ·04024	− 11	+ 190	+ 4	− 23	+ 28	+ 1	− 10
8	− ·04963	− 25	− 184	− 10	+ 28	− 20	− 5	+ 10
9	+ ·03380	− 40	+ 130	− 25	− 57	− 9	− 14	− 9
40	− ·01674	+ 2	− 155	− 8	+ 23	− 24	− 15	− 2
1	+ ·01321	+ 3	+ 136	− 9	− 37	+ 7	− 1	− 10
2	− ·00497	+ 14	− 131	+ 20	+ 37	+ 2	+ 12	+ 12
3	− ·00265	+ 16	+ 146	+ 17	+ 7	+ 24	+ 20	+ 12
4	− ·00645	+ 20	− 31	+ 23	+ 36	+ 27	+ 14	+ 21
45	+ ·01414	+ 22	+ 4	+ 10	− 8	+ 6	+ 20	+ 4
6	− ·00068	+ 7	+ 1	+ 18	+ 22	+ 15	+ 6	+ 7
7	− ·01404	+ 14	+ 65	− 10	+ 5	− 15	− 6	− 4
8	+ ·01682	− 34	− 73	− 15	− 31	− 10	− 16	− 12
9	− ·01310	− 33	− 63	− 34	− 32	− 29	− 19	− 11
50	+ ·00551	− 27	+ 54	− 5	+ 3	+ 1	− 16	− 9
1	− ·00204	+ 2	− 2	− 9	− 3	− 12	− 6	− 8
2	− ·00120	+ 11	− 60	− 1	− 19	− 7	+ 3	− 5
	Totals .	± ·00347	± ·01647	± ·00241	± ·00430	± ·00246	± ·00203	± ·00153

Looking at the sums of the differences, irrespective of sign, we see that for the most part these range themselves in the same way as the smoothing coefficients given in column 8 of Table 12, the magnitude of the values varying from $\pm$·01647, in the case of Woolhouse's formula, to less than 10 per-cent of that amount, $\pm$·00158, in the case of the 21-term formula, and it is interesting to note that these quantities show a tendency to vary among themselves much in the same way as the corresponding ratios in column 8 of Table 12. If we take Woolhouse's value as a standard, and alter the others in proportion to the ratios just mentioned, we should have

(1)	(2)	(3)	(4)	(5)	(6)	(7)
·00412	·01647	·00274	·00441	·00260	·00235	·00154

against which we may put the values actually obtained, namely,

·00347 ·01647 ·00241 ·00430 ·00246 ·00203 ·00158.

In making the above comparison we have taken a fairly large group of ages, and in this connection it may be pointed out that equally good results can hardly be expected to arise from an examination of individual values, or even from small age-groups. Indeed, it is not to be expected that a similarly close correspondence would necessarily be arrived at in every case, even if large groups were taken. There would perhaps be less likelihood of this in a table where the observed facts were already fairly regular, and free from the waves which are to be found in the Government Female Annuitants' Table, and accidental circumstances might easily give rise to a departure from what may be called the expected result, in the case of a particular formula. But that the correspondence brought out is not wholly accidental may be seen by taking another example. In his paper referred to above, Dr. Karup has given values of q_x in the case of the $H^{M(5)}$ Table as graduated by his own 19-term formula, and by Mr. J. A. Higham's simpler formula, and I reproduce the values for ages 30 to 75 in Table 15, and give in addition the rates resulting from the application of my 21-term formula. As a matter of interest I have also tabulated Dr. Sprague's graphic rates, and the values deduced by Dr. Karup according to his "New Mechanical Method", which consists of a preliminary graduation by Higham's simpler 17-term formula, a rectification of these values so as to bring the expected deaths into closer conformity to the actual, and a final graduation of log q, as thus corrected, by the 19-term formula.

Table 15.

$H^{M(5)}$ *Table.*

Graduated Values of q_x and their Third Differences.

Age	J. A. Higham's simpler formula		Karup's 19-term formula		Spencer's 21-term formula		Karup's Double Graduation		Sprague's Graphic Method	
	q_x	$\Delta^3 q_x$	q_x	$\Delta^3 q_x$	q_x	$\Delta^3 q_x$	q_x	$\Delta^3 q_x$	q_x	$\Delta^3 q_x$
30	·00923	·00002	·00927	·00000	·00923	−·00001	·00926	−·00001	·00910	·00010
1	919	15	917	3	920	− 3	917	0	880	− 10
2	919	− 4	918	− 2	924	0	920	− 3	870	0
3	925	− 20	930	− 9	939	− 4	934	− 5	890	− 20
4	952	− 2	956	− 7	962	− 7	959	− 4	930	− 10
35	996	− 2	994	− 6	993	− 2	992	− 6	990	− 10
6	·01037	− 3	·01035	1	·01028	0	·01028	1	·01050	0
7	1073	− 2	1072	3	1060	1	1063	0	1100	15
8	1102	16	1099	5	1087	2	1091	5	1130	5
9	1121	3	1117	8	1109	7	1113	6	1140	5
40	1128	3	1129	4	1127	4	1129	5	1145	0
1	1139	5	1140	5	1143	6	1144	3	1150	5
2	1157	4	1158	− 2	1164	− 2	1164	0	1160	15
3	1185	− 7	1187	− 1	1194	− 1	1194	0	1175	− 5
4	1228	− 7	1232	− 3	1239	− 4	1237	− 3	1200	0
45	1290	6	1291	− 5	1297	− 3	1293	− 3	1250	− 10
6	1364	− 11	1363	− 5	1367	− 3	1362	− 5	1320	− 10
7	1443	− 6	1445	− 1	1445	1	1441	0	1410	0
8	1533	5	1532	2	1528	0	1527	0	1510	0
9	1623	12	1619	3	1613	1	1615	0	1610	0
50	1707	− 6	1705	5	1701	2	1705	4	1710	0
1	1790	6	1792	3	1792	1	1797	2	1810	10
2	1884	3	1883	− 3	1887	3	1891	3	1910	10
3	1983	− 3	1983	3	1988	4	1991	2	2010	− 10
4	2093	− 3	2095	2	2096	3	2099	2	2120	0
55	2217	10	2216	2	2214	3	2218	3	2250	0
6	2352	9	2349	6	2346	− 1	2350	4	2390	10
7	2495	− 4	2496	5	2495	5	2497	3	2540	0
8	2656	4	2659	1	2664	4	2662	2	2700	0
9	2844	4	2844	1	2852	− 5	2849	− 4	2880	0
60	3055	− 5	3056	− 10	3064	− 5	3061	− 2	3080	0
1	3293	− 21	3296	− 8	3304	− 11	3300	− 9	3300	0
2	3562	− 1	3565	− 7	3567	− 4	3562	− 6	3540	0
3	3857	− 4	3853	− 6	3848	3	3845	3	3800	10
4	4157	4	4152	7	4136	19	4140	6	4080	0
65	4461	6	4455	28	4427	18	4441	19	4380	0
6	4765	73	4756	31	4724	28	4751	21	4710	10
7	5073	12	5062	35	5046	9	5076	22	5070	0
8	5391	32	5401	11	5411	2	5435	10	5460	20
9	5792	− 47	5804	− 17	5847	− 17	5849	− 1	5890	0
70	6288	5	6306	− 38	6363	− 11	6340	− 16	6360	10
1	6911	− 97	6918	− 38	6961	− 17	6918	− 22	6890	0
2	7614	8	7623	− 18	7624	5	7582	− 18	7480	− 10
3	8402	...	8383	...	8341	...	8316	...	8140	...
4	9178	...	9160	...	9095	...	9098	...	8870	...
75	9950	...	9936	...	9891	...	9910	...	9660	...
	Totals	±·00502	...	±·00360	...	±·00232	...	±·00234	...	±·00230

Dealing with the figures in the first three columns of Table 15, we see that as regards the order of smoothness of the different formulas the results confirm the figures previously given, and although, relatively to the others, the Higham graduation is in this instance slightly less unfavourable than we should have expected, the general tendency of the figures is again confirmatory of the result previously arrived at, and shows that considerable reliance may be placed on the smoothing coefficients. It is interesting to note that as regards smoothness the 21-term formula graduation compares very well with that resulting from the combination of the Karup and Higham formulas, although as might be expected, the fidelity of Dr. Karup's graduated values to the rough facts is somewhat closer. The graphic rates up to nearly age 50 are demonstrably less smooth than the corresponding values as brought out by Dr. Karup's processes or my own formula, though from age 50 to 70, Dr. Sprague has done an excellent piece of work.

The student who desires to investigate the question of smoothing power further will find more material in Table 16, from which a final illustration may be taken. The sums of the values of $\Delta^3 q_x$, irrespective of sign, in the G.F.A. Table, for ages 60 to 80, according to certain of the formulas referred to above, are

(1)	Spencer's (15-term) formula		·00488.
(2)	Woolhouse's	do.	·02115.
(3)	J. A. Higham's	do. (*b*)	·00604.
(4)	G. F. Hardy's	do.	·00344.
(5)	Spencer's (21-term)	do.	·00232.

These results again reflect very fairly the ratios indicated by the smoothing coefficients.

There are minor inconsistencies in the comparative figures given above which may be noticed. Comparing, for instance, the totals in Table 14, according to formulas (1) and (4), it will be seen that, notwithstanding the similarity of their smoothing coefficients, the result brought out by the former is decidedly superior, not only as to the magnitude of the third differences of q, but also as regards the steadiness of their progression. Again, in the same table, formula (3) gives a slightly better result than formula (5).

The conclusion suggested, which is supported by figures obtained experimentally in other cases, is that in comparing different formulas having similar smoothing coefficients the best results as regards the magnitude and smoothness of the third

differences of the ungraduated values may usually be looked for in the case of the formula embracing the smallest number of terms, or, speaking generally, the formula whose central terms have the largest coefficients. This feature must not be lost sight of in instituting a comparison, but, on the whole, I submit that the figures tabulated above show that the test suggested by Mr. Hardy is one of undoubted practical value, and that, if used with discrimination, it affords a simple means of comparing the smoothing power of summation formulas without actually performing the work of graduation.

TABLE 16.

Government Female Annuitants' (1883) *Ultimate Table.*
Graduated Values of q for ages 55–90 brought out by specified Formulas.

Age	Spencer's 15-term formula (*a*)	Woolhouse	J. A. Higham's simpler formula (*b*)	G. F. Hardy	Spencer's 21-term formula (*b*)
55	·01798	·01812	·01805	·01806	·01795
6	·01863	·01852	·01866	·01873	·01871
7	·01943	·01941	·01942	·01941	·01940
8	·02026	·02040	·02022	·02018	·02012
9	·02107	·02088	·02098	·02098	·02095
60	·02196	·02185	·02191	·02196	·02200
1	·02308	·02325	·02320	·02322	·02334
2	·02457	·02475	·02478	·02481	·02501
3	·02661	·02664	·02675	·02684	·02699
4	·02917	·02921	·02921	·02926	·02924
65	·03204	·03203	·03195	·03194	·03169
6	·03496	·03478	·03469	·03465	·03426
7	·03771	·03742	·03735	·03733	·03697
8	·04028	·04009	·04010	·04008	·03992
9	·04299	·04314	·04309	·04314	·04327
70	·04633	·04656	·04670	·04681	·04719
1	·05078	·05110	·05119	·05137	·05177
2	·05647	·05664	·05675	·05686	·05706
3	·06314	·06318	·06306	·06308	·06300
4	·07023	·06986	·06991	·06985	·06955
75	·07736	·07714	·07694	·07691	·07654
6	·08440	·08419	·08430	·08428	·08393
7	·09173	·09204	·09177	·09188	·09160
8	·09966	·09953	·09979	·09987	·09977
9	·10838	·10854	·10827	·10840	·10851
80	·11788	·11750	·11803	·11797	·11806
1	·12839	·12908	·12840	·12853	·12845
2	·13986	·13938	·13991	·13999	·14011
3	·15234	·15245	·15225	·15256	·15294
4	·16620	·16603	·16683	·16666	·16707
85	·18155	·18321	·18182	·18205	·18216
6	·19806	·19672	·19807	·19822	·19832
7	·21541	·21545	·21514	·21544	·21506
8	·23354	·23376	·23351	·23292	·23246
9	·25106	·25147	·25035	·25048	·24968
90	·26825	·26586	·26796	·26810	·26641

TABLE 17.

Government Female Annuitants' (1883) Ultimate Table.

Expected Deaths according to Graduations by specified Formulas and Deviations of these from the Actual Deaths.

Age	SPENCER'S 15-term formula (*a*)		WOOLHOUSE		J. A. HIGHAM (*b*)		G. F. HARDY		SPENCER'S 21-term formula (*b*)	
	Expected Deaths	Deviation	Expected Deaths	Deviation	Expected Deaths	Deviation	Expected Deaths	Deviation	Expected Deaths	Deviation
55	66·2	9·2	66·7	9·7	66·5	9·5	66·5	9·5	66·1	9·1
6	76·5	− 6·5	76·0	− 7·0	76·6	− 6·4	76·9	− 6·1	76·8	− 6·2
7	86·9	·9	86·8	·8	86·9	·9	86·8	·8	86·8	·8
8	99·0	9·0	99·7	9·7	98·8	8·8	98·6	8·6	98·3	8·3
9	111·3	−11·7	110·3	−12·7	110·8	−12·2	110·8	−12·2	110·6	−12·4
60	123·9	−14·1	123·3	−14·7	123·7	−14·3	123·9	−14·1	124·2	−13·8
1	139·4	23·4	140·4	24·4	140·1	24·1	140·2	24·2	140·9	24·9
2	157·8	·8	159·0	2·0	159·1	2·1	159·3	2·3	160·6	3·6
3	179·9	− 2·1	180·1	− 1·9	180·9	− 1·1	181·5	− ·5	182·5	·5
4	211·4	2·4	211·7	2·7	211·7	2·7	212·0	3·0	211·9	2·9
65	243·5	−14·5	243·4	−14·6	242·8	−15·2	242·7	−15·3	240·8	−17·2
6	274·9	20·9	273·5	19·5	272·8	18·8	272·4	18·4	269·4	15·4
7	304·0	−13·0	301·6	−15·4	301·1	−15·9	300·9	−16·1	298·0	−19·0
8	330·2	−22·8	328·6	−24·4	328·7	−24·3	328·5	−24·5	327·2	−25·8
9	357·1	14·1	358·4	15·4	358·0	15·0	358·4	15·4	359·4	16·4
70	387·9	− 8·1	389·8	− 6·2	391·0	− 5·0	391·9	− 4·1	395·1	− ·9
1	421·1	20·1	423·7	22·7	424·5	23·5	426·0	25·0	429·3	28·3
2	460·6	28·6	462·0	30·0	462·9	30·9	463·8	31·8	465·4	33·4
3	499·1	−18·9	499·4	−18·6	498·5	−19·5	498·6	−19·4	498·0	−20·0
4	532·9	·9	530·1	− 1·9	530·5	− 1·5	530·0	− 2·0	527·8	− 4·2
75	558·1	−33·9	556·5	−35·5	555·0	−37·0	554·8	−37·2	552·2	−39·8
6	566·6	− ·4	565·2	− 1·8	565·9	− 1·1	565·8	− 1·2	563·4	− 3·6
7	569·2	21·2	571·1	23·1	569·4	21·4	570·1	22·1	568·4	20·4
8	568·5	3·5	567·7	2·7	569·2	4·2	569·7	4·7	569·1	4·1
9	563·1	1·1	564·0	2·0	562·6	·6	562·4	·4	563·8	1·8
80	549·8	·8	548·0	− 1·0	550·5	1·5	550·2	1·2	550·6	1·6
1	531·8	−26·2	534·7	−23·3	531·8	−26·2	532·4	−25·6	532·0	−26·0
2	500·1	21·1	498·4	19·4	500·3	21·3	500·6	21·6	501·0	22·0
3	467·7	−13·3	468·0	−13·0	467·4	−13·6	468·4	−12·6	469·5	−11·5
4	428·0	29·0	427·5	28·5	429·6	30·6	429·2	30·2	430·2	31·2
85	389·6	−14·4	393·2	−10·8	390·2	−13·8	390·7	−13·3	390·9	−13·1
6	337·9	− 4·1	335·6	− 6·4	337·9	− 4·1	338·2	− 3·8	338·3	− 3·7
7	286·9	−12·1	287·0	−12·0	286·6	−12·4	287·0	−12·0	286·5	−12·5
8	237·7	27·7	238·0	28·0	237·7	27·7	237·1	27·1	236·6	26·6
9	197·8	−10·2	198·2	− 9·8	197·3	−10·7	197·4	−10·6	196·8	−11·2
90	152·6	− 7·4	151·3	− 8·7	152·5	− 7·5	152·5	− 7·5	151·6	− 8·4
...	11969·0	±468·4	11968·9	±480·3	11969·8	±485·4	11976·2	±484·4	11970·0	±500·6

Total Actual Deaths, 11,968.

We may now turn to another aspect of the subject and investigate how far different formulas vary as regards their capacity to yield results which follow the observed facts with sufficient closeness. The most obvious way of doing this would be to investigate the differences, irrespective of sign, between the ungraduated and the graduated results. As, however, has been pointed out, this method gives no effect to the weights of the observations, and it will be more satisfactory to consider how far the graduated rates will reproduce the deaths actually recorded when multiplied by the appropriate numbers exposed to risk. For the purpose of giving illustrative figures bearing on this important question I have selected the section of ages 55 to 90 in the G.F.A. Ultimate Table, where the preponderating portion of the observations in that table is situated. I give in Table 16 graduated values of q according to certain of the formulas previously dealt with, while Table 17 shows the expected deaths according to each set of graduated values with their deviations from the numbers actually recorded.

It would not be safe to generalize too widely from the data given in the last table, but we may at any rate notice what, I think, are the most important features of the results there presented, namely, (1) the great fidelity with which all the formulas reproduce on the whole the observed mortality, and (2) the close agreement between the sums of the deviations according to the different graduations. When one remembers the fundamental difference as regards graduating power between Woolhouse's formula and the 21-term formula it is a little surprising, to find that the sums of the respective deviations do not differ by more than about 20 on a total of 500. Looking at the total expected deaths in the section dealt with we see an extremely near agreement in the case of four of the formulas, Nos. (1), (2), (4), and (7) of Table 12, the greatest deviation being 2 from the total number observed, 11,968. The Friendly Society formula brings out a deviation of $+8$, and at first sight this result looks less satisfactory than the others. It does not, however, follow that it is in fact so, as the figures are dependent on the particular age group taken, and a result more favourable to the last-mentioned formula might have been arrived at by altering the grouping. I think it probable that over the whole of this particular table the Friendly Society formula would have led to an extremely satisfactory result. I have, however, purposely avoided dealing with the whole table here, as it is not clear that

by doing so we should have reached conclusions which could safely be taken as applicable to other tables.

By splitting up the facts of Table 17 into four groups we get the following comparison:

Table 18.

Government Female Annuitants' (1883) Ultimate Table.
Deviations of Expected from Actual Deaths.

Ages 55–90.

(1) Age Group	(2) Spencer's 15-term formula	(3) Woolhouse	(4) J. A. Higham (*b*)	(5) G. F. Hardy	(6) Spencer's 21-term formula
55–63	+ 8·9	+10·3	+11·4	+12·5	+14·8
64–72	+27·7	+29·7	+30·5	+33·6	+33·5
73–81	−51·9	−54·3	−57·6	−57·0	−65·7
82–90	+16·3	+15·2	+17·5	+19·1	+19·4

It must not be overlooked that different figures would be obtained by modifying the groupings, but this table shows quite clearly, I think, the divergences due to variations in the theoretical error of the formulas. These divergences are not peculiar to the particular table here taken, but will in general be observed in the case of other mortality tables. Comparing columns 2, 3, 4, and 6, the values are seen to vary (subject to trifling exceptions) with the fourth difference error of the formulas, the larger positive deviations arising from the more powerful formulas in some of the groups being counterbalanced by a correspondingly large negative deviation in another group. I exclude the Friendly Society formula from this statement, for the reason that its second difference error sometimes renders it difficult to compare this formula effectually with the others, a doubt arising in some cases as to exactly how far the second difference error will at any period counteract the error due to the fourth and higher orders of differences.

The figures given in Table 17 do not for the reason stated show the total deviation brought out by the different formulas over the whole table. Reference, however, to Table 6 on p. 375 will show that in the case of the 21-term formula the deficiency in the expected deaths up to age 90 was 3·9 only, and this fact, read with the figures given in Table 17, may, I think, be taken as indicating that, with most mortality tables, the deviation will almost invariably

be extremely small, but that this will, as a rule, be greatest, not only in different sections of the table, but also over the whole table, in the case of the formula having the largest theoretical error, notwithstanding the tendency of the deviations in different parts of the table to counterbalance each other. It seems fairly clear, moreover, from the results of Mr. King's investigations, that where the table we are dealing with follows the Makeham type, the effect of employing most of the formulas referred to above will, having regard to the distribution of the numbers exposed to risk, be in a minute degree to understate the actual deaths if q_x be the function operated on. The Friendly Society formula, however, forms an exception to this rule, as also will any other formula involving a similar positive second-difference error, as these will probably, if Makeham's law approximately apply, lead to a very slight overstatement of the observed deaths. If the particular table under investigation exhibit special features the results just indicated will not necessarily be arrived at, though it is very unlikely that the deviation of the graduated from the ungraduated table would ever be at all material, or such as would exert any appreciable influence on monetary values.

IV.

A NEW FORMULA FOR THE GRADUATION OF COLOG. p OR μ.

One of the most conspicuous of recent developments of actuarial method has been the rise into prominence of the function colog. p. Twenty years ago this function was little thought of, but latterly it has been very much in evidence, and has become much more familiar, owing to the simplicity of the form which it assumes when the Makeham hypothesis applies, and its extensive employment in the graduation of the British Offices' Tables. Having regard to the intimate relationship between colog. p and log. l, and the consequent facility with which it is possible to pass from the former to the annuity-values, Mr. King has, I think, done excellent service in suggesting the employment of colog. p for graduation by mechanical methods, and in showing how trifling is the extent to which the curve is distorted when values of colog. p, based on Makeham's law, are graduated by summation formulas. It is true that the figures given by Mr. King in his Tables 3 and 5 (vol. xli, pp. 76 and 78), where, however, he has not dealt very fully with ages below 80, show that, when Woolhouse's formula is in question, the closest

results of all are obtained by graduating q_x. This may be seen by comparing the "Error caused by the graduation of q_x" in Table 3, with the "Error in q caused by graduating colog.p" in Table 5. There is no appreciable difference between the two up to about age 60, but from this point up to age 93, when the error ceases to be of the least importance, the balance of advantage is distinctly in favour of the q graduation. The difference is, however, of not much practical importance, nor is it such as to militate against the conclusions arrived at as to the suitability of colog.p for graduation purposes. It is safe to assume that the employment of most other formulas of the type common in practice would, like Woolhouse's formula, lead in similar circumstances to results indicating the superiority of q over colog.p. Mr. King has, however, called attention to the interesting fact that, when a formula with a small positive second difference error is employed, the disturbance caused by the graduation of colog.p may be reduced, owing to the fact that the errors due to the second and the fourth differential coefficients are opposite in sign, and that they may thus, when Gompertz's law, or one of its developments, applies, tend to counteract each other. The figures given by Mr. King in illustration of this point show, however, that with his modification of Woolhouse's formula the error is not appreciably reduced, being in fact 90 per-cent, approximately, of that brought out by the latter formula unmodified, although it is in the opposite direction; and that, although the employment of the Friendly Society formula would give slightly better results, the resulting error would still amount to about two-thirds of that brought out by Woolhouse's formula, and would be roughly the same as that resulting from the employment of the formula (1) of Table 12. What is wanted is a formula in which the errors due on the one hand to the second and on the other to the fourth and higher differential coefficients exactly counterbalance each other, and one which at the same time supplies in other respects the essentials of a satisfactory method of graduation. The conditions to be complied with here are more or less precise. The ratio of the second to the fourth differential coefficient of colog.p when Makeham's law holds is as $1:(\log_e c)^2 = \cdot 008064$, if $\log_{10} c$ be taken as $\cdot 039$; and if we fix upon $+\frac{1}{12}$ as the coefficient of the second difference error in the formula to be deduced, we ought, in order to counter-balance this, to have $10\cdot 1$ approximately

as the coefficient of $-\frac{d^4}{dx^4}u$, assuming that the coefficient of $-\frac{d^6}{dx^6}u$ is roughly $2\frac{1}{2}$ times as great as that of $-\frac{d^4}{dx^4}u$. In my earlier paper I gave the formula resulting from the operations

$$\frac{[7][5]^2}{175}\{-u_{-2}+3u_0-u_{+2}\}$$

which was employed in graduating a section of the Manchester Unity (1893-97) Sickness Rates (*J.I.A.*, xxxviii, p. 337). By substituting for $\frac{[7][5]^2}{175}$ the approximately equivalent operations $\frac{[4][5]^2[6]}{600}$ we arrive at the easily applied formula:

$$u'_0=\frac{[4][5]^2[6]}{600}\{-u_{-2}+3u_0-u_{+2}\}$$

$$=\frac{1}{600}\{110u_0+102u_{\pm1}+81u_{\pm2}+54u_{\pm3}+27u_{\pm4}+6u_{\pm5}$$
$$-5u_{\pm6}-8u_{\pm7}-7u_{\pm8}-4u_{\pm9}-u_{\pm10}\},$$

another 21-term formula with a smoothing coefficient of $\frac{1}{141}$, and one, therefore, that is certain to yield excellent results in practice as regards the vital point of smoothness. Expressing the formula in terms of the central ungraduated value and its differential coefficients, we have—

$$u'_0=u_0+\frac{1}{12}\frac{d^2}{dx^2}u_0-10\cdot30\frac{d^4}{dx^4}u_0-27\cdot25\frac{d^6}{dx^6}u_0-\ldots$$

which is seen to meet our needs as regards accuracy with very great success.

The application of this formula to re-graduate values of colog.p, based on Gompertz' or Makeham's law, as also on Makeham's second modification of Gompertz, will, when $c=\log_{10}{}^{-1}\cdot039$, produce results not differing from absolute accuracy by more than $\frac{1}{80000}\beta c^x$, a quantity, which, at age 80, will amount to about a unit only in the *sixth* decimal place of colog.p, and even at age 100 will hardly affect the fifth decimal place. The formula is equally applicable to the function μ, which, of course, is identical in form with colog.p, the error, in this case, being

$\frac{1}{80000}\text{B}c^x$. If the table under examination were based on an appreciably higher value of c, say $\log_{10}^{-1}\cdot 04$, the error would form a larger proportion of βc^x or $\text{B}c^x$, though the absolute magnitude of the error would probably not be increased in the same proportion, as the increase in c would, in all likelihood, be accompanied by a decrease in β and B. In any event, however, the formula would still produce an exceedingly near result, and would, I think, be found decidedly superior for the particular purpose in view to other formulas of the same type that have been employed in practice.

In order to show the utility of the new process I have applied it to graduate colog.p according to the aggregate table representing the experience of the "New" portion of the O^M observations [*see* "Combined Experience of Assured Lives (1863–1893)", pp. 484–5]. The working process adopted was as follows:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Ungraduated colog p $= u$	$3u$	$u_{-2} + u_{+2}$	(2)−(3)	Sum in 5's	Sum in 5's	Sum in 4's	Divide by 600	Sum in 6's = graduated colog p

No difficulty was encountered at the beginning of the table, an average value of colog.p. deduced from the data for ages 10 to 14 being employed as far back as age 0, in order to obtain graduated values from age 10. The ungraduated rates of mortality above age 80, and, consequently, the corresponding values of colog.p, show violent irregularities, and with the view of arriving at the smoothest possible results in this section it was thought desirable in the first place to substitute for the ungraduated values of colog.p for ages 81 to 94 an alternative series based on Makeham's hypothesis. Following the spirit of the new method $\log_{10}c$ was assumed to be ·039, and the constants a and β were obtained by writing colog $p_{80} = \cdot 06604$, to agree with the ungraduated value, and by making the sums of the products of the "Exposed to Risk" into colog p at each age from 81 to 94 agree in both the graduated and ungraduated figures, colog p at age 94 being taken as unity. The resulting values of a and β were respectively ·00382 and ·0000472, the

latter by an odd coincidence agreeing with the value of β in Mr. Hardy's graduation of the $O^{M(5)}$ Table, though the value of α in the present instance is 50 per-cent greater than in that table. The values of colog.p up to age 100 derived from these constants were then inserted in the working sheet, being in fact treated as the unadjusted values at those ages, and the formula was applied in the usual way. It follows from the particular method adopted that at age 90 the main curve automatically merges into the terminal curve substituted for the unadjusted facts above age 80.

TABLE 19.

O^M *Table. "New" Data.*

Graduation of colog p by Spencer's Second 21-term Formula.

Adjusted Values of colog p_x and q_x.

(x)	colog p_x	q_x	(x)	colog p_x	q_x	(x)	colog p_x	q_x
10	·00126	·00289	40	·00380	·00871	70	·02698	·06024
1	·00127	·00292	1	·00398	·00912	1	·02936	·06536
2	·00130	·00298	2	·00417	·00955	2	·03200	·07102
3	·00134	·00309	3	·00436	·00998	3	·03492	·07725
4	·00140	·00322	4	·00454	·01041	4	·03815	·08410
15	·00147	·00337	45	·00474	·01085	75	·04173	·09161
6	·00153	·00353	6	·00494	·01131	6	·04566	·09981
7	·00160	·00367	7	·00517	·01183	7	·05000	·10874
8	·00165	·00380	8	·00544	·01244	8	·05476	·11846
9	·00171	·00392	9	·00576	·01317	9	·05995	·12894
20	·00176	·00404	50	·00614	·01403	80	·06559	·14017
1	·00181	·00415	1	·00658	·01503	1	·07168	·15214
2	·00186	·00427	2	·00707	·01615	2	·07824	·16486
3	·00191	·00439	3	·00760	·01735	3	·08532	·17837
4	·00196	·00451	4	·00816	·01860	4	·09300	·19276
25	·00201	·00462	55	·00872	·01987	85	·10136	·20815
6	·00207	·00475	6	·00929	·02116	6	·11049	·22463
7	·00213	·00489	7	·00987	·02247	7	·12048	·24226
8	·00221	·00506	8	·01048	·02385	8	·13143	·26112
9	·00230	·00528	9	·01116	·02536	9	·14342	·28126
30	·00242	·00555	60	·01192	·02708	90	·15653	·30264
1	·00255	·00585	1	·01281	·02907	1	·17089	·32530
2	·00269	·00617	2	·01384	·03137	2	·18659	·34926
3	·00283	·00650	3	·01502	·03400	3	·20376	·37448
4	·00297	·00682	4	·01636	·03696	4	·22255	·40097
35	·00310	·00711	65	·01782	·04019	95	·24310	·42865
6	·00322	·00739	6	·01939	·04367	6	·26558	·45747
7	·00335	·00768	7	·02108	·04738	7	·29018	·48735
8	·00348	·00798	8	·02288	·05133	8	·31708	·51814
9	·00363	·00833	9	·02484	·05559	9	·34652	·54972
						100	·37872	·58190

The graduated values of colog.p and q, shown in Table 19, are seen to proceed with very considerable smoothness, being in this respect hardly capable of material improvement.

TABLE 20.

Graduation of O^M *"New" Data by Spencer's Second* 21-*term Formula. Comparison of Expected and Actual Deaths.*

Age Group	Expected Deaths	Actual Deaths	Deviation	Accumulated Deviation
10–14	8	8	0	0
15–19	91	91	0	0
20–24	725	743	−18	−18
25–29	2,326	2,303	+23	+ 5
30–34	4,336	4,352	−16	−11
35–39	5,818	5,785	+33	+22
40–44	6,620	6,714	−94	−72
45–49	6,783	6,712	+71	− 1
50–54	6,801	6,745	+56	+55
55–59	6,244	6,320	−76	−21
60–64	5,261	5,252	+ 9	−12
65–69	4,142	4,152	−10	−22
70–74	2,720	2,708	+12	−10
75–79	1,401	1,399	+ 2	− 8
80–84	502	509	− 7	−15
85–89	108	97	+11	− 4
90–94	15	16	− 1	− 5
Totals	53,901	53,906	±439	±281

Table 20 gives the expected and actual deaths arranged in quinquennial groups. As regards this test also the table satisfies exacting requirements, the total deviation being −5 on nearly 54,000 deaths. It would be unprofitable to attempt to improve upon this close result, which confirms the expectations arrived at in building up the formula and studying its characteristics.

It may, I think, be taken for granted that the new method will in most cases furnish results as accurate as can possibly be desired, and if therefore colog.p or μ is to be graduated by a mechanical process this method has pre-eminent claims for consideration. At the same time it must not be thought that I regard the formula as superseding the first 21-term formula, which, applied to q with the corrections indicated in Table 9, will usually represent the facts with equal accuracy, while at the same time taking precedence as regards smoothness, judged by the magnitude of the third differences, and bringing somewhat greater pressure to bear on the wave-like excrescences in the rough data.

APPENDIX.

Note on Mr. G. F. Hardy's Friendly Society Formula.

It has not, I think, been noticed that the Friendly Society formula, usually applied in the form

$$\frac{[4]\,[5]\,[6]}{120}\{-u_{-2}+u_{-1}+u_0+u_1-u_2\},$$

is given, identically, by the operations

$$\frac{[2]\,[3]\,[4]\,[5]}{120}\{u_0-(b_{-2}+b_2)\},$$

a result which at first sight looks curious.

It will be found that $[2]\,[4]\{u_0-(b_{-2}+b_2)\}$

$$=\{[3]+[7]\}u_0-(u_{-5}+u_5),$$

and we thus arrive at the following alternative method of obtaining Mr. Hardy's results which, though interesting rather than of practical importance, avoids the interlinear summations.

(1) Ungraduated Rate $\div 12$ $=u$	(2) Sum in 3's	(3) Sum of u in 7's	(4) $(2)+(3)$	(5) $u_{-5}+u_5$	(6) $(4)-(5)$	(7) Sum in 3's	(8) Sum in 5's $\div 10$ = graduated value

The Friendly Society formula, in this modified shape, forms one of a remarkable series of formulas, the characteristics of which will, I think, be sufficiently evident, in the light of what has been written, from the following summary.

Formulas resulting from the operations $\dfrac{[2][3][4][5]}{120}$.

Second Difference Error $+\dfrac{1}{12}\dfrac{d^2}{dx^2}u_0$.

No.	Terms summed	Number of terms in formula	Fourth Difference Error Coefficient of $-\frac{d^4}{dx^4}u_0$	Sum of Coefficients of 5 central terms	Smoothing Coefficient
1	u_0-2b_0	13	2·50	·967	$\frac{1}{47}$
2	$u_0-(b_{-1}+b_1)$	15	3·50	·933	$\frac{1}{62}$
3	$u_0-(b_{-2}+b_2)$	17	6·50	·850	$\frac{1}{95}$
4	$u_0-(b_{-3}+b_3)$	19	11·50	·750	$\frac{1}{120}$
5	$u_0-(b_{-4}+b_4)$	21	18·50	·683	$\frac{1}{155}$

LEGAL NOTES.

By Arthur Rhys Barrand, F.I.A., *Barrister-at-Law.*

In accordance with the announcement made by the President of the Institute at the sessional meeting in January last, it is proposed, under the above heading, to call attention from time to time, in each successive issue of the *Journal,* to legal facts and cases of recent occurrence, and which may be of interest to actuaries and other officials of life assurance companies. It is not proposed to confine the notes strictly to the law of life assurance, but rather to refer to matters that are of importance in connection with the business of a life assurance office from any point of view. Thus, legal questions relating to certain aspects of such subjects as investments, joint stock companies, bankruptcy, executors and administrators, agency, and the various difficulties that arise by reason of dealings with policies, may all come under consideration occasionally. An endeavour will be made to refer to all cases of any importance that come within the scope of these notes, but it will no doubt frequently happen that, through an oversight, or by reason of the facts not having been reported, some cases of interest will escape notice here. In these circumstances it is hoped that any reader observing such an omission, will call the attention of the writer of these notes to the fact; and where the omission arises from a case being unreported, or relates to some incident or decision that has arisen in office practice, full particulars of the matter in question will be much appreciated.

For the benefit of those who may not be familiar with the abbreviations commonly used to indicate the various series of law reports, it may be stated that in general, reference will be made to what are known as The Law Reports, that is, to the series of reports published by The Incorporated Council of Law Reporting for England and Wales, a body representing the Inns of Court, the General Council of the Bar, and the Law Society. Six volumes are published in this series each year, namely one volume of House of Lords and Privy Council cases, known as Appeal cases, two volumes of Chancery cases, including those of the High Court and Court of Appeal, two volumes of cases in the King's Bench Division, also including the corresponding Court of Appeal cases, and one volume of cases decided in the Probate,

Divorce, and Admiralty Division. These reports are cited by the year in which they are reported, the volume in that year, the division to which they belong, and the page. Thus, taking page 100 as a sample page, the reports for 1906 are cited as follows:—[1906] A.C. 100, [1906] 1 Ch. 100, [1906] 2 Ch. 100, [1906] 1 K.B. 100, [1906] 2 K.B. 100, [1906] P. 100.

There are three other important series of reports published at the present time, to each of which reference may occasionally be made, namely the Law Journal Reports, the Law Times Reports and the Times Law Reports. The first of these is divided into three volumes for each year, namely Chancery, King's Bench and Bankruptcy, and a third volume containing statutes and reports on Privy Council, Probate, Divorce and Admiralty cases. They are cited by the number of the volume, the Court to which they refer, and the page, each volume in a given year bearing the same number. Thus, for the year corresponding to volume 68, the references would be—68 L.J. Ch. 100, 68 L.J. K.B. 100, 68 L.J. P. 100. The volumes of the Law Times Reports contain cases from all Courts, and are cited by the number and page, as 86 L.T. 100. The Times Law Reports, which are not so often cited in the Courts as the preceding, but which often contain cases not reported elsewhere, are cited by the volume and page, thus, 22 T.L.R. 100. In addition to these reports, there are two other series containing somewhat brief reports, and to which it may sometimes be necessary to refer, namely the Weekly Reporter and the Weekly Notes. The former of these is cited by the number of the volume and the page, thus, 45 W.R. 100, and the latter by the year of publication and the page, as (1906) W.N. 100.

Limits of space prevent reference to earlier series of law reports, particularly to the large and valuable series of private reports which preceded the Law Reports, and to the earlier form which these latter took; and also to reports dealing with cases of a particular class, such as Bankruptcy cases, Taxing cases, Commercial cases, &c. Those, however, who desire information on the subject will find a full list of the abbreviations used for all these reports in the appendix to Jelf's "Where to find your Law." Those who seek a short account of the history of Law Reports will find it in the chapter on the subject in Pollock's "First Book of Jurisprudence."

The subject of after-acquired property in bankruptcy has been referred to in an earlier part of this volume of the *Journal,*

and in connection therewith the case of *In re Bennett, J.I.A.*, xli, 169, was quoted. The case was decided too late to be dealt with at all fully there, but as it is now reported in the Law Reports [1907] 1 K.B. 149, and contains many points of interest, a more extended reference to it may be made here. The facts were as follows: A debtor was adjudged bankrupt in 1896. No assets were disclosed, and the Official Receiver became trustee. The debtor remained undischarged, and died in 1905. About a year before his death he effected two small policies on his life and paid the premiums out of his earnings. On his death his brother administered his estate which consisted solely of the amount due under the two policies. After payment of costs of administration the amount was divided among the next of kin of the deceased. The administrator was one of the next of kin, and not knowing of the bankruptcy, he distributed the estate, retaining his own share and the shares of two of the next of kin who were infants. Subsequently the Official Receiver heard of the death of the bankrupt, and that the moneys due under the policies had been received by the administrator, and he thereupon claimed payment of these moneys from the administrator. In June, 1906, Bigham, J., ordered the administrator to pay over to the Official Receiver his own share which he had retained, and also the two shares that he held for the infants, on the ground that such moneys formed part of the estate of the bankrupt; but held that the administrator was protected in respect of the shares he had, in good faith, paid away to the other next of kin, and ought not to be ordered to pay these over again.

The Official Receiver now applied that Herbert Bennett, one of the next of kin who had received his share, might be ordered to pay over such share to him on the ground that it formed part of the estate of the bankrupt. It was held by Bigham, J., that the principle laid down in Cohen *v.* Mitchell, *J.I.A.*, xli, 168, only applied where the transaction was for value, and that a person who receives after-acquired property from a bankrupt or his representative otherwise than for value, cannot retain it as against the trustee in bankruptcy. As therefore no value had been given for the share in the estate, the recipient was ordered to refund it to the Official Receiver.

Another bankruptcy case of some interest, and in which policies of life assurance were also involved, is that of *In re Tyler, ex parte The Official Receiver* [1907] 1 K.B. 865. Here

one William Tyler was adjudged bankrupt in 1896. At the commencement of the bankruptcy, the bankrupt was entitled to two policies of assurance, effected on his own life, which had been assigned to his bankers in 1893 to secure an overdraft. Towards the end of 1895 the bankrupt got into difficulties, and informed his wife that he was unable to pay the interest on his overdraft and the premiums on the policies, and requested her to make these payments for him, to prevent the premiums being paid by the mortgagees or the policies lapsing or being sold. This was done by the wife through the bankers until the death of the bankrupt in 1906, when the sum so paid by the wife amounted to £481. The policy moneys amounted to £936, and after the bankers had deducted the amount due to them for principal, interest and costs, there remained the sum of £514, and this was paid by the mortgagees to the Official Receiver. The widow then claimed to be paid out of this amount the sum of £481 paid by her for premiums and interest. At the commencement of the bankruptcy the amount due to the bankers on the security of the policies was in excess of their surrender-value. The Official Receiver asked for a declaration that the sum of £514 formed part of the property of the bankrupt divisible amongst his creditors, but Bigham, J., held that, although the widow had no legal right, and probably also, no equitable right, to be repaid the £481, yet the trustee, as an officer of the Court, must do what was just and right. He therefore ordered that £481, out of the £514, should be paid to the widow. The Official Receiver appealed against this decision, and on his behalf it was contended that the principle as to the officer of the Court doing what was fair and just, irrespective of the strict legal rights of the parties, was confined to the special case of money paid under a mistake of law. The Court of Appeal, however, upheld the decision of Bigham, J., and declined to limit the application of the principle in the way suggested. Vaughan Williams, L.J., in delivering judgment to this effect, quoted the words of James, L.J., "I am "of opinion that a trustee in bankruptcy is an officer of the "Court. . . . The Court then, finding that he has in his hands "money which in equity belongs to someone else, ought to set "an example to the world by paying it to the person really "entitled to it. In my opinion the Court of Bankruptcy ought "to be as honest as other people." He went on to point out that, in addition to the above mentioned principle in the widow's favour, "in this particular case, having regard to the fact that

"these policies were entered into by the bankrupt before the "commencement of the bankruptcy, and then the wife, at his "request, advanced the money to pay the premiums, it seems "tolerably plain . . . that she had such an interest in the "policies as entitled her to prevent the trustee or other "representative getting the benefit of those policies without "recouping the premiums which she had paid." Farwell and Buckley, L.JJ., delivered judgment to the same effect. The principle on which this case was decided is a very interesting one, but considerable care must be exercised in applying it to another set of facts not standing on all fours with the present case. The need for this caution can be seen by reference to the case of *In re Hall, ex parte The Official Receiver* [1907] 1 K.B. 875, which was decided at the same time as the preceding case, but where, in somewhat different circumstances, the Court declined to apply the principle in question, and gave judgment in favour of the Official Receiver, reversing the decision of the County Court and the Divisional Court which had followed that principle.

There appears to be an increasing tendency on the part of assurance companies to invest some portion of their funds in the debentures of trading companies. In these circumstances it is perhaps as well that attention should be called to three recent cases in connection with the re-issue of debentures, as they raise a very real difficulty in regard to the security of such investments. The first case is that of *In re George Routledge & Sons, Limited* [1904] 2 Ch. 474. Here the company had power to borrow by means of first mortgage debentures which should have priority over all other mortgages or securities of the company. Such debentures were issued to the full amount authorized. The company subsequently purchased some of them in the open market and took a transfer of them to itself, and afterwards sold them to various persons and executed transfers in favour of the purchasers. In a debenture holder's action, it was held by Buckley, J., that the debentures so re-issued had ceased to be securities, and that they did not therefore rank *pari passu* with the other debentures of the same series. In delivering judgment, he said: "The company had become the assignee of its own "undertaking, by way of charge, to secure the payment. The "result, to my mind, is that the debt and the security are both "absolutely gone. A man cannot be the assignee of his own

" debt, and cannot be the mortgagee of property of which he is " also mortgagor. The debt was gone and the security was also " gone. Subsequently the company transferred these debentures. " . . . To my mind that transaction had no effect. The " purchasers were transferees of nothing. There was no debt in " existence; there was no security in existence at the date of the " transfer to them."

The second case is that of *In re W. Tasker & Sons, Limited* [1905] 2 Ch. 587. Here the company had power to borrow by means of debentures, and in pursuance of such powers certain debentures were duly issued in 1896. In 1899 the company borrowed certain sums, and as security for these advances deposited with the lenders unissued debentures of the nominal value of double the amount advanced, such debentures being registered in the names of the lenders. The company repaid portions of the amount advanced, and on such repayments, took a blank transfer of debentures of double the amount so repaid, in accordance with the original agreement for the loan. From time to time, as the company received applications for debentures, these blank transfers were filled in with the names of the applicants, who paid to the company the full nominal value of the debentures; and these transferees were then entered on the register of debentures in place of the original lenders. On the company going into liquidation, the transferees claimed to rank *pari passu* with the original debenture holders, but Kekewich, J., held that they were not so entitled, and this decision was upheld on appeal. Cozens-Hardy, L.J., in delivering judgment in the Court of Appeal, said: " It seems to me that the redemption of " the debentures by payment-off of the loans must involve " precisely the same consequences as if the debentures had " been redeemed by payment-off of the amount due on the " debentures themselves. In either case the debentures were " spent. . . . . I think the debentures, when redeemed, must " be considered as dead and gone for all purposes, and as " incapable of transfer . . . . The result is that the instant the " debentures were redeemed by the company, the redemption " enured for the benefit of all the persons entitled to the *pari* " *passu* charge."

The third case, that of *In re Perth Electric Tramways Company, Limited* [1906] 2 Ch. 216, is even more far-reaching in its effect. Here the company, in 1903, assigned its property and assets to trustees (subject to certain prior charges) to secure an

issue of debentures. In 1905, the company arranged to borrow from its bankers upon the security of the deposit of certain of these debentures, and these were accordingly signed and sealed by the company. By arrangement with the bank, they were signed and sealed in blank, that is, without the holder's name or the date, and they were not registered in the debenture register or at Somerset House. The company repaid the loan and received back the blank debentures, and immediately afterwards issued certain of them, filling in the name of the holder and the date, but without resealing the debentures, and the holders of them were duly registered as such, and the debentures themselves were registered at Somerset House. These holders had no reason to suppose that there had been any prior dealings with the debentures. On a question being raised as to their validity, it was held that the debentures were not validly issued. In delivering judgment to this effect, Swinfen Eady, J., said: "The debentures "have served their purpose. They have been used as security "for the loan, albeit a temporary loan, and the loan has run its "course, and has been paid off. Under those circumstances I "am of opinion that the debentures in question were issued or "agreed to be issued, and that as the company has no power, "according to the trust deed under which the debentures were "created, to re-issue debentures, it is not now in a position to deal "with . . (these) debentures."

The difficulty raised by these cases is a very serious one to investors in debentures, since in all cases it will be difficult, and in many cases practically impossible, to ascertain whether any particular debentures which it is proposed to purchase, have been re-issued within the meaning of these decisions. This and other difficulties relating to debentures, such as questions raised in connection with the validity of perpetual debentures, power to give a mortgagee of debentures an option of purchase, and power to obtain specific performance of an undertaking to subscribe for debentures, are dealt with in the proposed new Companies Bill, introduced by Earl Granard, and also in the Companies (Debentures and Debenture Stock) Bill introduced by Lord Avebury, but pending the passing of one or other of these Bills, considerable caution should be exercised in purchasing, or subscribing for, debentures; and it may perhaps be considered advisable to confine such investments to debentures where there is an express provision, contained either in the debentures themselves or in the trust deed by which they are secured, permitting

of their re-issue, and making the charge available for any debentures re-issued, up to a certain amount. Where such a provision for re-issue exists, the cases referred to above have no application, at any rate as far as the endangering of the security is concerned.

Another group of recent cases of great importance, also dealing with the subject of investments, may next be noticed. Unlike the preceding group, which deals with the nature of the security, this group is concerned with the transfer of ownership of securities. The two first cases to be noticed are well known, and can therefore, although very important, be passed over with but a slight reference. The third, however, is a more recent case, and for this reason, as well as on account of its importance, calls, perhaps, for a more extended notice.

The first of the three cases in the group is that of *Starkey* v. *Bank of England* [1903] A.C. 114. Here a broker obtained a transfer of Consols and Bank Stock, standing in the names of two trustees, by means of a power of attorney purporting to be signed by the two trustees, and which he believed to be so signed. As a matter of fact, however, the signature of one of the trustees had been forged. On this being discovered after the death of the trustee who had actually signed the power, the Bank was ordered by Kekewich, J., to replace the stock, and the broker was ordered to indemnify the Bank in respect of such replacement. The broker appealed against this decision, but it was confirmed by the Court of Appeal and ultimately by the House of Lords, it being held that the broker, by applying for, and tendering the power of attorney, must be taken to have given an implied warranty that he had authority to act for the owners of the stock, and that he was therefore liable to indemnify the Bank against the claim of the owners to have the stock replaced.

The second of these cases is the very well-known one of *Sheffield Corporation* v. *Barclay* [1905] A.C. 393. This also was a case of stock, in this instance that of the Sheffield Corporation, standing in the names of two trustees, and a transfer in favour of Barclay, purporting to be signed by both trustees, was tendered by Barclay to the Sheffield Corporation for registration, and he was duly registered as the holder of the stock. It was afterwards transferred by him, and passed into the hands of a *bonâ fide* holder for value. As a matter of fact, the signature of one of the trustees had been forged, and on this fact coming to light after

the death of the other, the Corporation was ordered by the Court to replace the stock. They did so, and then called upon Barclay to indemnify them. Judgment was given in favour of the Corporation in the first instance, but this decision was reversed by the Court of Appeal who decided in favour of Barclay. This latter decision was, however, reversed by the House of Lords who decided in favour of Sheffield Corporation, and held that there was an implied contract that the transfer was genuine, and therefore, as between the two innocent parties, the loss should be borne by Barclay who caused the Corporation to act upon an instrument which turned out to be invalid. The object of these notes is rather to call attention to cases than to comment upon them, but in passing it may be well to call attention to the far-reaching effect of this decision. Barclay only held this stock for a few weeks and had ceased to hold it for some years before this action was brought, so that any individual or company who may buy and sell Stock Exchange securities may, years after a security has been sold, be called upon to replace its value, on the ground that the transfer under which it was originally obtained was forged, and it may well happen that after such a lapse of time, there will be no effectual remedy against the person from whom the security was attained. This fact, coupled with the great difficulty in an ordinary Stock Exchange transaction of ascertaining that a transfer is actually executed by the person or persons purporting to execute it, may well give rise to anxious thought and care on the part of those who have to deal with such securities.

The third, and most recent case of the group is that of *The Bank of England* v. *Cutler* [1907] 1 K.B. 889. Here certain inscribed stock stood in the name of one Marian Pearson. Such stock is only transferable by an entry in the register of the stock kept by the Bank of England, duly signed by the transferor or his attorney. For the purpose of enabling such transferor to be identified by the Bank, it is the custom of the latter to keep a list of stockbrokers whose identification of the transferor is accepted. In 1903, Annie Pearson and Jeanie Pearson, relatives of the holder of the stock, having obtained possession of the stock receipt, went to a solicitor, Jeanie Pearson introducing herself as Marian Pearson, and instructed him to procure a transfer of the stock to one Abraham Loftus Tottenham, whose son, it was stated, was about to advance a sum of money to Marian Pearson. The solicitor sent his clerk with the two women to the defendant,

a stockbroker whose name was on the Bank's identification list, and Jeanie Pearson was introduced to him as Marian Pearson, and he was instructed to prepare a transfer to Abraham Tottenham as upon a nominal consideration. The defendant accordingly prepared the usual ticket giving particulars of the proposed transfer, and sent it to the Bank with notice that he would attend in the course of the afternoon and identify the transferor. He attended accordingly, when the transfer was executed by Jeanie Pearson signing the name of Marian May Pearson in the transfer book of the Bank, the defendant signing his name in the margin of the transfer with the words "Witness to the identity of Marian May Pearson"; and a new stock receipt was made out in the name of Abraham Tottenham. The stock ultimately passed into the hands of a Mr. Gibbs, who bought it in the market, and it was transferred to him. In 1905 the forgery was discovered, and Annie and Jeanie Pearson were prosecuted and convicted. Marian Pearson then called upon the Bank to reinstate her upon the register, which the Bank did, purchasing stock for that purpose, and they also paid her certain dividends which she had not received. The Bank then sued the defendant to recover the amount paid by them for the new stock and arrears of dividends, as damages for breach of an implied warranty that the person identified by the defendant was Marian May Pearson, in consequence of which they had been compelled to replace the stock in her name. The defendant claimed an indemnity from the solicitor, whom he brought in as third party. In delivering a considered judgment in favour of the Bank, A. T. Lawrence, J., said: "I come therefore to the conclusion, "as a matter of fact, that the defendant did request the Bank to "permit Marian May Pearson to make this transfer, and did "identify this woman Jeanie as Marian May Pearson, the true "owner of the stock. The legal effect of this was to warrant that "she was Marian May Pearson, and promise to indemnify the "Bank if she was not. I see no essential difference between this "case and that of Starkey *v.* Bank of England, or Sheffield "Corporation *v.* Barclay. What was warranted in those cases "was authority, but authority is involved in identity. The latter "contains the former." With regard to the contention of defendant's counsel that the Bank was not bound to replace the stock, he said: "*Primâ facie,* a transfer which the Bank has "permitted is valid, and if acted upon *bonâ fide* and for "value, the Bank is estopped as well from touching it as from

"saying that it is invalid." The case has been entered for appeal, but the appeal had not been heard when these notes were written.

Another case relating to the subject of investments may be briefly noticed, that of *Speyer Brothers* v. *The Commissioners of Inland Revenue* [1907] 1 K.B. 246. Here securities were issued by the Mexican Government in the shape of Gold Coupon Treasury Notes, practically in the form of promissory notes payable in two years' time, with coupons attached for half-yearly payments of interest, and with no security except the promise of the Government to pay the amount named in the notes. The Commissioners found that the instruments in question were capable of being sold on the London Stock Exchange, and held that they were marketable securities issued by a foreign state within the meaning of section 82, subsection 1 (*b*) of the Stamp Act, 1891, and should therefore be assessed to stamp duty as marketable securities. On a case being stated for the opinion of the Court, Walton, J., held that the instrument was a promissory note, and was sufficiently stamped as such. The Commissioners appealed, and the Court of Appeal decided in their favour, holding that the instruments, although capable of being described as promissory notes, were also properly described as marketable securities, and as such attracted the appropriate stamp duty.

In connection with the subject of the liability of an assurance company for the acts of its agent, particularly in the matter of filling up the proposal, discussed in an earlier part of this volume, the case of *M'Millan* v. *The Accident Assurance Company, Limited* (*J.I.A.*, xli, 124) was referred to, but no particulars were given as they were not then available. The case has now been carried to a higher Court, and the decision of the latter will now be found in (1907) 44 S.L.R. 334. The particulars of the case are as follows: The defenders issued a policy under which they became liable to indemnify the pursuer in respect of any compensation payable to workmen who should be injured while in his employment. The policy contained the usual clause making the proposal the basis of the contract and incorporating it in the policy. It went on to provide that "the company shall not be "held liable in respect of any knowledge of, or notice to, an "agent which shall not have been communicated to, and have "been acknowledged in writing by, the company at its registered

"office." The proposal contained the usual question as to whether any proposal for this class of risk had previously been declined or withdrawn. This was answered in the negative, and it was admitted that this was not a true answer, as a previous proposal to another office had been declined; but it was contended on the pursuer's behalf that the company's agent, and also their inspector who filled up the proposal, knew of the previous proposal having been declined, and that the proposer supplied full and accurate information to the inspector and relied upon his filling up the proposal form correctly. The proposer further said that he did not, himself, read over the proposal before he signed it, being led to believe that it was in accordance with the information which he had supplied. The first Court decided in favour of the assurance company, Biggar's case (*J.I.A.*, xli, 122) being approved and Bawden's case (*J.I.A.*, xli, 123) distinguished, and on appeal this decision was confirmed. The Lord Justice Clerk, in delivering judgment to this effect, said: "Upon the question whether, in the circum-"stances of the case, it can be held that the company is "bound by the knowledge of their agent, of facts contrary "to what is stated in the proposal, I am clearly of opinion "that it cannot. . . . The pursuer contends that the defenders' "agent inserted the false statement in the proposal, acting "as agent of the company. I cannot accept that suggestion. "The proposal is the pursuer's proposal. It is his duty to "see that his proposal is true in all substantial particulars. "If he chooses to allow another person to fill it up, then "such a person, in doing so, is acting not in the course of "his duty to any third party. He is acting as the agent for the "proposer and for nobody else." With regard to the assertion that the proposer did not read the proposal before signing it, he said: "If he chooses to take the risk of trusting that another has "drawn up the document as he desired it, he must take the "consequences if there be statements above his signature which "are false." Bawden's case was again doubted and distinguished, and Biggar's case was approved. The decision of the Supreme Court of the United States in the case of The New York Life Insurance Company *v.* Fletcher (1885) 117 U.S. Rep. 519 was referred to with approval.

The well-known case of Roberts *v.* The Security Company (*J.I.A.*, xli, 125) came under the consideration of the Judicial

Committee of the Privy Council recently in connection with the case of *The Equitable Fire and Accident Office, Limited,* v. *The Ching Wo Hong* [1907] A.C. 96, and in view of the importance attaching to the first-named case, particulars of the latter case may be of interest. A fire policy in force contained a provision that no additional insurance was allowed, except with the consent of the company, who had issued the policy. A fire occurred, and a policy issued by the Western Assurance Company and apparently duly executed was found in the insured's safe. This policy contained a statement in common form to the effect that the insured having paid the amount of the premium for insuring against loss or damage by fire, the company agreed to hold the property described therein insured against fire. The policy went on to state, that (condition 11) "This insurance will not be in force " nor will the company be liable in respect of any loss or " damage happening, before the premium, or a deposit on account " thereof, is actually paid." It was found, as a matter of fact, that no premium was paid on it. It was contended, nevertheless, on behalf of the insurance company, who were the appellants, that in spite of this fact, the insurance in question was in force when the fire occurred, and thus invalidated the existing insurance, and Roberts *v.* The Security Company was quoted in support of this contention. Lord Davey, in delivering the judgment of the Privy Council, dismissing the appeal, said: "The question therefore is whether the premium not " having been paid wholly or partially, the policy executed by " the Western Assurance Company ever became effective, and this " must be decided in the same way as if an action had been " brought by the respondents on that policy . . . It is plain from " the language of the condition (condition 11 quoted above) " that it applies . . . to the first premium . . . indeed it may be " said that it applies primarily to the first premium . . . Their " Lordships are of opinion that the 11th condition qualifies " and restricts the engagement of the company, and converts " what would otherwise be an absolute engagement into a " conditional one, and that the words 'having paid' to the " company are common form words or words of style for " expressing the consideration for the company's engagement, " which would become accurate when that engagement became " effective. . . . Their Lordships cannot treat the fact of the " executed policy having been handed to the respondents as a

"waiver of the condition, or attach any importance to the "circumstance. What was handed to the respondents was the "instrument with this clause in it, and that was notice to them, "and made it part of the contract, that there would be no "liability until the premium was paid. It was not a matter of "conditional execution but of the construction of what was "executed. The learned counsel for the appellant company "cited and relied on a decision of the Court of Appeal in "England, in Roberts *v.* The Security Company. It is enough "for their Lordships to say that the words of the instrument in "that case were different from those which their Lordships have "to construe, and they are relieved from saying whether they "would otherwise have been prepared to follow it."

An interesting and important case relating to the right to the custody of deeds is that of *In re Palmer, Lancashire and Yorkshire Reversionary Interest Company,* v. *Burke* [1907] 1 Ch. 486. Here, by certain mortgages and assignments, the plaintiffs became absolutely entitled to two shares in a reversion under a settlement. On the reversion falling into possession, the defendant, who was the sole surviving trustee of the settlement, admitted the plaintiffs' title to the shares in question, but refused to pay over the amounts unless the mortgage deeds and assignments constituting the plaintiffs' title were delivered to him on completion. The plaintiffs refused to hand them over, and applied for an order that the defendant should pay the shares in question to them, and for a declaration that he was not entitled to insist on their delivering up the deeds relating to their title. Swinfen Eady, J., in deciding that the trustee could not insist on the deeds being handed over, said—"no authority "has been produced in support of the trustee's contention. The "deeds do not belong to the trustee in point of law, nor has he "paid for them. The deeds purport to be mortgages and "assignments to the company. The trustee is not purchasing "from the company anything like an interest in land which the "deeds would go with. The company has offered the usual "acknowledgment and undertaking, but that is not enough for "the trustee, he insists on the delivery up of the deeds. In "my opinion the trustee has not established any right to the "deeds. . . . When money is paid by a trustee to a person who "receives it under a power of attorney, the trustee cannot claim "that the power of attorney should be given up to him. It was

"said that the trustee would be in a very unfortunate position if "an action were brought against him for the fund, and he had "not the deeds; but it might equally be said that the company "would be in a very unfortunate position if they handed over "the deeds, and the assignee disputed the assignment and "brought an action against the company to set it aside." The case is of interest to assurance companies in respect of their investments in reversions, as showing that they need not hand over their title deeds to the trustees when the reversion falls in. The chief importance, however, lies in the fact that it apparently applies also to the ordinary case of payments made by an assurance company under its policies, at any rate, to the payment of claims, unless, as is not infrequently the case, it is a term of the contract that all deeds relating solely to the policy shall be handed over to the assurance company on payment of the claim.

The case of *In re Colonel Eyre* (*Deceased*) [1907] 1 K.B. 331, is concerned with the rate of estate duty payable on reversions, and is therefore of interest to all who invest in such securities. The case turned on the effect of section 7, subsection 6, of the Finance Act, 1894. This subsection provides that "where an "estate includes an interest in expectancy, estate duty in respect "of that interest shall be paid, at the option of the person "accountable for the duty, either with the duty in respect of the "rest of the estate, or when the interest falls into possession; "and if the duty is not paid with the estate duty in respect of "the rest of the estate, then—(*a*) for the purpose of determining "the rate of estate duty in respect of the rest of the estate, the "value of the interest shall be its value at the date of the death "of the deceased; and (*b*) the rate of estate duty in respect of "the interest when it falls into possession shall be calculated "according to its value when it falls into possession, together "with the value of the rest of the estate as previously "ascertained." Here the parties accountable for the estate duty on a reversionary interest passing on the death of Colonel Eyre elected to pay the duty when it fell into possession, and on that event happening, contended that although in the circumstances the rate of estate duty was fixed, in accordance with the section of the Finance Act 1894 quoted above, by the value of the reversion when it fell into possession, yet that the amount upon which such duty should be paid was the value of the reversion as at the date of Colonel Eyre's death. The value at the latter

date, in 1902, was £13,758, whereas the value when in fell into possession in 1906 was £28,939. The Crown claimed duty on the latter amount, and it was held by Bray, J., that although the statute did not state in clear terms what was to be the amount on which the duty was to be payable at each of the times given by the option, nevertheless it must be held that the contention of the Crown was correct, and that duty was, in the circumstances, payable on the value of the interest when it fell into possession.

Another case relating to estate duty, and in which policies of assurance were involved, is that of *Lethbridge* v. *The Attorney-General* [1907] A.C. 19. Here the father of the appellant effected fifteen policies on his life, and after maintaining them for some time, assigned them to his son (the appellant) under a family arrangement entered into in order to relieve the father from certain pecuniary difficulties. The material part of the arrangement was that the son, who was tenant-in-tail in remainder, mortgaged the inheritance in the family estate to assist his father, who was tenant for life, and the father in return assigned these policies to his son, together with an annual sum out of his life interest in the estate sufficient to pay the premiums. In these circumstances it was contended by the Crown that estate duty was payable on the policy moneys on the death of the father, as constituting an interest purchased or provided by the father by an arrangement with the son within the meaning of section 2, subsection 1 (*d*) of the Finance Act, 1894. Judgment was given against the Crown by Phillimore, J., but this decision was reversed by the Court of Appeal, only to be restored by the House of Lords. Lord Macnaghten, in giving judgment against the Crown, said: "The policies, which had been effected in support of a charge on "the father's life interest, were redeemed by moneys raised by a "charge upon the inheritance. The son, in my opinion, acquired "them by purchase. He gave more than an equivalent for "what he received. Any respectable insurance company would "have dealt with him on much easier terms. If pecuniary "considerations alone are to be regarded, the transaction seems "to have been a most disadvantageous bargain for the son. "However that may be, the son undoubtedly gave valuable "consideration for the policies, and they were made over to him "absolutely. . . . The only other question is, What was the "position of the son with regard to the annual sums which, in

"pursuance of the family arrangement, . . . were to be devoted "to keeping up the policies. In my opinion, those annual sums "belonged to the son, and to no one else."

In connection with the testamentary dispositions of a deceased policyholder, a difficulty sometimes occurs, more often perhaps in connection with the small policies of industrial assurance than with ordinary policies, by reason of the fact that, in addition to executing an ordinary will dealing with all his property, the assured has also executed a will dealing only with his policy, and a question arises as to the relative force and effect of the two wills. Such a case came before the Court recently in *Simpson* v. *Foxon* [1907] P. 54. Here a testator, in 1898, executed a will, drawn up by a solicitor, disposing of all his property and appointing his daughter sole executrix, and in 1905 he, through another solicitor, executed a codicil which was described as "a codicil to the last will", by which, *inter alia,* he revoked all previous appointments of executors and trustees. Between the dates of these two documents the testator, who had insured his life for a small sum, executed a printed form of will which he obtained from the assurance company. This form, which was executed in 1903, according to its terms purported to be "the last and only will and testament" of the testator. It dealt only with the policy of assurance, and appointed an executor. It was duly executed as a will. In granting probate of all three documents, the President, Sir Gorell Barnes, said, "There is no doubt to my mind that, as "a matter of fact, the deceased cannot really have intended the "policy form of will to have been a revocation of his general "dispositions, and to have left himself intestate as to the greater "part of his property. . . . In my opinion all three documents "should be admitted to probate. I do not think, having regard "to the circumstances which these testamentary papers disclose, "that the words 'last and only' can be taken as revoking what "had been done by the previous will. The document which "contains the words in question is a printed form, evidently "drawn up for the purpose of disposing of a policy of assurance "only, and appointing an executor to deal with that matter only. "It is very unfortunate that it should have been drawn in this "way, but it is, notwithstanding the words 'and only', not "intended to be a complete disposition of the testator's "property."

In view of the considerable interest taken in life assurance circles as to the effect of the decisions in the cases of *The Victoria Daylesford Syndicate, Limited* v. *Dott* [1905] 2 Ch. 624, and *Bonnard* v. *Dott* [1906] 1 Ch. 740, particulars of another case dealing with the same subject may not be out of place. For the benefit of those who are not familiar with the above-named cases, it may be said that they both relate to the position of an unregistered money-lender with regard to a contract into which he has entered in that capacity. In the Victoria Daylesford case it was held by Buckley, J., that the effect of such non-registration was to make any contract entered into by the unregistered money-lender in that capacity an illegal one, upon which he could not sue. In Bonnard's case, the Court of Appeal took a similar view, and the situation was clearly expressed by Collins, M.R., who, in the course of his judgment, said, "The "defendant is a person who is declared by the Court to be a "money-lender, and who, by his omission to register himself, "finds himself under a statutory incapacity to enforce the bargain "he has made. The consequence of that is that whether it is "the borrower or the lender who brings the matter before the "Court, the transaction is absolutely void. The lender cannot "compel the borrower to return the money lent, while the "borrower . . . can compel the lender to return the securities "for the loan, at any rate on the terms of repaying "the amount lent." Following on this decision the appeal in the Victoria Daylesford case was dismissed without argument ([1906] 1 Ch. 747 n.). These decisions caused some alarm among life assurance companies, and The Life Offices' Association in consequence took the opinion of Mr. C. M. Warmington, K.C., on the situation thus created. This opinion was of a reassuring nature, and indicated that no change in practice was called for by these decisions. Quite recently another case, dealing with the same subject, has been decided, that of Lodge *v.* National Union Investment Company [1907] 1 Ch. 300. Here the plaintiff assigned a reversion and a policy of assurance to the defendants, who were money-lenders within the meaning of the Money-lenders Act, 1900, but were not registered under that Act. The deeds were in the form of absolute assignments, but it was admitted that the transaction was really one of mortgage. The reversion was subject to a prior mortgage which the defendants subsequently paid off, and this mortgage, and a policy of assurance held by the prior mortgagee as further security, were transferred

to them. The plaintiff now applied, *inter alia*, for (1) a declaration that the contracts and transactions were illegal and void as against him, and (2) for the delivery up to him of the assignments and securities. Parker, J., in delivering judgment, referred to the old Usury Laws, and said: "It seems reasonably "clear that at any rate in equity, if not also at law, a person "taking advantage of the exception arising from the fact that he "belonged to the class for whose protection the statutes were "passed, could not assert any right unless he was himself "prepared to do what the Court considered fair to the defendant." He therefore held that upon the plaintiff refunding to the defendants the amount actually received by him or paid on his account, they should deliver up the conveyance and policy in respect of the first transaction, but that in respect of the transfer of the prior mortgage and policy, which were transferred on the defendants paying off such prior mortgage, they were entitled to the benefit of that security.

An actuary is not often called upon to quote a rate for an annuity on the life of a person who is already dead, if one may so express it. That he may be required to do so, however, can be seen from the decision in the case of *In re Robbins, Robbins* v. *Legge* [1906] 2 Ch. 648. Here a testator directed his trustees, out of the proceeds of his estate, to purchase a Government annuity for his wife, on her life, of £400 per annum. The testator died on 11 October 1905, and his will was proved on 25 November 1905. The wife, on whose life and for whose benefit the annuity was to be purchased, survived the testator, but died on 27 October 1905, before the will was proved, and without having made any election to take the value of the annuity in cash. Her administrators then claimed that they were entitled to the sum which would, at the testator's death, have purchased the annuity in question. Swinfen Eady, J., in delivering judgment in favour of the representatives of the widow, said—"The law was stated by Sir "Thomas Plummer, M.R., in Palmer *v.* Crauford 3 Swans., 482, "487, in this way. . . . 'Where money is bequeathed to be "'invested in the purchase of an annuity for the life of the "'legatee, and the legatee dies before it is laid out or even . . . "'before the fund is available, as during the life of the person, "'after whose death the investment is to be made, yet still it is "'a vested legacy, and the legatee for whose benefit it was "'intended, having survived the testator, may elect either to take

" 'the sum or have it laid out in an annuity.' . . . I determine " that the legal personal representatives (of the widow) are " entitled to such a sum as at the date of the death of the testator " would have purchased a Government annuity of £400 on the " life of the widow." This decision has since been confirmed by the Court of Appeal.

In an earlier part of this volume (*J.I.A.*, xli, 225) reference is made to the fact that an order for foreclosure absolute is chargeable with ad valorem conveyance duty on the amount owing for principal, interest and costs, and section 6 of the Finance Act, 1898, which deals with the matter, is there quoted. The matter recently came before the Court in the case of *In re Lovell and Collard's contract* [1907] 1 Ch. 249. This was a vendor and purchaser summons, taken out because part of the title consisted of an order for foreclosure absolute dated 30 January 1896, in respect of a legal mortgage for £22,000, which was only stamped with 5*s*. The purchaser objected to the stamping of this document, and the vendor contended that as the foreclosure order was made before the Finance Act, 1898, it did not require an ad valorem stamp. Swinfen Eady, J., however, decided in favour of the purchaser, and held that section 6 of that Act was declaratory and therefore retrospective, so that the order of 1896 required stamping as a conveyance on sale, under section 54 of the Stamp Act, 1891.

It will no doubt at once strike the readers of these notes that one large class of cases, which would appear to come within their scope, is conspicuous by its absence. Reference is, of course, made to cases arising in connection with the Workmen's Compensation Acts, and in view of the considerable number of life assurance companies which now transact business of this class, the omission to notice cases of this description may perhaps call for explanation. It need hardly be said that the omission has not arisen from any lack of material. It is rather due to the superabundance of it. The space available for these notes is necessarily limited, and having regard to the fact that ample provision has been made in other directions for furnishing full reports of Workmen's Compensation cases, as for example in the admirable series of reports edited by the late Mr. R. M. Minton-Senhouse, it seems better that cases of this class should be omitted altogether from notice here. It may not

be out of place, however, in view of the fact that the new Workmen's Compensation Act, 1906, comes into operation on July 1st, to remind the readers of these notes that the Act extends, *inter alia,* to all the employees of assurance companies whose remuneration does not exceed £250 per annum, whether they be members of the inside or outside staff; and that the exemption from liability hitherto existing, where the injury is caused by the serious and wilful misconduct of the employee, no longer exists if the injury results in death or serious and permanent disablement.

Frequency-Curves and Moments. *By* ROBERT HENDERSON, F.I.A., *Assistant Actuary of the Equitable Life Assurance Society of the United States.*

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SO much has recently been said on the subject of the use of frequency-curves in all sorts of statistical work that it seems desirable that a concise statement of their general theory should be readily available for the actuarial student.

Perhaps for our purposes it would be more accurate to speak of a frequency function, because the curve only enters into the question as a convenient graphical representation of the function, and all the computations are made by the use of the algebraic function without any necessary reference to the geometrical curve. It has, however, become a well-established practice to speak of the frequency-curve, and no material advantage would be gained by departing from customary usage in the matter.

If a variable may have any value within certain limits, the curve, in which the ordinate y is proportional to the chance of the variable having the value x represented by the abscissa, is called the curve of frequency. For example, let the variable be the length of a human life which may have any value from zero to the limits of the mortality table. The chance of the duration falling between x and $x+dx$ is evidently $(l_x-l_{x+dx})/l_0$ or $l_x\mu_x dx/l_0$, so that the equation of the frequency-curve in this case is $y=l_x\mu_x$.

Similarly, if two points be taken at random on a straight line, the curve of frequency of the distance of the furthest of the two

from a specified end is represented by the straight line $y=x$ between the limits 0 and a where a is the length of the line.

Where in any two cases the variables are comparable quantities, such as the heights of individuals in two different nations or the durations of life in two different groups of individuals, it is convenient to have some short method of comparison between the two; some coefficient or factor which will indicate whether one curve or the other falls, on the whole, on the higher values of the variable, and if so by how much.

The quantity most frequently used for this purpose is the *mean* value of the variable obtained by multiplying each value by its probability and summing. In other words, if m_1 be the mean value and a and b the limits of the curve, we have

$$m_1=\frac{\int_a^b yxdx}{\int_a^b ydx}.$$

It is evident from the above method that if in one curve the value of m_1 is greater than in another, the former falls on the average on higher values of the variable. It is also evident that represented geometrically, m_1 is the abscissa of the centre of position of the area included between the curve and the base.

Other functions which have been used for this purpose are the *median* and the *mode*. The median is that value of the variable which it is as likely as not to exceed. Its value h is determined from the equation $\int_a^h ydx=\int_h^b ydx$ where a is the lower and b the upper limit of the value of x. The mode is the value of the variable whose probability is the greatest. For instance: in the case where the curve is represented by the equation $y=f(t)=l_{x+t}\mu_{x+t}$, t being the variable, we see that the mean value of t is the complete expectation of life at age x, that the median value is what is known as the *vie probable* or equation of life, and that the mode corresponds to the most probable after-lifetime.

When the mean value of the variable has been determined, the next question is, "How closely do the values of the variable cluster about this mean value or how widely dispersed are they?" Several methods might be proposed of measuring the degree of dispersion, but the most natural one in connection with the mean is the mean square of the departure designated by μ_2 when the departure is measured from the mean.

The mean square of the departure from any given value is known as the second moment about that value. The value of the second moment m_2 about any given origin can be readily expressed in terms of m_1, the mean value of the variable, and μ_2 as follows :

Designating the operation of taking the mean value by writing M in front of the expression, we have

$$\mu_2 = \mathrm{M}(x-m_1)^2 = \mathrm{M}x^2 - 2m_1\mathrm{M}x + m_1^2 = m_2 - 2m_1^2 + m_1^2 = m_2 - m_1^2$$

or

$$m_2 = \mu_2 + m_1^2.$$

It is thus evident that the second moment about any other value of the variable is greater than that about the mean value. In other words, taking the mean value as point of reference makes the second moment a minimum. It thus appears that the second moment has a natural connection with the mean value of the variable. The mean absolute departure, departure in either direction being considered positive, has a similar connection with the median.

Other measures in terms of μ_2 are sometimes substituted for it in order to express the dispersion as a linear magnitude. The measures most frequently so used in connection with frequency-curves in general are the *standard deviation* and the *modulus*. The standard deviation, commonly denoted by σ, is a quantity whose square is equal to the mean square of departure. In other words $\sigma^2 = \mu_2$. The modulus, sometimes denoted by c, is determined by the equation $c^2 = 2\mu_2$, and the name is derived by analogy from the normal exponential frequency-curve whose equation is $y = ke^{-\frac{x^2}{c^2}}$ in the case of which curve $\mu_2 = \frac{c^2}{2}$.

Having determined the mean value of the variable and the degree of dispersion from that mean value, the next question is whether the various possible values of the variable are dispersed symmetrically about the mean value or whether the curve is heaped up on one side and drawn out on the other. And as the first moment necessarily vanishes and the second moment can give us no information on the subject because departures in both directions enter into it positively, we are forced to look to the third moment, or the mean value of the cube of the departure, for a criterion. It is evident that if the curve is symmetrical, each positive departure will be balanced by a corresponding negative one, and so the mean value of the cube will vanish.

It is thus evident that a value of the third moment, other than zero, is an indication of a lack of symmetry. Denoting by μ_3 the third moment about the mean and by m_3 the corresponding moment about any other point taken as origin, we have

$$\mu_3 = \mathrm{M}(x-m_1)^3$$

$$= \mathrm{M}x^3 - 3m_1\mathrm{M}x^2 + 3m_1^2\mathrm{M}x - m_1^3 = m_3 - 3m_1m_2 + 2m_1^3.$$

$$= m_3 - 3m_1\mu_2 - m_1^3.$$

or $\quad m_3 = \mu_3 + 3m_1\mu_2 + m_1^3.$

Of course, the curve is not necessarily absolutely symmetrical if μ_3 vanishes, but any marked lack of symmetry would likely show itself in the value of μ_3. The value of μ_3 is usually taken as a measure of the skewness or lack of symmetry, being divided by c^3 in order that the measure may be always numerical, and the quotient being designated by j so that we have $j = \mu_3 | c^3$. Another function entering into the theory of curves of frequency is the quotient of the square of the third moment by the cube of the second moment and denoted by β_1, so that we have $\beta_1 = \mu_3^2 | \mu_2^3$. But we have $j^2 = \mu^3{}_2 | c^6 = \mu_3^2 | 8\mu_2^3$ so that $\beta_1 = 8j^2$.

μ_3^2

Similarly, further information in relation to the curve can be secured by determining the moments of higher order, but we shall in this investigation only take into account the fourth moment μ_4 or the mean value of the fourth power of the departure from the mean, and the quantity β_2 determined from the equation $\beta_2 = \mu_4 | \mu_2^2$.

The value of m_4, the fourth moment about any other point as origin, may be readily seen to be connected with that of μ_4 by the following relation

$$\mu_4 = m_4 - 4m_1m_3 + 6m_1^2m_2 - 3m_1^4 \text{ or}$$

$$m_4 = \mu_4 + 4m_1\mu_3 + 6m_1^2\mu_2 + m_1^4.$$

In practical cases it frequently occurs that the form of the curve of frequency cannot be determined exactly, but must be approximated to by observation of a limited number of cases. It is therefore of interest to investigate the method of determining approximately the values of the constants above defined, in cases where the number of observations is limited.

The similarity between the average of a finite number of cases and the mean value of an indefinitely large number, including the fact that the latter is the limit of the former when the number of cases is indefinitely increased, indicates the average as an approximate measure of the mean value. As each of the individual cases is, however, liable to variation, it is evident that the average is also so liable, and in fact has a curve of frequency of its own and the constants of this curve of frequency can be determined in terms of those of the primary curve. It is evident that the mean value of the average of n variable quantities is equal to the average of the n mean values of those quantities, and it follows that, if those n quantities follow the same law of distribution and have the same mean value, such mean value is also that of the average of the n quantities, so that the mean value in the derived curve is the same as in the primary curve. With regard to the other moments, we have, taking the mean as origin and designating moments in the derived curve by accented letters,

$$\mu_2{}^1 = \mathrm{M}\left(\frac{x_1+x_2+\ldots+x_n}{n}\right)^2 = \mathrm{M}\frac{x_1{}^2+x_2{}^2+\ldots x_n{}^2}{n^2}$$

$$+\mathrm{M}\frac{2x_1x_2+2x_1x_3+\ldots}{n^2}$$

$$=\frac{n}{n^2}\mathrm{M}x_1{}^2+\frac{n(n-1)}{n^2}\mathrm{M}x_1x_2=\frac{1}{n}\mu_2$$

$$\mu_3{}^1=\mathrm{M}\left(\frac{x_1+x_2+\ldots x_n}{n}\right)^3=\frac{1}{n^3}\mathrm{M}(x_1{}^3+x_2{}^3+\ldots x_n{}^3)$$

$$+\frac{1}{n^3}\mathrm{M}(3\Sigma x_1{}^2x_2+6\Sigma x_1x_2x_3)=\frac{1}{n^2}\mu_3$$

$$\mu_4{}^1=\mathrm{M}\left(\frac{x_1+x_2+\ldots+x_n}{n}\right)^4=\frac{1}{n^4}\mathrm{M}\{(x_1{}^4+x_2{}^4+\ldots x_n{}^4)$$

$$+4\Sigma x_1{}^3x_2+6\Sigma x_1{}^2x_2{}^2+12\Sigma x_1{}^2x_2x_3+24\Sigma x_1x_2x_3x_4\}$$

$$=\frac{1}{n^4}\{n\mu_4+3n(n-1)\mu_2{}^2\}=\frac{1}{n^3}\mu_4+\frac{3(n-1)}{n^3}\mu_2{}^2$$

The above reductions follow from the principle that the mean value of the product of two independent variables is equal to the product of their respective mean values, and from the fact that the mean value of the first powers of the variables is zero. It is

evident from the above that in relation to the other constants above mentioned we have

$$\sigma^1 = \frac{1}{\sqrt{n}}\sigma;$$

$$c^1 = \frac{1}{\sqrt{n}}c;$$

$$\beta_1{}^1 = \frac{\mu_3{}^2}{n^4} \div \frac{\mu_2{}^3}{n^3} = \frac{1}{n}\beta_1;$$

$$j^1 = \frac{1}{\sqrt{n}}j; \text{ and}$$

$$\beta_2{}^1 = \frac{\mu_4 + 3(n-1)\mu_2{}^2}{n^3} \div \frac{\mu_2{}^2}{n^2}$$

$$= \frac{1}{n}\{\beta_2 + 3(n-1)\} = 3 + \frac{1}{n}(\beta_2 - 3).$$

From these equations it is evident that as we take the average of a larger and larger number of cases, the values of $\beta_1{}^1$ and j^1 tend to vanish, that is, the curve becomes more and more symmetrical, and the value of $\beta_2{}^1$ tends to approach the value 3, which will be seen later to be the value of this function in the normal symmetrical exponential curve of frequency.

The second and higher moments, as derived from a finite number of observations, are subject to the additional source of error that besides the variability of the individual values, the true position of the mean is not known and we are forced to use the average of the known values as a substitute. The second moment, as derived from the observations, will be

$$\frac{1}{n}\Sigma\left(x_1 - \frac{\Sigma x_1}{n}\right)^2 = \frac{1}{n}\left\{\Sigma x_1{}^2 - 2\Sigma x_1\frac{\Sigma x_1}{n} + n\left(\frac{\Sigma x_1}{n}\right)^2\right\}$$

$$= \frac{\Sigma x_1{}^2}{n} - \frac{(\Sigma x_1)^2}{n^2}$$

$$= \left(\frac{1}{n} - \frac{1}{n^2}\right)\Sigma x_1{}^2 - \frac{2}{n^2}\Sigma x_1 x_2.$$

The mean value of this is evidently $\frac{n-1}{n}\mu_2$, so that the second moment as derived from n observations is on the average less

than the true second moment by $\frac{1}{n}$ of itself. The third moment is, similarly,

$$\frac{1}{n}\Sigma\left(x_1-\frac{\Sigma x_1}{n}\right)^3=\frac{1}{n}\left\{\Sigma x_1^3-3\Sigma x_1^2\frac{\Sigma x_1}{n}+3\Sigma x_1\left(\frac{\Sigma x_1}{n}\right)^2-n\left(\frac{\Sigma x_1}{n}\right)^3\right\}$$

$$=\frac{1}{n}\Sigma x_1^3-\frac{3}{n^2}(\Sigma x_1^3+\Sigma x_1^2x_2)+\frac{2}{n^3}(\Sigma x_1^3+3\Sigma x_1^2x_2+6\Sigma x_1x_2x_3).$$

The mean value of this is

$$\left(1-\frac{3}{n}+\frac{2}{n^2}\right)\mu_3=\frac{(n-1)(n-2)}{n^2}\mu_3.$$

The fourth moment, as derived from the observations, is

$$\frac{1}{n}\Sigma\left(x_1-\frac{\Sigma x_1}{n}\right)^4=\frac{1}{n}\left\{\Sigma x_1^4-\frac{4}{n}\Sigma x_1^3\Sigma x_1+\frac{6}{n^2}\Sigma x_1^2(\Sigma x_1)^2-\frac{3}{n^3}(\Sigma x_1)^4\right\}$$

$$=\frac{1}{n}\Sigma x_1^4-\frac{4}{n^2}(\Sigma x_1^4+\Sigma x_1^3x_2)+\frac{6}{n^3}(\Sigma x_1^4+\Sigma x_1^2x_2^2$$

$$+2\Sigma x_1^3x_2+2\Sigma x_1^2x_2x_3)-\frac{3}{n^4}(\Sigma x_1^4+4\Sigma x_1^3x_2$$

$$+6\Sigma x_1^2x_2^2+12\Sigma x_1^2x_2x_3+24\Sigma x_1x_2x_3x_4).$$

The mean value of this is

$$\mu_4-\frac{4}{n}\mu_4+\frac{6}{n^2}\left(\mu_4+\frac{n-1}{2}\mu_2^2\right)-\frac{3}{n^3}[\mu_4+3(n-1)\mu_2^2]$$

$$=\mu_4\left(1-\frac{4}{n}+\frac{6}{n^2}-\frac{3}{n^3}\right)+\frac{3(n-1)}{n^2}\mu_2^2\left(1-\frac{3}{n}\right)$$

$$=\frac{(n-1)(n-2)}{n^2}\mu_4-\frac{(n-1)(n-3)}{n^3}(\mu_4-3\mu_2^2).$$

Where n, the number of observations, is large, this correction is, of course, practically negligible.

There are other cases where we are seeking, not the ideal ultimate moments which would result from an indefinitely large number of observations, but a concrete method of representing the results of a finite number of observations, and, in such, the uncorrected moments, as derived from those observations should be used.

A problem in connection with the calculation of these moments arises from the fact that where the number of observations is large, we are usually given, not the exact value of the variable resulting from each observation, but the total number of cases in which that value fell within each of a number of specified intervals in value, usually proceeding by equal differences, the problem being to determine the moments from the data so given. For example: in the case of a mortality experience, we are given the total number exposed to risk in each year of age and also the number of deaths in the same interval. In such cases it is convenient to work with a function representing the total number of cases greater than a given value, as the exact value of this function can be determined for the values of the variable corresponding to the points of division between the various intervals. If this function be denoted by $\mathrm{F}(x)$, then the equation of the curve of frequency of the variable will be

$$y = -\frac{d\mathrm{F}(x)}{dx}.$$

We have then generally

$$\int_a^b (x-a)^n y\,dx = -\int_a^b (x-a)^n d\mathrm{F}(x)$$

$$= -\left[(x-a)^n \mathrm{F}(x)\right]_a^b + n\int_a^b (x-a)^{n-1}\mathrm{F}(x)\,dx.$$

But the first term on the right-hand side vanishes for both limits, if n be greater than zero, so that we have

$$\int_a^b (x-a)^n y\,dx = n\int_a^b (x-a)^{n-1}\mathrm{F}(x)\,dx$$

where a and b are limits of value within which all the observed values of the variable lie.

But we have approximately

$$\int_a^b u_x dx = \Sigma_a^b u_x - \tfrac{1}{2}(u_a + u_b)$$

$$+\tfrac{1}{12}\left(\frac{du_a}{dx} - \frac{du_b}{dx}\right) - \tfrac{1}{720}\left(\frac{d^3u_a}{dx^3} - \frac{d^3u_b}{dx^3}\right).$$

Calculating then the respective moments about $x=a$ we have, if we assume that the various derived functions of $\mathrm{F}(x)$ vanish for

$x = a$ and $x = b$ or, in other words, that the curve of frequency has high contact with the curve at the limiting values—

$$m_1 F(a) = \int_a^b (x-a)y\,dx = \int_a^b F(x)\,dx = \Sigma_a^b F(x) - \tfrac{1}{2}F(a)$$

$$= \Sigma_a^{b-1}(x-a+\tfrac{1}{2})\{F(x)-F(x+1)\}, \text{ since } F(b) \text{ vanishes.}$$

$$m_2 F(a) = 2\int_a^b (x-a)F(x)\,dx = 2\Sigma_a^b (x-a)F(x) + \tfrac{1}{6}F(a)$$

$$= \Sigma_a^{b-1}(x-a+\tfrac{1}{2})^2\{F(x)-F(x+1)\} - \tfrac{1}{12}F(a).$$

$$m_3 F(a) = 3\int_a^b (x-a)^2 F(x)\,dx = 3\Sigma_a^b (x-a)^2 F(x)$$

$$= \Sigma_a^{b-1}\{(x-a+\tfrac{1}{2})^3 - \tfrac{1}{4}(x-a+\tfrac{1}{2})\}\{F(x)-F(x+1)\}$$

$$= \Sigma_a^{b-1}(x-a+\tfrac{1}{2})^3\{F(x)-F(x+1)\} - \tfrac{1}{4}m_1 F(a).$$

$$m_4 F(a) = 4\int_a^b (x-a)^3 F(x)\,dx = 4\Sigma_a^b (x-a)^3 F(x) - \tfrac{1}{30}F(a)$$

$$= \Sigma_a^{b-1}(x-a+\tfrac{1}{2})^4\{F(x)-F(x+1)\}$$
$$\qquad - \tfrac{1}{2}\Sigma_a^{b-1}(x-a+\tfrac{1}{2})^2\{F(x)-F(x+1)\} + \tfrac{7}{240}F(a)$$

$$= \Sigma_a^{b-1}(x-a+\tfrac{1}{2})^4\{F(x)-F(x+1)\} - \tfrac{1}{2}m_2 F(a) - \tfrac{1}{80}F(a).$$

It is evident in the above equations that $F(x)-F(x+1)$ is the number of cases in which the value of the variable falls between x and $x+1$, and from the first equation we see that the mean value can be correctly determined by assuming all the cases in each interval to be concentrated at the middle point of the interval. If we designate then by accented letters the second and higher moments computed on that assumption, we have from the remaining equations—

$$m_2 = m_2^1 - \tfrac{1}{12}$$
$$m_3 = m_3^1 - \tfrac{1}{4}m_1$$
$$m_4 = m_4^1 - \tfrac{1}{2}m_2^1 + \tfrac{7}{240}.$$

Or, reducing to the mean value as point of reference

$$\mu_2 = m_2 - m_1^2 = m_2^1 - m_1^2 - \tfrac{1}{12} = \mu_2^1 - \tfrac{1}{12}$$

$$\mu_3 = m_3 - 3m_1 m_2 + 2m_1^3 = m_3^1 - \tfrac{1}{4}m_1 - 3m_1 m_2^1$$
$$\qquad + \tfrac{1}{4}m_1 + 2m_1^3 = \mu_3^1$$

$$\mu_4 = m_4 - 4m_1 m_3 + 6m_1^2 m_2 - 3m_1^4$$
$$= m_4^1 - \tfrac{1}{2}m_2^1 + \tfrac{7}{240} - 4m_1 m_3^1 + m_1^2 + 6m_1^2 m_2^1$$
$$\qquad - \tfrac{1}{2}m_1^2 - 3m_1^4$$
$$= m_4^1 - 4m_1 m_3^1 + 6m_1^2 m_2^1 - 3m_1^4 - \tfrac{1}{2}m_2^1 + \tfrac{1}{2}m_1^2 + \tfrac{7}{240}$$
$$= \mu_4^1 - \tfrac{1}{2}\mu_2^1 + \tfrac{7}{240} = \mu_4^1 - \tfrac{1}{2}\mu_2 - \tfrac{1}{80}.$$

These formulas give the corrections to be applied to the calculated second and fourth moments in order to determine their true value and show that the third moment is obtained correctly without adjustment.

Hitherto we have dealt with general principles applicable to any curve of frequency, but there is a special family of such curves which have in recent years almost appropriated the title to themselves. I refer to the family of curves whose general differential equation takes the form, when the mean value is taken as the origin of x, of $\frac{1}{y}\frac{dy}{dx}=\frac{-(b+x)}{c+bx+ax^2}$. This family of curves has come into prominence partly because it includes as one of its members the curve of frequency, as given in the theory of inverse probability, of the probability of an event which has been observed to happen a specified number of times out of a given number of trials, and as another member the well-known exponential curve of error. But its principal claim to notice arises from the fact that a great number of the distributions occurring in nature appear to closely resemble some one of the types of curve included in his family.

It is evident from the form of the equation, that *log y* becomes, in general, infinite either positively or negatively along with x and also for values of x for which the denominator vanishes, so that these values of x will be the limits of the curve, y either vanishing or becoming infinite. We have, therefore, three essentially different cases according as the roots of the equation $ax^2+bx+c=0$ are real and of different signs, real and of the same sign or complex. In the first case, the curve is limited in both directions, the two roots of the equation being the limiting values. In the second case, it is limited in one direction by the numerically least of the two roots, but extends to infinity in the other direction, while in the last case the curve is unlimited in both directions. In passing from the first to the second curve, we have a limiting case where one root is infinite and in passing from the second to the third, we have the case where the two roots are equal; in both of these cases the curve is limited in one direction and unlimited in the other. Of sufficient importance to be distinguished from these are the three cases where b vanishes, the cases being distinguished according as a/c is negative, positive or zero. The first of these is a particular case of the first type above mentioned, and the second of them is a particular case of the third type, while the last is a limiting case through which one

passes into the other. In this last case, the equation takes the form

$$\frac{1}{y}\frac{dy}{dx}=\frac{d}{dx}\log y=-\frac{x}{c}$$

Hence
$$\log y=\log y_0-\frac{x^2}{2c}$$

$$\text{or } y=y_0e^{-\frac{x^2}{2c}}.$$

This is the well-known exponential law of error. We have thus, in all, eight distinct types of curve belonging to this family.

Independently, however, of the type of curve involved, we can, subject only to certain conditions, express the coefficients of the differential equation in terms of the various moments, and it will be seen by an examination of these conditions that they are really only the conditions that the various moments involved should not become infinite.

From the fundamental equation we have—

$$(b+x)y=-(c+bx+ax^2)\frac{dy}{dx}.$$

Suppose now l and m are the limiting values of the curve, these values being not necessarily finite.

Then
$$\int_l^m (b+x)x^{n-1}ydx$$
$$=-\int_l^m (cx^{n-1}+bx^n+ax^{n+1})\frac{dy}{dx}dx$$
$$=-\left[(cx^{n-1}+bx^n+ax^{n+1})y\right]_l^m$$
$$+\int_l^m \{(n-1)cx^{n-2}+nbx^{n-1}+(n+1)ax^n\}ydx$$

or, if $(c+bx+ax^2)x^{n-1}y$ vanishes for both limits.

$$b\mu_{n-1}+\mu_n=(n-1)c\mu_{n-2}+nb\mu_{n-1}+(n+1)a\mu_n$$

or, $\quad \{1-(n+1)a\}\mu_n=(n-1)b\mu_{n-1}+(n-1)c\mu_{n-2}.$

Putting then n equal in succession to 1, 2, 3, and 4, we get, since $\mu_0=1$, $(1-2a)\mu_1=0$, as we should, since the mean value has been taken as the origin,

$$(1-3a)\mu_2=c$$
$$(1-4a)\mu_3=2b\mu_2$$
$$(1-5a)\mu_4=3b\mu_3+3c\mu_2.$$

Eliminating b and c from these equations, we have—

$$2(1-5a)\mu_4\mu_2-3(1-4a)\mu_3^2-6(1-3a)\mu_2^3=0$$

or, dividing through by μ_2^3,

$$2(1-5a)\beta_2-3(1-4a)\beta_1-6(1-3a)=0$$

whence $a=\dfrac{2\beta_2-3\beta_1-6}{10\beta_2-12\beta_1-18}=\dfrac{2(\beta_2+3)-3(\beta_1+4)}{10(\beta_2+3)-12(\beta_1+4)}=\dfrac{2-3\gamma}{10-12\gamma}$

if we put γ for $\dfrac{\beta_1+4}{\beta_2+3}$.

Substituting then in the first two equations, we have—

$$b=\frac{1-4a}{2}\frac{\mu_3}{\mu_2}=\frac{1}{10-12\gamma}\frac{\mu_3}{\mu_2}$$

$$c=(1-3a)\mu_2=\frac{4-3\gamma}{10-12\gamma}\mu_2.$$

The criterion of the nature of the roots of the equation $ax^2+bx+c=0$ is

$$k_2=\frac{b^2}{4ac}=\frac{1}{4(2-3\gamma)(4-3\gamma)}\frac{\mu_3^2}{\mu_2^3}=\frac{\beta_1}{4(2-3\gamma)(4-3\gamma)}.$$

If this is negative, the roots are real and of different signs; if it is positive and greater than unity, they are real and of the same sign; while if it is positive and less than unity, they are complex; if k_2 is infinite, one root of the equation is infinite; while if it is equal to unity, the two roots are equal. The case where k_2 and consequently b vanishes requires a further criterion and we have seen that $\dfrac{a}{c}$ or $\dfrac{2-3\gamma}{4-3\gamma}\dfrac{1}{\mu_2}$ forms this criterion. Since μ_2 is essentially a positive quantity, this is negative if γ lies between $\frac{2}{3}$ and $\frac{4}{3}$, zero or infinite according as γ has one or the other of those values, and positive otherwise. It is evident from the form of the expression for k_2 that it is negative if $9(1-\gamma)^2$ is less than unity, positive and greater than unity if $9(1-\gamma)^2$ is greater than unity and less than $1+\dfrac{\beta_1}{4}$, and positive and less than unity if $9(1-\gamma)^2$ is greater than $1+\dfrac{\beta_1}{4}$; also that $\dfrac{a}{c}$ is negative if $9(1-\gamma^2)$ is less than unity and positive if it is greater, so that the value of γ relatively to that of β_1 forms a criterion of the type of curve which will give the required moments.

We have then the following eight types:

	Value of $9(1-\gamma)^2$	Value of β_1
I	$=1$	$=0$
II	$=1$	>0
III	<1	>0
IV	<1	$=0$
V	$>1<1+\frac{\beta_1}{4}$	>0
VI	$=1+\frac{\beta_1}{4}$	>0
VII	$>1+\frac{\beta_1}{4}$	>0
VIII	>1	$=0$

With the integration of this function and the consequent determination of the equation it is unnecessary to deal here, as it reduces readily to one of the fundamental forms taken up in all treatises on the Infinitesimal Calculus. I shall, therefore, content myself with appending a table, showing the equation of each type in its simplest form together with a statement of the mean value of x, of the values of μ_2, β_1, γ and k_2 and of the limitations to which the constants in the equation are subject in order that the conditions on p. 440 may be satisfied. On examination it will be seen that these are also the conditions that the area of the curve should be finite without k vanishing and that the values of the various moments up to the fourth inclusive should be finite (*see* p. 437). I have assumed, in the case of the curves which are limited in one direction, that the skewness is positive and consequently that the direction which is limited is the negative one. The equation of curves of negative skewness can be obtained by putting $-x$ for x in the formula and changing the signs in the column giving the limits of the value of x and also in the mean value column.

When it is desired to fit a curve, following any other general law, to a given set of observations, the general method would be similar to that above indicated for this particular family of curves. The values of the various moments of any selected point should be expressed in terms of the arbitrary constants involved in the general form of the equation. A sufficient number of orders of moments must be used to give equations equal in number to the arbitrary constants involved. From these simultaneous equations the values of the arbitrary constants would then be determined. Of course, it is readily seen that an essential condition of the applicability of this method is the possibility of expressing the various moments as some known function of the arbitrary constants.

Types of Frequency-Curves.

Type	Equation $y=$	Limits of x		Mean m_1	μ_2	β_1	γ	κ_2	Limitations of Constants
		Lower	Upper						
I	$ke^{-\frac{x^3}{a^2}}$	$-\infty$	$+\infty$	0	$\frac{1}{2}a^2$	0	$\frac{2}{3}$	0	...
II	$kx^{m-1}e^{-\frac{x}{a}}$	0	$+\infty$	ma	ma^2	$\frac{4}{m}$	$\frac{2}{3}$	∞	$m>0;\ a>0$
III	$(p+q=1)$ $k(a-x)^{np-1}(a+x)^{nq-1}$	$-a$	$+a$	$(q-p)a$	$\frac{4pq}{n+1}a^2$	$\frac{4(n+1)(p-q)^2}{(n+2)^2pq}$	$\frac{2}{3}\cdot\frac{n+3}{n+2}$	$-\left(\frac{1}{4pq}-1\right)$	$p>0;\ q>0;\ n>0;\ a>0$
IV	$k(a^2-x^2)^{\frac{n}{2}-1}$	$-a$	$+a$	0	$\frac{a^2}{n+1}$	0	$\frac{2}{3}\cdot\frac{n+3}{n+2}$	0	$n>0;\ a>0$
V	$(q-p=1)$ $k(x-a)^{np-1}(x+a)^{-(nq+1)}$	$+a$	$+\infty$	$(p+q)a$	$\frac{4pq}{n-1}a^2$	$\frac{4(n-1)(p+q)^2}{(n-2)^2pq}$	$\frac{2}{3}\cdot\frac{n-3}{n-2}$	$\frac{1}{4pq}+1$	$p>0;\ n>3;\ a>0$
VI	$kx^{-(n+2)}e^{-\frac{a}{x}}$	0	$+\infty$	$\frac{a}{n}$	$\frac{a^2}{n^2(n-1)}$	$\frac{16(n-1)}{(n-2)^2}$	$\frac{2}{3}\cdot\frac{n-3}{n-2}$	1	$n>3;\ a>0$
VII	$k(a^2+x^2)^{-\left(\frac{n}{2}+1\right)}e^{\nu\tan^{-1}\frac{x}{a}}$	$-\infty$	$+\infty$	$\frac{\nu a}{n}$	$\frac{n^2+\nu^2}{n^2(n-1)}a^2$	$\frac{16(n-1)}{(n-2)^2}\cdot\frac{\nu^2}{n^2+\nu^2}$	$\frac{2}{3}\cdot\frac{n-3}{n-2}$	$\frac{\nu^2}{n^2+\nu^2}$	$n>3$
VIII	$k(a^2+x^2)^{-\left(\frac{n}{2}+1\right)}$	$-\infty$	$+\infty$	0	$\frac{a^2}{n-1}$	0	$\frac{2}{3}\cdot\frac{n-3}{n-2}$	0	$n>3$

REVIEWS.

Frequency-Curves and Correlation. By W. Palin Elderton. Published for the Institute of Actuaries by C. & E. Layton, London.

ALTHOUGH it has been customary in actuarial literature to recognize three classes of graduation-methods—the mechanical or summation, the graphic, and the mathematical or analytical—it is only by courtesy that the mathematical methods have been given the dignity of a class, for they have in fact been generally identified with a solitary representative, namely, the Gompertz-Makeham and its developments, and more particularly with the special application of that method described in the *Text-Book*, Part II. It is true that occasional references to the important work that has been done of late years in scientific curve-fitting have found their way into the *Journal* and other actuarial publications, and the expression "frequency-curve" has even come to have a familiar sound at Staple Inn Hall if it has remained for most a "word like Basingstoke teeming with hidden meaning." Mr. G. F. Hardy's review of Prof. Pearson's "Chances of Death" (*J.I.A.*, xxxiii, 530), the courses of lectures given by Mr. Bowley and Mr. Hardy at the Institute, Prof. Glover's Makeham graduation of the American Table by the moments method (*Transactions of the Actuarial Society of America*, vii, 339), Mr. Henderson's short but most useful paper on frequency-curves and moments* (*Transactions of the Actuarial Society of America*, 1904), Mr. Elderton's "Temporary Assurances" and "Spurious Selection" (*J.I.A.*, xxxvii, 501 and xl, 221)—these, and perhaps even more Mr. Hardy's graduation of the British Offices Tables, have doubtless led a few enterprising members of the profession to the pages of *Biometrika* and other original sources. But what they have done for the few, the book which forms the subject of this notice does for the many. The book is in the main a compilation, but it is a compilation with practical actuarial examples, and this makes all the difference to an actuarial reader. Processes which seemed mysterious and abstruse when applied to the Müllerian glands of 2,000 male swine or the stature of St. Louis school girls, assume at once a practical meaning to the actuarial student when exemplified by the familiar exposed to risk and deaths. It is scarcely too much to say, therefore, that Mr. Elderton has rendered possible a wider conception of mathematical methods of graduation than has hitherto been generally entertained in actuarial circles.

In the present stage of statistical knowledge any work on scientific curve-fitting must necessarily fall far short of the ideal—if such a thing be attainable. The ultimate treatise on curve-fitting containing a complete classification of the various mathematical formulas which can be used to represent or graduate statistical data, with an indication of the classes of statistics to which they are respectively adapted and with diagrams showing the various shapes

* At the suggestion of several members of the Institute, Mr. Henderson's paper has been reprinted (by permission) on pp. 429–442 of the present volume.—[ED. *J.I.A.*]

they assume for different values of the constants, is not yet possible. But in Prof. Pearson's method of moments and family of frequency-curves there is the material for two very important chapters in that treatise, and these two chapters are admirably written in Part I of Mr. Elderton's book. The method of moments is of course of general application. Given any series of statistical observations u and any mathematical expression $f(x)$—to take the case of one variable only—by which it is proposed to represent or graduate them, all that is necessary is to equate the values of $\int x^r u dx$ and $\int x^r f(x) dx$ for $r=0, 1, 2$, &c., until as many equations are obtained as there are constants in $f(x)$, and then to solve the resulting simultaneous equations. The question naturally arises as to how the method compares with other methods—such as the method of least squares, the method employed by Mr. King and Mr. Hardy, in the *Text-Book*, Chap. VI or Dr. Cantelli's very similar method of areas, the successive summation or accumulated deviation method, or the special combination of the latter method and the moments method suggested by Mr. Henderson (*Transactions of the Actuarial Society of America*, vii, p. 485). As compared with any of these methods the moments method seems to have the advantage of being more generally applicable, and—as compared with the *Text-Book* or Dr. Cantelli's methods—of avoiding the necessity of trial-groupings of the data. It is not clear, however, that it is to be preferred to the least squares or summation methods in cases where these methods can be readily employed. If applied to a parabolic curve by finite integration in the form $\Sigma x^r u = \Sigma x^r f(x)$ it is identical with the method of least squares, as Prof. Pearson has shown—or with the summation methods—but if applied in the usual way it gives results which differ materially from those obtained by the other methods. This may be illustrated by the following comparisons with the graduations given by Mr. Elderton on pp. 32 and 34.

Unadjusted Values	Graduation by $y=a+bx+cx^2$				Unadjusted Values	Graduation by $y=a+bx+cx^2+dx^3$			
	By Moments	Deviation	By Least Squares or Summation	Deviation		By Moments	Deviation	By Least Squares or Summation	Deviation
51·81	52·21	·40	52·59	·78	21·20	21·13	−·07	21·09	−·11
43·74	43·19	−·55	43·35	−·39	19·91	20·28	·37	20·21	·30
35·58	34·91	−·67	34·90	−·68	19·34	19·31	−·03	19·24	−·10
27·80	27·37	−·43	27·24	−·56	18·58	18·22	−·36	18·19	−·39
20·42	20·56	·14	20·39	−·03	16·74	17·02	·28	17·06	·32
13·79	14·49	·70	14·32	·53	15·69	15·76	·07	15·82	·13
8·26	9·16	·90	9·06	·80	14·70	14·43	−·27	14·49	−·21
4·29	4·56	·27	4·60	·31	12·99	13·05	·06	13·05	·06
1·69	·70	−·99	·93	−·76					
207·38	207·15	±5·05	207·38	±4·84	139·15	139·20	±1·51	139·15	±1·62

It is not suggested that the least squares or summation graduation is necessarily the better representation of the entire curve. But it gives the most probable values of the particular ordinates represented by the data, and it avoids the error resulting from the necessity of adjusting the statistical moments. With such an expression as $a+bc^x$ the equations given by the method of least squares are intractable, but given the value of c (whether determined by trial or by the moments method), the values of a and b may be found by least squares or finite integration—in other, words, by Mr. Hardy's method of equating the sum and accumulated sum of the deviations to zero—more satisfactorily, perhaps, than by moments. The method of moments has in fact the defects of its qualities. It has the great merits of extensive applicability and of taking account in a systematic way of the whole of the observations, but as a necessary consequence it admits of no conception of the objects of a graduation except that of obtaining on the whole a good representation of the data, and it treats the ends of the range of observation (which may for the particular purpose in view be of little or no importance) as entitled to equal weight (relatively to their magnitude) with any other part; in fact, owing to the high values of x^n at the ends of the range, the tails of the experience may exercise a disproportionate influence on the values of the constants. Mr. Elderton is apparently disposed to think that the advantage of systematic treatment outweighs any other consideration. That may be so in general statistical work; but in the special examples with which actuaries are more particularly concerned a case might perhaps be made out for more elastic methods.

In Mr. Elderton's discussion of the method of moments the two most interesting features are his ingenious adaptation of Mr. Hardy's summation method to the calculation of the moments, and the application of his new quadrature formulas to their adjustment when the statistics represent a series of ordinates without high contact. In comparison with Weddle's formula (for example), Mr. Elderton's seems simplicity itself. In fact, it appears to remove the only serious difficulty in the application of the method to a series of ordinates. The only doubt that suggests itself (more especially when the statistics comprise a few terms only, or when the end terms are relatively large) is whether the use of one and the same quadrature formula for the area and the successive moments can produce a consistent set of simultaneous equations. With regard to cases in which the statistics represent a system of areas and there is not high contact, Mr. Elderton recommends, for want at present of any satisfactory adjustment, the use of the unadjusted moments. An alternative would appear to be to deal with the areas in such cases as a system of ordinates, and to take the ordinates of the resulting curve as the adjusted values of the original areas. This would, no doubt, be inappropriate in the case of a genuine frequency distribution. But in the majority of cases the process of curve-fitting can hardly be regarded as anything more than the graduation of a series of numbers, and in these circumstances it is

immaterial whether the numbers represent ordinates or areas. Unfortunately, the course suggested, although practicable in the case of graduation by a parabolic curve (where the integration can be performed between suitable limits, namely, over a range extending one-half an interval beyond each end of the data), does not appear to be applicable in the case of a graduation by a frequency-curve. In fact it is not clear that the adjustments where there is not high contact can be properly used in the graduation of a series of ordinates by a frequency-curve, because the assumption upon which they depend, namely, that the integration of the curve is over the range $-\frac{1}{2}$ to $n-\frac{1}{2}$ breaks down. Thus in the Type I example if the data be treated as ordinates the curve starts at age 17·074 and the first group disappears altogether. In this and similar cases Mr. Elderton seems inclined to abandon the strict severity of the systematic method, and to fix the start or the start and range of the curve, when possible, by general considerations. This suggestion admits an element of elasticity which certainly leads to an improved result in the case of the Type I example (p. 102). In the amended graduation, however, of the Type III example (p. 103), there seems to be some mistake, the value of the first moment being incorrect. In this case ·75 would appear on general grounds to be the proper starting point for the curve. This reduces the criterion (numerically) to $-3·792$, so that a Type I curve seems to be indicated. Neither Type I nor Type III, however, leads to an altogether satisfactory result, the former, if applied in the most general way giving a curve starting at 1·1409 (which is out of the question), and the latter with the start at ·75 giving 51, 117, 50, 20, 8, $3\frac{1}{2}$, $1\frac{1}{2}$, 0. It seems almost necessary in this case to introduce a further element of elasticity by omitting the awkward final term of the series. Type III then gives 41, 129, 51, 18, 6, 2, 1. The example seems, however, to come very near a distribution to which a frequency-curve is inapplicable.

Various definitions of a "frequency distribution" have been given, from the extremely wide definition which practically identifies such a distribution with any function of an independent variable to the restricted one which confines it to the distribution of the occurrences of a given single event in a given number of trials. Mr. Elderton's definition is of the more restrictive variety—an arrangement of statistics showing the frequency with which an event happens in a particular way. It seems of less importance, however, to agree upon a definition of a frequency distribution or a frequency-curve, than to bear in mind that mere definition will not in itself create a fit. It is of academic interest that the differential equation from which the group of curves known as frequency-curves is derived was suggested by certain general features of chance distributions—the maximum at the position of greatest frequency and the zero values (often with high contact) at the ends of the range. It is instructive, also, and suggestive of the possibility of wider generalization in the future, to recognize that the group includes the representation of the occurrences of events dependent

on certain elementary probabilities. But at present, and for practical purposes, the case for frequency-curves rests on the results they produce. And this case is a remarkably strong one. That they adapt themselves to actuarial data of a certain class—no less than to the botanical, zoological and anthropometrical observations to which they have been principally applied—is shown by the examples with which Mr. Elderton has illustrated the several types. In all these examples with the exception of those of Type III (to which reference has already been made) and Type V (in which case a better result would no doubt have been obtained by the use of the strictly appropriate type of curve) a good graduation is produced whether tested roughly by the eye or inspection of the adjusted and unadjusted figures, or measured by the method of Chapter IX. It would be interesting, however, to know whether Mr. Elderton has come across any (or many) cases in which the family has failed to graduate satisfactorily an apparently suitable distribution. All mathematical methods of adjustment are open to the objection that they place a certain constraint upon the data. But the method of frequency-curves meets this objection at least half way. It exercises, no doubt, a certain measure of constraint. For example, it imposes on the adjusted results the obligation that $y(b_0+b_1x+b_2x^2)$—not necessarily y as stated by Mr. Elderton on p. 39—shall vanish at the ends of the range (a condition involving certain restrictions on the constants of the type equations) and admits of no secondary maxima and minima (except indirectly by superposition of curves). But at the same time it offers the data a very considerable latitude in the choice between seven curves—or between eight, if we follow Mr. Henderson in making two separate types of the case in which $\kappa=0$ and β_2 is respectively $>$ and <3.

From a practical standpoint the most important section of Mr. Elderton's book is that in which he shows how the method of frequency-curves can be applied to the graduation of rates. In the discussion on Professor Glover's graduation of the American Table, Mr. Henderson suggested that instead of operating on log l_x—as Professor Glover following Professor Pearson (*Biometrika*, Vol. I, p. 298) had done—it would be better to use a function more closely resembling the actual distribution of the deaths. Mr. Elderton's method of taking a normal distribution of exposed and graduating the deaths obtained by multiplying the hypothetical exposed by the observed rates of mortality, marriage, &c., as the case may be, seems to be based on a very similar idea. It has, moreover, the necessary effect, as regards the use of the frequency-curve method, of giving the whole of a distribution for adjustment independently of whether the range of the original observations is complete or not. Of special interest is the application of the method to a Makeham graduation, in which case a distribution analogous to the deaths is obtained in the form of the sum of two normal distributions, and the Makeham constants can be expressed in terms of the total frequency and the first two moments. The whole process is extremely simple, and has the recommendation of taking account (roughly) of the weights of

the observations. Two graduations of the $O^{NM(5)}$ Table (in 5-year groups) are given—one by this method and the other by a frequency-curve. The latter is apparently the better graduation but has not of course the practical advantages of a Makeham graduation. In fact, a frequency-curve graduation of rates by reference to a hypothetical distribution of exposed (or by graduation of the actual exposed and deaths separately) seems necessarily to involve the loss of any incidental advantages accruing from the representation of rates by an exact mathematical formula, because the rates are determined as the ratios of areas. The force of mortality could apparently be expressed mathematically, but the formula would be of a somewhat complicated nature.

Mr. Elderton expresses some doubt in his introduction as to the bearing of Part II on actuarial work. It seems open to question, however, whether the subjects of Part II are not really of much greater practical importance to the actuary than that of Part I. The single practical application of Part I is to the problem of graduation—a problem of considerable historical and academic interest, but one which the average actuary rarely requires to solve in any but the roughest and readiest way—whereas the applications of probable error in office work and actuarial finance and of correlation in the comparative study of actuarial statistics are of frequent occurrence. Mr. Elderton's examples of the use of the probable error in providing a safety-loading on the rate for a special risk (p. 133), and of the correlation in certain vaccination statistics are cases in point. It is to be hoped, therefore, that readers of the book will not regard the above-mentioned expression of doubt as a reason for omitting the second Part, nor be deterred from studying it by the author's warning that it is harder than Part I. This is a misfortune that might, perhaps, have been avoided if Mr. Elderton had not thought that the subject would be regarded by actuaries as unpractical, and that it was, therefore, useless to maintain the effort of writing down to the level of the ordinary student. Chapters VII–X are comparatively simple and straightforward—the last two, in fact, being exceptionally lucid and instructive—but the Part unfortunately opens with Chapter VI, which, with its "complex of measurable characteristics" and formal propositions on spurious correlation, seems unnecessarily difficult. The elementary theory of correlation might perhaps have been more simply developed on the lines followed in Mr. Bowley's *Elements of Statistics* or *Measurement of Groups and Series*, or in conformity with the method of Part I by deducing the equation to the normal correlation surface from the partial differential equations $\frac{1}{z}\frac{\delta z}{\delta x} = a_0x + a_1$ and $\frac{1}{z}\frac{\delta z}{\delta y} = b_0y + b_1$ (the normal curves representing the x and y arrays respectively), and determining the constants by equating moments. A good diagram of the surface, showing the axes of a horizontal elliptical section and the maximum ordinate of a vertical section representing an array would also have been helpful.

Throughout the book the mathematical analysis is excellent and

often elegant, and the reader's indebtedness for the numerical examples would be even greater than it, in fact, is were it not that Mr. Elderton is credited with the faculty of being able to do these things on the proverbial half-sheet of notepaper. Of the analytical work the only criticism that may be suggested is that it has the defect—from the student's point of view—of the author's exceptional facility in work of this nature. Mr. Elderton's mental athleticism renders him independent at times of such things as origins and limits of integration. On page 39 the limits of integration are not clearly indicated, at the top of the following page the origin is changed without any change of constants, and throughout the subsequent analysis it is continually on the move; nor is its position shown in the type-diagrams. Again, in the proofs of the several type-formulas the values of the constants are worked out quite independently, with the result that the student may lose sight of the fact that with the exception of y_0 all the constants can be obtained directly from Formula III on page 40, and the way in which the condition $y(b_0+b_1x+b_2x^2)=0$ at the ends of the range is involved in the derivation of the constants is not explained. The Type IV constants in particular appear to be obtainable more simply by the method of page 40. A minor point that may be mentioned is that it would have been convenient if the types could have been numbered from left to right from $\kappa = -\infty$ to $\kappa = \infty$.

As is usual and unavoidable in books of this nature a few slips or misprints appear to have escaped the proof-readers. For instance, on page 30 the adjustment of the statistics in Example II is stated to be advisable, whereas on page 55 these statistics are used without adjustment, on page 102 the second moment about age 17 is stated to be 29·208 instead of 25·093, and on pp. 111 and 112 $\frac{1}{2\sigma_1^2}$ and $\frac{1}{2\sigma_2^2}$ should be $\frac{1}{\sigma_1^2}$ and $\frac{1}{\sigma_2^2}$ respectively, and r should be $-\frac{c_{12}}{\sqrt{c_1c_2}}$. As the book is not too easy to read without unintentional obstacles, it is to be hoped that a list of errata may be published in an early number of the *Journal.*

R. T.

Mr. Altenburger on Graduation.

Beiträge zum Problem der Ausgleichung von Sterblichkeitstafeln. Von Julius Altenburger. Mitteilungen des Österreichisch-ungarischen Verbandes der Privat-Versicherungs-Anstalten. New Series, vol. I.

Versuch einer allgemeiner Theorie der Mechanischen Ausgleichungs-Methoden. Von Julius Altenburger. Mitteilungen des Österreichisch-ungarischen Verbandes der Privat-Versicherungs-Anstalten. New Series, vol. III.

It cannot be said that the graduation formulas given in these papers are in themselves likely to materially enhance the reputation of the "mechanical" method. For practical purposes they seem

decidedly inferior to Mr. G. F. Hardy's Friendly Society formula or the formulas more recently devised by Mr. Spencer. But the principles underlying them and the processes by which they are obtained are of considerable interest, and may lead hereafter to results of greater practical utility. Mr. Altenburger's idea seems to be that mechanical graduation is essentially a process of progressive improvement, and that the graduation formula should be so constructed as to lead, by repeated application or by the inclusion of additional terms, to successively closer approximations to a satisfactory graduation. Thus, in the three formulas given in the first paper, namely, $\left[\frac{1}{2^8}(175S_2 - 28S_4 + 3S_6)\right]^{2n}$, $\left[\frac{1}{2^4}(10S_2 - S_4)\right]^{2n}$, and $\frac{1}{2^{2n}}S_2^{2n}$—correct to 5th, 3rd and 1st differences respectively—the index n denotes that the operation is to be repeated until further repetition ceases to produce any material alteration, or until the graduator is satisfied with the degree of smoothness attained, subject, of course, to the limitation imposed by the loss of terms by each operation. Similarly, in the case of the illustrative formula given at the end of the second paper, namely, $\frac{1}{5^6}S_5^6 - 6\Delta + 21\Delta^2 -$ &c., where Δ denotes $\frac{1}{5^7}S_5^7 - \frac{1}{5^6}S_5^6$, the intention is that successive graduations should be obtained by taking into account the successive differences, $\frac{1}{5^6}S_5^6$ being correct to 1st differences (of u), $\frac{1}{5^6}S_5^6 - 6\Delta$ to 3rd differences, $\frac{1}{5^6}S_5^6 - 6\Delta + 21\Delta^2$ to 5th differences, and so on.

The way in which the formulas of the first paper are obtained is rather curious. Mr. Altenburger appears to have thought, when he began his researches, that the deformation of the mortality curve resulting from the assumption of constant third differences was a serious defect in Woolhouse's, Higham's and Karup's methods, and that a better formula would be obtained if this error could be eliminated. Accordingly he proceeds to investigate the conditions under which the terms of a series can be reproduced by summation. It seems fairly obvious that an actual term of a series cannot be exactly expressed by a summation formula without the imposition of some law on the terms included in the range of summation. In fact, the general investigation shows that in order that u may be expressed as a function of S_3, S_{33}, S_{333}, &c. (to take the simple case of summation in threes), 5 terms must be on a curve of the 3rd degree, 7 on a curve of the 5th degree, or, generally $2n+3$ on a curve of the $(2n+1)$th degree; in the case of summation in fives 9 terms must be on a curve of the 3rd degree, or, generally, $4n+5$ on a curve of the $(2n+1)$th degree. With the idea, however, of avoiding any assumption of this kind, Mr. Altenburger has recourse to summation in twos, which

admits of the interpolated term situated in the middle of a range of $2n+2$ terms being expressed as a linear function of S_2, S_{222} . ., S_2^{2n}, on the assumption that the $2n+2$ terms lie on a curve of the $(2n+1)$th degree. Thus the interpolated term in the middle of a range of 6 terms lying on a curve of the 5th degree $= \frac{1}{8}\left(\frac{15S_2}{2} - \frac{10S_{222}}{8} + \frac{3S_{22222}}{32}\right)$, which, after expansion and rearrangement of the terms may be written in the form $\frac{1}{2^8}(175S_2 - 28S_4 + 3S_6)$; similarly, for 4 terms on a curve of the 3rd degree $u = \frac{1}{2^4}(10S_2 - S_4)$; and for 2 terms $u = \frac{1}{2}S_2$. But this gives *interpolated* values only. In order to obtain graduated values of the terms of the original series the operations must be repeated an even number of times (leading to the symbolical formulas given above), which has the effect of introducing the very assumption that it was the object of the process to avoid. Notwithstanding this apparently unsatisfactory result, Mr. Altenburger regards the formulas with some parental pride. For practical purposes he rejects the first (which is correct to 5th differences) because it does not graduate sufficiently, the coefficient of the central term with $n=1$ being too predominant (·706) and repeated application being too laborious and reducing the number of terms too rapidly, but he recommends the third—which has an error of no less than $\frac{n}{4}(\Delta^2 - \Delta^3)$—for very rough data on account of its simplicity, its natural basis, and its smoothing power. The second he uses in the form $\left[\frac{1}{16}(10S_2 - S_4)\right]^6 -$ a 19-term formula with an error of $\frac{9}{64}(\Delta^4 - 2\Delta^5)$ and coefficients descending from ·4883992 to ·0000001 with six changes of sign. A graduation of the Gotha [(7)] Table by this formula appears to compare favourably with Karup's as regards expected and actual deaths (in consequence, no doubt, of the small 4th difference error), but the graduation is far from smooth—as might be expected from the steepness of the coefficient-curve and from the consequent slight reduction of the probable error. All the formulas admit of an interesting geometrical interpretation. The second, for example, represents the result of drawing curves of the 3rd degree through every succeeding 4 points, taking the middle points as a first adjustment and repeating the process; the third of taking the middle points of the straight lines, joining every succeeding two points and repeating the process with the new points thus obtained, the nature of the consequent deformation being thus clearly indicated.

In the second paper, Mr. Altenburger takes up a more satisfactory line of research. Dividing the ungraduated value η into an unknown true value y and an error ϵ, he shows that if A_r be a symmetrical linear operator (*i.e.*, an operator of the form

$\Sigma_{k=-n}^{k=n} a_k \eta_{x+k}$) which reproduces y correctly to the rth differential coefficient, then by combination of different powers of A_r (where the power denotes symbolically the repetition of the operation), or by combination of different operations A_r, B_r, &c., formulas may be obtained which will reproduce y correctly to the $(pr+p-1)$th differential coefficient. In general, $y = A^m - m\Delta + \frac{(m+1)m}{2}\Delta^2 - \text{\&c.}$, where Δ represents $A^{m+1} - A^m$, and m may have any positive integral value. Since $\frac{1}{2n+1} S_{2n+1}$ is a symmetrical linear operator which accurately reproduces a series of the first order, the substitution of $\frac{1}{2n+1} S_{2n+1}$ for A in the above expression gives a family of graduation formulas of the familiar summation type correct to 1st, 3rd, 5th, &c., differences, according to the number of terms retained. Mr. Altenburger proceeds to discuss the effect of these formulas on the ϵ of the ungraduated values. Adopting the usual criterion, namely, the sum of the squares of the coefficients, he shows that the correction of the arithmetic means in order to reproduce y more accurately involves a loss of smoothing power—in fact, that generally reproduction and smoothing are opposing forces. He thinks, however, that they may be sufficiently reconciled for practical purposes by a suitable choice of m, n and p. As an illustration of the application of the method, four graduations of a distribution of deaths are given—correct respectively to 1st, 3rd, 5th and 7th differences—the first by $\frac{1}{5^6} S_5^6$, and the second, third, and fourth by inclusion successively of the terms -6Δ, $+21\Delta^2$, and $-56\Delta^3$. The improvement in the reproductive effect of the formula at each operation, and the deterioration of its smoothing power are well shown by a diagram. For practical purposes, the only one of the four graduations that seems to merit special mention is the third-difference one by $\frac{7}{5^6} S_5^6 - \frac{6}{5^7} S_5^7$. The constants of this formula, when plotted out, give a very satisfactory curve, and as the coefficient of the central term is under ·16, its smoothing power is no doubt considerable, but it has the disadvantage of comprising 29 terms. Mr. Altenburger holds that these summation formulas can be properly applied to asymmetrical or "one-sided" operations, so that no beginning or end terms need be lost, but his reasons for this opinion are not clear. In the four graduations referred to above, the graduated values of the beginning and the end terms appear to have been obtained by the assumption of zero values beyond the ends of the range, but as the data consists of a complete distribution of deaths the values beyond the range would naturally be zero, and consequently the result of the graduation does not afford any test of the validity of the assumption.

A considerable portion of the first paper is devoted to the discussion of weights of observation and of the question how they can

best be taken into account in graduation. It is shown that if the weight $E_{[x-n]+n}$ be assigned to the probability $q_{[x-n]+n}$, the value of the aggregate q_x can be logically deduced from the values of the component select q's, but the underlying assumption that the select q's can be regarded as so many independent observations of one and the same aggregate q appears hardly justifiable—at any rate unless the first few years of assurance are excluded. It seems better to regard q_x as a blend (conventionally taken to represent the probability of dying in a year at age x) of the various select q's in certain fortuitous proportions. Proceeding to consider the weights of the aggregate q's, Mr. Altenburger holds that the theoretical weight of q, namely, $\dfrac{\lambda E_x}{p_x q_x}$, where λ is a constant, cannot be employed, because the true values of p and q are unknown, and the use of the adjusted values would assign too high a weight to those q's of which the observed values are too great and too low a weight to those of which the observed values are too small (an infinite weight, for example being assigned to the zero-value of q at an age at which no deaths are recorded). It is not clear, however, why for the purpose of obtaining approximate theoretical weights (if required), recourse should not be had to some already graduated table for the values of p and q. Mr. Altenburger, in fact, adopts this plan in a later section of the paper in order to calculate the weighted mean square of the error in a Makeham graduation of a table already graduated by his formula $\frac{1}{2^6}S_2^6$; in this case he uses the p and q of the preliminary graduation. However, for ordinary graduation purposes he considers that as q changes slowly with the age, the weights of successive values over a moderate range may be taken as proportional to E, and on this basis effect may be given to the weights of the observations by a separate graduation of the deaths and the exposures. But the double application of the formula $\left[\frac{1}{16}(10S_2 - S_4)\right]^6$ is laborious, and the formula $\frac{1}{2^{2n}}S_2^{2n}$ is even more unsuitable than when applied to q, because since the points of inflexion of the deaths-curve do not occur at the same ages as those of the exposures-curve the deformations of the deaths and exposures by the application of this formula are in opposite directions over considerable stretches of the curve. It is suggested, therefore, that the deaths should be graduated by repeated application of the formula $\theta^{(n)} = \dfrac{E}{2}\left(\dfrac{\theta_{-1}^{(n-1)} + \theta^{(n-1)}}{E_{-1} + E} + \dfrac{\theta^{(n-1)} + \theta_1^{(n-1)}}{E + E_1}\right)$; this formula has a geometrical basis similar to that of $\frac{1}{5^2}S_{22}$, and by a conventional treatment of the ends of the table makes the total graduated deaths equal to the total ungraduated. The graduated values of q would then be obtained by dividing the $\theta^{(n)}$ by the ungraduated values of E. A specimen graduation of the Gotha (7) Table by this method seems fairly satisfactory (although apparently slightly increasing the values of q throughout the Table), but the practical conclusion is that the weights are of no importance except at the ends of the Table, where E is rapidly increasing or decreasing.

R. T.

CORRESPONDENCE.

THE GENERAL THEORY OF SUMMATION FORMULAS.

[We have received from Mr. Julius Altenburger a letter giving a resumé of the second paper referred to in the review on p. 449. Owing to the pressure on our space we are unable to print the letter in full, but the following slightly modified extracts will indicate the general course of Mr. Altenburger's investigation.—ED. *J.I.A.*]

Mechanical operations for graduation can be represented by the general formula $A(u_x)=\Sigma_{-n}^{n}a_\kappa u_{x+\kappa}$, where A is the symbol of operation, u the series of values to be graduated, and a the coefficients of the formula of operation. If $a_\kappa=a_{-\kappa}$ (as is usual in practice), the operation is a symmetrical one. The investigation, however, is general, and applicable to asymmetrical as well as symmetrical operations.

Let $\eta_x=y_x+\epsilon_x$, where η_x is the observed value, y the (unknown) true value, and ϵ the (unknown) error of observation.

Then $A(\eta_x)=A(y_x)+A(\epsilon_x)$, and A will be a graduating operation if $A(y_x)=y_x$, and $\Sigma[A(\epsilon)_x]^2$ is $<\Sigma[\epsilon_x]^2$.

The second condition will be fulfilled if Σa^2_κ is <1, and the less the sum of the squares of a the greater will be the smoothing effect of the formula. Since Σa^2 cannot be zero, no operation can give the true value y, the result of graduation being merely to restrict the limits of the errors of observation.

The first condition leads to the general problem of the reproduction of functions by mechanical operations. On the assumption that $y_{x+\kappa}$ can be expanded in the series—

$$y_x+\kappa y'_x+\frac{\kappa^2}{2!}y''_x+\ldots$$

where $y^{(\nu)}$ denotes $\dfrac{d^\nu y}{dx^\nu}$.

$$A(y_x)=y_x\Sigma a_\kappa+y'_x\Sigma\kappa a_\kappa+\ldots+\frac{y^\nu}{\nu!}\Sigma\kappa^\nu a_\kappa+\ldots$$

and the conditions of reproduction are

$$\Sigma a_\kappa=1\text{ ; and}$$

$$\Sigma\kappa^\nu a_\kappa=0\text{ for }\nu=1,\ 2,\ 3,\ldots$$

The number of coefficients a being limited, these conditions can be satisfied for only a limited value of ν, so that only rational integral functions are exactly reproducible.

Let A_r denote an operation of degree r, for which the condition $\Sigma\kappa^\nu a_\kappa = 0$ is satisfied for $\nu = 1, 2 \ldots r$. Then, if y be of the sth degree,

$$A_r(y_x) = y_x + \sum_{\nu=r+1}^{\nu=s}\left[\frac{y_x^{(\nu)}}{\nu\,!}\Sigma\kappa^\nu a_\kappa\right].$$

$$= y_x + \phi_r, \text{ say.}$$

The expression ϕ_r is of the $(s-r-1)$th degree; and, consequently,

$$A_r(\phi_r) = \phi_r + \phi_{2r+1} \text{ (say),}$$

where ϕ_{2r+1} is of the $\{s-2(r+1)\}$th degree.

Hence, $A_r^2(y_x) = y_x + 2\phi_r + \phi_{2r+1}$,

and generally

$$A_r^m(y_x) = y_x + m\phi_r + \frac{m(m-1)}{2\,!}\phi_{2r+1} + \ldots + \frac{m\,!}{(m-p)\,!\,p\,!}\phi_{p(r+1)-1}$$

The elimination of the expressions ϕ leads to the general formula of approximation—

$$y = A^m - m\Delta_m + \frac{(m+1)\,!}{2\,!\,(m-1)\,!}\Delta^2{}_m - \ldots + (-1)^i\frac{(m+i-1)\,!}{i\,!\,(m-1)\,!}\Delta^i{}_m + \ldots$$

where A^m is written for $A_r{}^m(y_x)$ and $\Delta_m = A^{m+1} - A^m$.

[This result may be readily obtained by the method of operators. Since $\phi_r = A(y) - y = \Delta y$; $\phi_{2r+1} = A(\phi_r) - \phi_r = \Delta\phi_r = \Delta^2 y$, &c.; where the differences have reference to the series y, $A(y)$, $A^2(y)$, &c., the expression for $A^m(y)$ may be written in the form $(1+\Delta)^m y$. Hence, $y = (1+\Delta)^{-m}A^m = A^m - m\Delta_m + \frac{(m+1)m}{2}\Delta_m^2 - \ldots$—Ed. *J.I.A.*]

Let $$A(u_x) = \frac{1}{2n+1}\,\Sigma_{-n}^n u_{x+\kappa}.$$

Then A is an operation of the first degree, because $\Sigma a_\kappa = 1$, $\Sigma\kappa a_\kappa = 0$,

and $\Sigma\kappa^2 a_\kappa = \frac{2}{2n+1}\Sigma_1^n\kappa^2$. Since $\Sigma\left(\frac{1}{2n+1}\right)^2 = \frac{1}{2n+1}$, which is <1,

A is a smoothing operation. The substitution of this operation in the general formula gives a very simple process of graduating to any requisite degree of reproduction and smoothing. The expression A^m then gives an approximate representation of the function on the

assumption that *first* differences (of η) are constant, $A^m - m\Delta_m$, on the assumption that *third* differences are constant, $A^m - m\Delta_m + \frac{(m+1)m}{2}\Delta^2{}_m$, on the assumption that *fifth* differences are constant, and so on.

The resulting errors may be represented generally by $\eta - y^{(N)}$. The degree of approximation (N) may be considered sufficient if these errors are distributed about zero. This may be tested by the application to the errors of a strong graduating operation of the first degree. If the $A(\epsilon^N)$ form a series of positive or negative values, the reproduction is not sufficiently accurate, and an approximation of higher degree must be obtained.

[Mr. Altenburger proceeds to show that formulas of reproduction can be obtained by combining different operations A, B, instead of by repetition of the single operation A, and deduces Woolhouse's, Higham's, and Karup's formulas. He also investigates the effect of the operator A^m on a series following Makeham's law.]

The general process admits of the graduation of θ_x and E_x separately. The ratio of the graduated values then gives a graduated value of q, where account is taken of the weights of the observations.

The method may be applied to functions of two or more variables (*e.g.*, Select Tables). The process then consists of two or more graduations, one of the variables at a time being regarded as varying while the others remain constant.

THE LIFE ASSURANCE COMPANIES OF THE UNITED KINGDOM.

Summary of the Life Assurance and Annuity Revenue Accounts.

[Extracted from the Parliamentary Returns for 1906, published in 1907.]

INCOME	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Balance at the beginning of the Year	287,699,364	29,389,579	317,088,943
Adjustment in connection with the amalgamation of the "Commercial Union" and the "Hand-in-Hand"	+137,683	...	+137,683
	287,837,047	29,389,579	317,226,626
Premiums	25,332,993	11,619,303	36,952,296
Consideration for Annuities . .	2,185,859	6,722	2,192,581
Interest and Dividends (less Tax)	10,860,172	1,032,835	11,893,007
Increase in value of Investments .	181,738	638	182,376
Fines, Fees, &c.	15,766	1,568	17,334
Capital Paid-up	201,000	89,413	290,413
Customs Timber Measuring, &c. .	3,205	...	3,205
Transfers from other Accounts .	26,139	83,482	109,621
Miscellaneous	31,149	...	31,149
	326,675,068	42,223,540	368,898,608

OUTGO	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Claims	17,527,265	4,409,035	21,936,300
Cash Bonuses and Reduction of Premiums	1,132,576	334	1,132,910
Surrenders	1,706,448	103,811	1,810,259
Annuities	2,137,657	6,925	2,144,582
Commission	1,337,415	2,910,670	4,248,085
Expenses of Management . .	2,094,516	2,128,687	4,223,203
Bad Debts	1,923	576	2,499
Decrease in value of Investments .	581,095	6,804	587,899
Interest on Capital and Dividends and Bonuses to Shareholders .	479,922	579,819	1,059,741
Transfers to other Accounts . .	302,426	308,337	610,763
Miscellaneous	23,912	3,305	27,217
Balance* at the end of the Year .	299,349,913	31,765,237	331,115,150
	326,675,068	42,223,540	368,898,608

* This Balance includes the whole of the Life and Annuity Funds (£326,949,605), and, in addition, the Capital, &c., of Companies whose business is limited to Life Assurance only.

Summary of the Balance Sheets

LIABILITIES	Ordinary Companies	Industrial Companies*	TOTAL
	£	£	£
Paid-up Capital (including sundry Shareholders' Balances)	11,694,209	2,164,118	13,858,327
Life and Annuity Funds	296,003,001	30,946,604	326,949,605
Fire Funds of Companies transacting Life Business	14,929,018	...	14,929,018
Marine Funds of Companies transacting Life Business	1,394,066	...	1,394,066
Reserve Funds	4,573,663	1,802,448	6,376,151
Other Funds	3,533,282	247,692	3,780,974
Profit and Loss Balances	6,520,572	5,554	6,526,126
Depreciation and Investment Balances	2,434,584	70,581	2,505,165
Globe Annuitants (Liverpool and London)	1,654,200	...	1,654,200
Outstanding Claims	4,205,042	7,079	4,212,121
Outstanding Accounts	1,145,433	35,294	1,180,727
Temporary Loans	1,025,863	6,055	1,031,918
	349,112,933	35,285,465	384,398,398

ASSETS	Ordinary Companies	Industrial Companies*	TOTAL
	£	£	£
Mortgages	92,854,076	4,051,298	96,905,374
Loans on Policies	18,168,337	148,011	18,316,348
,, Rates	33,714,471	11,081,300	44,795,771
British Government Securities	7,134,321	2,068,979	9,203,300
Indian and Colonial Government Securities	19,240,971	836,047	20,077,018
Foreign Government Securities	11,157,145	568,671	11,725,816
Debentures	68,413,937	3,877,544	72,291,481
Shares and Stocks	40,700,115	1,440,140	42,140,255
Companies' own Shares	615,889	...	615,889
Land and House Property and Ground Rents	28,093,227	9,183,145	37,276,372
Life Interests and Reversions	9,971,404	2,276	9,973,680
Loans on Personal Security	2,267,776	6,637	2,274,413
Agents' Balances and Outstanding Premiums	6,637,324	635,319	7,272,643
Outstanding Interest	3,275,840	313,108	3,588,948
Cash, Deposits, Stamps, &c.	6,280,078	404,148	6,684,226
Deficiencies, Establishment Expenses, &c.	588,022	668,842	1,256,864
	349,112,933	35,285,465	384,398,398

* In the case of one or two Companies transacting both Ordinary and Industrial business, but not returning separate Balance Sheets, the Liability and Assets given in the above columns under the heading "Industrial Companies", are those appertaining to the *combined* operations of such companies.—[ED. *J.I.A.*]

INCREASE (+) *or* DECREASE (−) *in the Chief Items of this Year's* SUMMARY *as compared with the corresponding Items for the previous Year.*

	Ordinary Companies	Industrial Companies
INCOME.	£	£
Premiums	+ 785,120	+ 526,035
Consideration for Annuities . . .	+ 392,123	+ 2,876
Interest and Dividends (less Tax) . .	+ 403,869	+ 98,129
OUTGO.		
Claims	− 339,204	+ 153,069
Annuities	+ 55,184	− 5,765
Surrenders	+ 70,075	+ 24,036
Commission	+ 58,734	+ 122,355
Expenses of Management	+ 59,892	+ 89,352
Net Decrease in value of Investments, &c.	399,357	6,166
LIABILITIES.		
Paid-up Capital (including sundry Shareholders' Balances)	− 57,308	+ 213,104
Life and Annuity Funds	+11,469,392	+2,401,980
ASSETS.		
Mortgages (including Loans on Rates) .	+ 2,487,081	+ 483,243
Life Interests and Reversions . . .	+ 581,073	− 4,316
Loans on Policies	+ 990,381	+ 25,882
British Government Securities . . .	− 166,105	+ 3,868
Indian and Colonial Government Securities	− 10,143	+ 230,813
Foreign Government Securities . . .	+ 1,098,945	+ 240,225
Debentures	+ 5,955,301	+1,009,787
Shares and Stocks	+ 1,290,456	+ 355,111
Companies' own Shares	− 15,398	...
Land and House Property and Ground Rents	+ 1,115,236	+ 358,482
Loans on Personal Security . . .	+ 207,701	+ 2,687

NUMBER OF COMPANIES.

The total number of Companies appearing in the above Summary is 95, of which 77 are classed as Ordinary, 9 as Industrial, and 9 appear in both Classes, the returns of these Companies showing the Ordinary and Industrial business separately. The accounts of the Gresham Continental and Popular are included for the first time.

During the year two names have been removed from the Official List of Companies, viz.: Ladies of the Maccabees of the World; and Mutual Reserve of New York, which have discontinued business within the United Kingdom. And four names have been added, viz.: Confederation Life Association (of Toronto); General Accident, Fire and Life Assurance Corporation, Limited; London and Provincial Assurance Company, Limited; and National Standard Life Assurance Corporation, Limited; in which cases the Board of Trade have issued their Warrant under the provisions of Section 1 of "The Life Assurance Companies Act, 1872."

Summary of the Assurances in Force, *as shown by the last Returns of the Companies*

ORDINARY BUSINESS.

	With Profits		Without Profits		Total		Re-assurances	Net Amount
	No.	Amount	No.	Amount	No.	Amount	Amount	
Assurances.		£		£		£	£	£
Whole Term of Life	791,014	375,934,114	152,531	70,976,804	943,545	446,910,918	28,351,200	418,559,718
Limited number of Premiums . .	58,184	36,572,156	14,859	6,060,094	73,043	42,632,250	2,367,751	40,264,499
	849,198	412,506,270	167,390	77,036,898	1,016,588	489,543,168	30,718,951	458,824,217
Endowments . .	1,519	365,391	22,346	5,428,405	23,865	5,793,796	18,500	5,775,296
Endowment Assurances . . .	1,195,544	195,100,116	105,905	25,760,306	1,301,449	220,860,422	3,196,704	217,663,718
Joint Lives . .	16,385	3,204,100	2,546	903,500	18,931	4,107,600	247,006	3,860,594
Last Survivor . .	757	639,507	1,022	1,375,305	1,779	2,014,812	314,207	1,700,605
Contingent . .	30	66,076	4,306	6,768,008	4,336	6,834,084	1,792,060	5,042,024
Issue . . .	21	46,055	1,820	6,478,061	1,841	6,524,116	2,016,604	4,507,512
Miscellaneous . .	5,424	2,446,453	23,702	16,128,030	29,126	18,574,483	2,456,666	16,117,817
	2,068,878	614,373,968	329,037	139,878,513	2,397,915	754,252,481	40,760,698	713,491,783
Annuities.								
Immediate . .	...	...	...	...	39,840	2,102,607	53,906	2,048,701
Deferred . .	...	...	...	...	15,622	423,882	26,430	397,452
	...	...	...	...	55,462	2,526,489	80,336	2,446,153

INDUSTRIAL BUSINESS—(Sickness and Friendly Society Contracts not included).

	With Profits		Without Profits		Total		Re-assurances	Net Amount
	No.	Amount	No.	Amount	No.	Amount	Amount	
Assurances.		£		£		£	£	£
Whole Term of Life	252	16,381	22,681,832	220,685,374	22,682,084	220,701,755	1,670	220,700,085
Limited number of Premiums . .	...	...	317	6,300	317	6,300	...	6,300
	252	16,381	22,682,149	220,691,674	22,682,401	220,708,055	1,670	220,706,385
Endowments . .	140	42,475	1,835,116	17,997,016	1,835,256	18,039,491	...	18,039,491
Endowment Assurances . . .	45	2,599	608,695	6,086,306	608,740	6,088,905	88	6,088,817
Joint Lives . .	...	...	417,640	6,714,996	417,640	6,714,996	...	6,714,996
Contingent . .	...	...	5	2,160	5	2,160	400	1,760
Miscellaneous . .	...	...	3	2,500	3	2,500	...	2,500
	437	61,455	25,543,608	251,494,652	25,544,045	251,556,107	2,158	251,553,949
Annuities.								
Immediate . .	...	...	...	...	51	1,761	...	1,761
Deferred . .	...	...	...	...	7	132	...	132
	...	...	...	...	58	1,893	...	1,893

The above figures are based on Returns deposited, for the most part, during the past five years, and are, therefore, merely an approximation to the amount of contracts in force at the present time. The figures of the Colonial and Foreign Companies have been excluded, as their Returns do not separately show the extent of business in the United Kingdom.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On Extra Premiums. *By* Harold Edward William Lutt, F.I.A., *of the Northern Assurance Company.*

[Read before the Institute, 29 April 1907.]

THE question of the premiums which should be charged to meet the cases of lives proposed for assurance admittedly incurring, or about to incur, risks more hazardous than those of the ordinary assured life, has from time to time engaged the attention of the Institute.

More stress has, perhaps, been laid upon the mortality of persons living in tropical countries, where the conditions of life are very different from those to which we are here accustomed, and statistics, obtained either from official sources or from business experience, have been furnished; whilst the only occupations to which attention has been directed are the Naval, Military, and Mercantile Marine Services, and those connected with the sale of alcohol.

With the view of obtaining, if possible, information respecting the modern practice of Life Assurance in connection with extra risks, I ventured to address to all the larger offices a Schedule intended to elicit such replies as would enable me to present a general idea of the methods applied to these cases.

In this I hope I have partially succeeded, but I need not add

that the information must be regarded in a very general light. Many offices were particular to point out that, in the absence of any ascertained experience, each case of this nature would be considered upon its individual merits, and rated accordingly.

I should, however, like to offer my thanks to those offices which furnished me with the available information, and to acknowledge the very considerable trouble to which they put themselves on my behalf. If the result should be an extension of our general knowledge upon some of the points raised in the Schedule, I shall consider that the effort has not wholly failed.

I obtained replies from forty offices, but, naturally, they could not all return full answers to the various questions in the Schedule, and, where no experience was indicated, the Company has been excluded from the summary relating to the particular occupation or climate risk.

For the purpose of easier reference, I divide the paper into sections relating to (i), Occupation Extras; (ii), Climate Extras; (iii), Inferior Eligibility; and (iv), the Methods of dealing with extra premiums when imposed; whilst the theoretical application of Extra Risk Tables is subsequently discussed.

Occupation Extras.

The chief source of information as to the varying degrees of mortality associated with different occupations is undoubtedly the periodical Supplement to the Annual Returns of the Registrar General, but, beyond indicating the occupations which involve a hazard to life greater than that incurred by the average person, they are, unfortunately, not well adapted to use by the Actuary in his official investigations amongst assured lives. The latter are, more or less, a class to themselves, and though the Registrar General's Returns may indicate a preponderance of certain diseases which operate unfavourably in particular occupations, yet the greater mortality therein recorded must perforce fall upon the weaker and poorer lives of the general class, whilst those with whom the actuary has to deal are probably of superior vitality and power of resistance to disease generally—a fact which is brought out by their passing the medical test necessary for the purpose.

Nevertheless, we can obtain an indication from these Reports of certain occupations to which it were well to direct attention, to ascertain if the assured lives so engaged exhibit these features

in any marked degree, and this is conveniently, if somewhat roughly, afforded by the "Comparative Mortality Figure" given in the tables of the Supplement. It may be as well to define this:

Amongst 61,215 men living between ages 25 and 65 at the Census of 1891,—I am quoting from the Supplement to the 55th Report,—it was found that, on the average of the 3 years 1890–2, 1,000 deaths occurred; these totals were proportioned into the respective figures for the four decennial age groups of central age 30, 40, 50, and 60. The annual rates of mortality amongst males of various occupations, deduced from the Census and Death Returns for the same age groups, were applied to these proportions of the 61,215 men, and the total of the resulting deaths in the four groups furnished the comparative mortality figure in relation to the 1,000 deaths amongst the general population constituted in the same way.

If we applied the $O^{M(5)}$ mortality rates in this manner our comparative mortality figure would be about 776.

Taking from the Report the experience of those engaged in the following occupations, which exhibit a comparative mortality figure of over 1,250, we have:

Carman, Carrier, &c.	1,284
Iron and Steel Manufacture	1,301
Lead Mining	1,310
Slate and Tile Industry	1,322
Seamen (Merchant Service)	1,352
Wool, Silk, and Cotton Dyeing and Printing	1,370
Copper Working	1,381
Chemical Manufacture	1,392
Tin Mining	1,409
Tool, Scissors, File, Saw, and Needle Making	1,412
Brewing	1,427
Glass Manufacture	1,487
Coalheaving	1,528
Innkeeper (Publican, Spirit, Wine, and Beer Dealer)	1,642
Hawking, Costermonger, &c.	1,652
Pottery and Earthenware Manufacture	1,706
Inn, Hotel, &c., Service	1,725
Lead Working	1,783
Dock Labouring	1,829

For the following occupations, however, we have:

Railway Service (Engine Drivers, Guards, Porters, &c.)	818
Fishermen	845
Malting	884
Baking	920
Coal Mining (generally)	925
Do. (Lancashire)	1,069
Do. (South Wales)	1,145
Bricklaying, Building, Masons, &c.	1,001
House Painting, &c.	1,120

We may take from the first section innkeepers, brewers, and seafaring men as the only classes of which assurance companies who do not transact "industrial" business have much practical experience, whilst the occupations mentioned in the second group do not exhibit, on the whole, any marked extra risk, though it must be borne in mind that the comparison is with the general population and not with assured lives.

There are, however, two points to be noted in connection with these figures—first, that the mortality figure represents the total risk between ages 25 and 65, and the comparisons for smaller age periods may vary greatly, whilst those for ages over 65 are not available; and, secondly, that the Returns are based upon occupations at the time of death, and therefore do not include in a particular occupation the deaths of those who have been compelled to relinquish it on account of ill-health derived from its previous pursuit. The latter is an important feature, and, though the selection exercised by a life office probably more than counterbalances the effect, it is not possible to obtain definite evidence on the subject. With regard to the former, the Returns do afford some information, and, speaking quite generally, we may say that the occupations of baker, butcher, maltster, mason (building trade generally), painter, railway employee, and even coal miner, do not exhibit any very increased mortality up to ages 55 or 60.

There is, however, one set of data deduced from the experience of assurance companies which has been published, though not, unfortunately, relating to this country. I refer to the "Experience of Thirty-four Life Companies upon Ninety-eight Special Classes of Risk", compiled by the Actuarial Society of

America, and reviewed in vol. xxxviii, p. 363, of the *Journal* of the Institute. This experience includes an investigation into the mortality of lives with under-average personal or family history, as well as into the influence of occupation; but, referring for the time to the latter portion only, we may extract the following mortality percentages, based on the standard of 100 as representing Farr's Healthy English Life Table (somewhat modified at certain ages), and excluding from the comparison the first 5 years' experience. This is a more severe standard than the general population, and even than the $O^{M(5)}$ after age 37; the comparative mortality figure as above would be about 718 only:

Butcher	89
Painter	83
Railway Engineer	105
Do. Fireman	123
Brewer	138
Liquor Seller (abstainer)	128
Do. (non-abstainer)	136
Hotel Keeper (not attending bar)	121
Merchant Marine Officer	110
Seaman or Fisherman	91

We see that, with the exception of railway risks, which are somewhat different in America, the only marked extra mortality is that connected with the liquor trade.

As a matter of interest, it may be pointed out that the occupations which in the American experience have shown generally increased risk include:

Seafaring Risks (officers of sailing vessels, though not seamen and fishermen)	126
Glass Workers	114
Police Force	115
Members of the Theatrical Profession	139

Seeing that the comparison is one with the Healthy English Life Table, we may notice here the influence of medical selection in an improved mortality, which renders the assured person superior to the average of his class amongst the general population.

To turn from this to the practice of British companies, I

extract the following information from the particulars furnished in response to my enquiries :

(1) Bakers.

Of 28 companies :—
15 make no charge ;
4 have no fixed rule, but state that they deal with cases on their merits.
The others impose an addition varying from 5*s*. (or 3 years) to 10*s*. % (or 7 years) ; and
1 office charges 20*s*. % .

(2) Butchers.

Of 27 companies :—
11 make no charge ;
4 deal with cases on their merits.
The rates of the remainder are similar to those for bakers ; but
2 discourage these cases.

(3) Painters.

Of 26 companies :—
12 make no charge ; but the extra, where imposed, ranges from 5*s*. to 10*s*. % .

(4) Stonemasons.

Of 22 companies :—
5 charge no extra ; and
2 deal with each case on its merits ;
but 5*s*. to 10*s*. % seems a general charge.

(5) Railway Risks : Guards.

Of 24 companies :—
6 make no charge as a rule ;
A general extra would appear to be 10*s*. % ; and
4 discourage such cases.

Do. Porters.

Of 21 companies :—
11 make no charge if no shunting is undertaken ; but a not unusual rate to cover this is 10*s*. % , or a little less ; and
3 discourage such risks.

Do. Engine Drivers.

Of 24 companies :—
Nearly all charge an extra premium, several requiring 20*s*. % , whilst
3 discourage such cases.

In considering occupations (1) to (4) it is not unusual to diminish or waive the extra in the case of endowment assurances, and one or two companies only issue the latter form of policy on these lives. It would seem to be a fair way of meeting the risk ; the extra mortality is not, apparently, excessive in the early years, and if the eligibility is carefully scrutinized in connection with the known dangers of the particular occupation, it does not seem that a first-class life need be surcharged merely on that

account for endowment assurances maturing at ages 55 or 60. This, I believe, is a favourite method of dealing with occupation hazards on the Continent.

(6) Mining Engineers (who supervise underground operations).

Of 22 companies:—

About one half do not require an extra; the others charge from 5*s.* % upwards, 10*s.* being a very general rate; whilst
1 discourages such risks.

(7) Motor Drivers.

Of 19 companies:—

7 charge no extra, but generally exclude racing;
1 charges only 5*s.* %;
9 charge 10*s.* or 20*s.* %. In some of the latter cases racing is excluded, but others do not specifically refer to the point.
2 discourage such cases.

(8) Mining.

Coal, lead, or tin mining involves a usual charge of from 10*s.* to 20*s.* %, whilst for gold or silver mining a similar extra is imposed according to locality.

(9) Naval and Military Services.

The question of naval and military risks has been dealt with in a voluminous report made in 1890 by Messrs. Ackland and Smee, part of which is published in *J.I.A.* xxxiv, 358, and in a Paper by Mr. McLauchlan in *J.I.A.* xxxiv, 251.

The conclusions arrived at by the former pointed to an annual extra mortality of ·0075, which they assumed to be incurred until age 50, and then to decrease until it vanished at age 55. The extra premiums for whole life assurances based upon their assumptions were about 11*s.* % at age 20, decreasing to 8*s.* at age 40; nearly the same rates were applicable to endowment assurances maturing at 60 or death. These extras, it may be noted, would be payable throughout the duration of the policy, and relate to officers and men combined. Mr. McLauchlan came to the conclusion that, though the extra risk for the naval service was somewhat less, that for the army was considerably greater.

Officers of the Army.

Of 37 companies:—

21 charge military officers at home an annual extra of 10*s.* %, payable until retirement only in most cases, whilst 1 offers a permanent extra of 7*s.* 6*d.* instead, and 4 give the option of a policy at ordinary, or in one case very slightly increased, participating rates, but not commencing to share in profits

until after retirement from the service. One office charges the extra for 10 years only, and 1 charges a little less (7*s*. 6*d*. %) for endowment assurances;
5 charge 12*s*. 6*d*. %, of which 1 gives the option of postponing the bonus;
3 charge 15*s*. %, of which 1 offers an alternative permanent 7*s*. 6*d*. %;
3 charge 17*s*. 6*d*. %, of which 1 makes the charge permanent, but allots to such policies a correspondingly increased bonus;
1 offers the alternative of 20*s*. % continuous to cover all risks, or 10*s*. % continuous and a climate extra whenever necessary;
2 incorporate in the premiums a varying (permanent) charge of 10*s*. to 15*s*. %, of which 1 bases its distribution of profits upon the full premiums paid, and the other offers an alternative of a continuous military rate of about 10*s*. % with climate extra as incurred;
2 offer only the postponement of participation in profits until permanent retirement, 1 increasing the ordinary participating rates by a small permanent addition (2*s*.).

For Military Officers proceeding to or stationed in India.

1 office charges 5*s*. % additional for climate, or a single amount of 15*s*. %;
1 office charges 7*s*. 6*d*. % additional, making 20*s*. % in all;
2 of the offices charging 17*s*. 6*d*. % above make no change;
7 offices charge 10*s*. % additional, *i.e.*, a reduced climatic extra;
6 offices charge 12*s*. 6*d*. or 15*s*. %, of which 1 makes 5*s*. only continuous out of a total extra of 25*s*. %;
6 offices charge 20*s*. %, or the full climatic extra;
6 offices charge Indian rates, with 6*s*. to 10*s*. % permanent military extra;
1 office, which at home postpones the bonus, charges a special extra without postponement of 25*s*. %.
6 offices do not state the extra charged in such cases;
1 office quotes 30*s*. % for naval and military officers, excluding war service.

The postponed bonus scheme generally involves the usual, but sometimes a smaller, climatic extra.

In connection with war extras, Monsieur Fleury states in his paper submitted to the Berlin Congress last year that French companies, as a rule, do not cover by their policies the risks of warfare, but make a special contract at the outbreak of hostilities. Either the extra is charged as a single premium on the amount at risk, and liability for the whole sum assured is undertaken, or else the assured pay special premiums, according to scale dependent upon their rank and employment, and one-third only of the sum assured is guaranteed, the remainder being dependent upon the fund created by the war extras so far as it is possible; whilst in either method, if any surplus remain, it is refunded to the survivors in proportion to their payments (*Transactions*, Fifth Congress, vol. i, pp. 199–204).

Officers of the Royal Navy.

Most offices deal with these risks on the same, or very similar, lines to those adopted for the Army.

23 charge 10*s*. % ;
2 charge 12*s*. 6*d*. % until retirement, to which is generally added a special climatic extra when the officer is under orders for a foreign station (Asia or Africa), the extra being sometimes reduced, or limited to a few years with no further liability for an additional charge, and, in one case, commutation by three years' purchase is offered;
1 charges 15*s*. % continuous to cover all risks, or 7*s*. 6*d*. % and a climate extra when incurred;
2 charge 15*s*. % continuous till retirement, or 7*s*. 6*d*. % permanent;
2 charge 17*s*. 6*d*. or 20*s*. %, and, apparently, would regard this as covering any ordinary climate risk.
The postponed bonus scheme is here again utilized by the same offices, as in the case of military officers.
2 offer these policies only, but the ordinary participation rate, without the small addition, is charged by both for naval officers;
4 offices quote permanent extras (7*s*. 6*d*. to 15*s*. %); 2 of these include special participation in profits;
1 quotes 5*s*. % permanent for war extra, and 12*s*. 6*d*. % for marine risk whilst afloat;
One or two companies advertise that they are prepared to accept naval and military officers under double endowment assurances without extra.

Sea Risk, Mercantile Marine, &c.

In the mortality experience of the Marine and General Office, submitted to the Institute in 1887 (*J.I.A.*, xxvi, 413), the results for the period 1852–1879 indicated an excess mortality of about ·01 for chief officers, and about ·015 for engineers and stewards. This experience showed a mortality up to age 55 of about $2\frac{1}{2}$ times the H^{M} rate for the former and 3 times for the latter. As regards the present practice:

Officers.

Of 30 companies:—
11 do not charge for first-class lines trading between ports within free (or temperate) limits; whilst
3 charge a continuous extra of 5*s*. % (one *nil* for endowment assurances); these would generally charge about 10*s*. % continuous till retirement for ports of call outside the limits, or otherwise a temporary climate extra;
2 charge 10*s*. to 15*s*. % incorporated in the premium and sharing in profits;
6 charge 10*s*. %; and
1 charges 12*s*. 6*d*. %, to include all parts except the West Coast of Africa, but sometimes modified for endowment assurances;

4 charge 20*s.* % with rebates for periods passed at home (one offering a continuous 15*s.* % in lieu) to cover most parts of the world;

3 give postponed bonus schemes at ordinary rates.

One or two companies accepting lives abroad through established branches charge the ordinary climatic rates without special addition for sea risk.

Engineers.

For engineers the extra is generally the same as above, but with less tendency to waive or reduce the rate for endowment assurances; two offices charge an *additional* 10*s.* %.

Stewards.

Though the offices making no, or a small, extra charge for the above very generally adhere to the same practice, six of the companies specify additional amounts of 5*s.* to 10*s.* % for stewards, and doubtless such proposals are very carefully scrutinized. One or two offices discourage them altogether.

In view of the possibility of stewards, on retiring from the sea, taking up the licensed victualling business, it would seem advisable to provide for a suitable extra in such cases if necessary by endorsement on the policy when issued, and perhaps generally to encourage endowment assurances.

Sea Risk generally.

For the ordinary seafaring risk, including pilots and fishermen, the general custom is to quote about 10*s.* %, but a fair proportion of offices charge as much as 15*s.* or 20*s.*

Several offices have special postponed bonus schemes also for the mercantile marine, restricted, however, in one case to endowment assurances.

The later experience of sea risks should be well worth obtaining.

(10) Publicans, and the Wine, Beer and Spirit Trade.

Here we come to a portion of our subject wherein the existence of a greater hazard has long been recognized, and some organized efforts have been made to ascertain its extent.

"The Investigation of the Mortality of Persons engaged in the sale of Intoxicating Liquors", undertaken by the Associated Scottish Life Offices, whose report is published in the *Journal* (vol. xxxiii, p. 245), is the best known record of the experience of this class. The recommendations of the Committee were:

For Publicans, an extra premium of 25*s.* % per annum.

For Innkeepers and Hotelkeepers, an extra premium of 20*s.* % per annum.

For Licensed Grocers in Ireland, an extra premium of 15*s.* % per annum.

For Licensed Grocers in Scotland, an extra premium of 10*s.* % per annum.

The mortality experience over the age of 55 (which was rather limited) seemed to indicate only a small excess, but it would be of great interest to ascertain if this were confirmed by another similar investigation. It does not seem probable that the diseases that are more or less directly ascribed to the occupation would tend to diminish in intensity after retirement, if susceptibility to them were once acquired; and an experience reported by Mr. A. R. Barrand, in a paper submitted to the Paris Congress in 1900, certainly indicated no diminution of the extra mortality with increasing age as far as 65. (*Transactions*, Third Congress, vol. i, p. 449.) A further paper relating to the same experience was submitted to the Berlin Congress, (*Transactions*, Fifth Congress, vol. i, p. 517) by Mr. J. McDonald.*

The practice of companies at present can be summarized as follows:

Publicans.

26 offices charge 20*s*. %, though a few charge less under endowment assurances;
1 office maintains the 25*s*. % rate;
5 offices charge even 40*s*. % (and 30*s*. % for endowment assurances), though these are mostly of a special character;
4 offices do not entertain the risk.
On the other hand,
1 office does not charge specially for this occupation where the health and habits are perfectly satisfactory; and
1 office charges generally 7 years' addition, with, however, a minimum of 10*s*. %.

Licensed Grocers.

10 offices generally charge *nil* in approved cases;
9 offices generally charge *nil*, unless liquor is sold in open vessels, when a charge of about 10*s*. % is made;
5 offices charge *nil* for England, of which 2 also charge *nil* for Scotland, and the remainder 10*s*. %, whilst for Ireland 10*s*. to 20*s*. % is imposed;
4 offices charge 10*s*. %;
5 offices charge up to 20*s*. %, of whom 3 charge only 10*s*. for endowment assurances.

Brewers.

Most offices do not impose any extra, but here again we find 6 companies transacting ordinary business which charge from 7*s*. 6*d*. to 20*s*. %.

First Class Hotel Proprietors.

19 companies do not as a rule make a charge, but they discriminate when there is any connection with a "bar" trade.
11 companies charge upon the merits of the case up to 20*s*. %.

* N.B.—This paper was referred to by Mr. E. A. Rusher in the Discussion (*see* p. 521).

Removal of Extra Premiums for Occupations.

The rates above quoted for naval and military service and sea risks are usually removed on permanent retirement unless otherwise specified, but for those risks, which, like that of wine, spirit, and beer dealing, are regarded as specially injurious to health, the extra rate is not removed, or is removed only on satisfactory evidence of health and habits. Companies which do not incorporate the extra with the ordinary premium at the time of issuing a policy usually permit its removal in other cases without evidence of health, but a few insist upon such evidence.

(11) Female Lives.

The incidence of the mortality of female lives in relation to that of males was for long obscured owing to the comparison between different tables of investigations being based upon the expectation of life; the favourable mortality experienced in later life almost invariably caused the expectation of females to be in excess.

Mr. Walford (*J.I.A.*, xix, 174) gave a history of the experiences that had been published from time to time. He informs us that about 1827, the "Eagle and United Empire" Office was actually charging lower premiums for females at all ages; and that this system was adopted by some other offices.

In 1854, Mr. Jellicoe reported that the experience of the Eagle Office showed a comparatively high rate of mortality among female lives at the earlier ages.

In 1861, Messrs. Bailey and Day, in their paper "On the Rate of Mortality prevailing amongst Families of the Peerage during the 19th Century", pointed out that the evidence was in favour of the general conclusion that at the two extremes of life female mortality was superior, and in the intervening period, inferior, to that of the male.

The publication in 1869 of the Institute of Actuaries' Tables confirmed this in respect of assured lives up to age 50.

The managers of the Associated Scottish Life Offices in 1875 recommended the imposition of an extra premium of 5*s.* % per annum for assurances upon female lives aged under 50, to be discontinued on attaining that age; whilst, when a female proposed for assurance was actually pregnant, a further charge of 10*s.* % for the first year was suggested.

The Actuaries' Club enquired into and summarized the practice of offices in 1891 (*J.I.A.*, xxix, 75); and the later publication of

the New Mortality Experience, though showing a considerable improvement in female mortality, still indicates the continuance of an excess over the rates of mortality for males at assuring ages up to 44.

The conditions under which assurances upon female lives are effected have again materially altered within the last 15 years. Endowment assurances have become very popular amongst females engaged in business, and it would seem that the vigour and independence which are exhibited by the large number of women now supporting themselves in such a manner would be favourably reflected in the mortality experience of this class of assurance.

At the present time, out of 51 offices :—

- 26 charge no extra as a general rule; though, if recently married,
  - 13 charge a single extra of 20*s*. %, and
  - 2 charge a single extra of 10*s*. % ; whilst, of the remaining 11,
  - 4 charge 10*s*. to 20*s*. %, if pregnant, one of them charging 20*s*. % for a first confinement and 10*s*. % in subsequent cases ;
  - 2 postpone, and
  - 5 have no fixed rule;
- 7 offices charge single extras, of which
  - 3 charge 20*s*. % to be paid on marriage, or with the first premium if recently married ;
  - 1 charges 10*s*. % if under 45, with a further 10*s*. if recently married ;
  - 1 charges 20*s*. % if the life is under 50;
  - 1 charges 20*s*. % if the life has not borne children ;
  - 1 charges 20*s*. %, but *nil* if unmarried and not engaged ;
- 18 offices charge annual extras ;
  - 15 charge 5*s*. % until age 50 (two discriminating in favour of endowment assurances by a smaller addition), of which
    - 7 charge 15*s*. or 20*s*. % additional for the first year if recently married ;
    - 2 postpone or charge 20*s*. % extra if pregnant ;
    - 2 postpone or charge 10*s*. % extra if pregnant;
    - 4 have no fixed rule ;
  - 3 charge 5*s*. % per annum, (i) to all, (ii) if under 35, (iii) if under 45, which is as a general rule continued throughout the duration of the policy.

Climate Extras.

East and West Indies.

The earliest and, in extent, the most important contributions to this part of the subject were the reports, by Messrs. S. Brown and A. J. Finlaison respectively, on the Covenanted and Uncovenanted Service Funds then in operation in India (*J.I.A.*, xvi, 208, and xviii, 153).

The Covenanted Service Fund Experience was that of the European civil servants engaged in the Presidencies of Bengal

and Madras, both whilst on service and after retirement. It may be said to have reflected the mortality of the best class of residents—those who obtained every advantage in the way of leave and means of recruiting their health, and who would ultimately return to England upon pension. The experience included the period after retirement, when the mortality was not found to be greatly in excess of home rates, though the small numbers at the older ages rendered any general deductions of this nature unreliable.

The Uncovenanted Service Fund related to some of the Christian employees of the Honourable East India Company who would not possess the above advantages, and who most frequently settled down in India, where they remained after retirement. The resulting rates of mortality, especially at the older ages, reflected these conditions.

Indian mortality has doubtless improved since about 1872, when the above reports were published. The distinction between lives permanently resident and those only temporarily residing in such climates is, however, one which still continues. It probably accounts for a difference of practice in charging extra premiums between companies which transact a general business through local agencies in these countries and those which, operating chiefly in Great Britain, nevertheless accept a good many proposals from persons proceeding abroad for the purpose of making a living.

Mr. Chatham, in his paper submitted to the Paris Congress in 1900 (*Transactions*, Third Congress, pp. 304–364), gives a very complete summary of the available experiences of climate risks, including tables of the extra premiums then charged by the "Standard" Life Assurance Company under their various prospectuses. I notice, however, that some of the extra premiums quoted therein have since been revised; and in the West Indies and Tropical America a different sub-division of the localities is employed.

To the public spirit of offices like the "Standard," which from time to time have placed their experience of foreign climate risks at the disposal of the profession, the modern life assurance world owes practically the whole of the information which it possesses upon the subject. It may be of interest if I refer to some of the sources of information. In addition to the particulars of Indian mortality afforded by the Covenanted and and Uncovenanted Service Funds, we have in Mr. Chatham's paper an experience of the "Oriental Government Security Life Assurance Company" (mostly of native lives) down to the year

1891, showing for Europeans a ratio of actual to expected (H^M) deaths of 1·52, and for Natives, 1·21 (*idem*, p. 344).

In 1903, Mr. Spencer Thomson gave to the New York Congress (*Transactions*, Fourth Congress, pp. 111–119) particulars of the Standard Office Experience in the years 1870 to 1885, and 1885 to 1900; the latter exhibited a considerable improvement in the mortality, which, (including all classes of policies and lives), showed an excess in actual deaths of about 30 % over the H^M standard.

He also furnished an experience amongst lives resident in temperate climates who had previously lived in India for not less than twelve consecutive months, and the continuance of a considerable extra risk is apparent in these observations. The latter is a most important point, and I make the following extract (pp. 114–115):

"It has further to be observed that the deaths recorded in "these tables" (*i.e., the tables relating to the experience abroad*) "took place, with a very few exceptions, in India, and that the "rate of mortality at home is swollen by the return of invalids "to this country, a matter to be kept in mind in determining "Indian premiums which are reducible to home rates on the life "coming to reside in Europe."

In 1906, Mr. A. T. Winter submitted to the Berlin Congress (*Transactions*, Fifth Congress, vol. ii, pp. 61–77) an interesting experience of the assured lives of the "British Empire" and "Positive" Offices, showing for Europeans an excess of 44 % in deaths over the H^M mortality. He found that the effect of selection was not apparent in the observations, and that the extra premium was very well represented by an addition to the age, supplemented by a constant.

These contributions combine in indicating that, so far from ceasing after some years, the extra mortality continues to increase with the age of the life, and, instead of seeming to involve a constant addition to the "home" rate of mortality, would be better represented by the addition of a constant percentage.

Similar experiences have been published concerning the West Indies. We have (*J.I.A.*, xxi, 153) the paper by Mr. Stott upon the results of an investigation of lives assured in the "Scottish Amicable" Office (1846–1876), wherein the great difference in the mortality prevailing in different parts of the West Indies is shown; that of Messrs. Hardy and Rothery in connection with the Barbados Mutual Office (*J.I.A.*, xxvii, 161), indicating an aggregate excess up to age 55 of nearly 80 % over the expected

deaths. The latter paper also refers to the results of another Standard Office experience, which included a supplementary investigation of the lives after their return to Europe. Here, also, I make the following extract (p. 183):

"The number of the observations is small, but the facts are "sufficient to indicate a considerable improvement in the mortality "consequent upon the cessation of the extra risk, although the "rates still remain higher than those prevailing among assured "lives in this country." (*N.B.*—The excess was 28 %.)

Mr. Chatham has also given us in the paper of 1900, a further experience of the "Scottish Amicable", 1876–1891, indicating a considerable improvement in the mortality, which is confirmed by Mr. Thomson in his paper above referred to from the "Standard" Experience of 1895–1900 as compared with that of 1846–1885. The later observations indicated an excess mortality of about 50 %.

In connection with **Africa** we have been furnished with the following experiences:

Dr. T. B. Sprague's Investigation of the Mortality amongst White Employees on the Congo River in 1879–1885 (*J.I.A.*, xxv, 437). The general conclusion warranted, in his opinion, an extra of £10. 10*s*. % per annum.

Dr. T. Glover Lyon's Table (*J.I.A.*, xxix, 541) of the Deaths of European Residents on the West Coast between 1878 and 1890.

Mr. J. R. Hart brought out an average annual mortality of 5·2 per-cent, or an excess stated to be 4 per-cent per annum, in an investigation (*J.I.A.*, xxxiii, 310, and *Insurance Record* of 2 October 1896) concerning 296 Government officials of the four West African Colonies in the years 1881–1890.

Dr. A. E. Sprague, in a "Note upon the Rate of Mortality in Sierra Leone" (*Transactions of the Actuarial Society of Edinburgh*, iii, No. 12), dealt with the experience of a number of missionaries, and evolved a death rate of about 7 per-cent; and again in a very interesting paper (*J.I.A.*, xxxiii, 285) he reviewed the mortality in the following districts, which exhibit certain marked differences, the result being:

	District	Mortality per-cent	Suggested Lowest Extra
(i)	Congo	9·4	6 to 7 %
(ii)	Central Africa (excluding Congo)	7·8	5 to 6 %
(iii)	West Coast	4·7	3 to 4 %
(iv)	South West Coast	3·5	2½ to 3½ and 4 %

and he suggested also an increase in these rates for the first three districts in the early years of assurance.

The "Star" and "London and Lancashire" experience to 31 December 1896 indicated, however, a mortality of less than 3 % per annum for European assured lives in West Africa (*J.I.A.*, xxxiii, 516).

On page 255 of the present volume of the *Journal*, Mr. Paul Bergholm reports "An Investigation into the Mortality among Scandinavian Emigrants to the Congo", and in the letter from Dr. A. E. Sprague which follows (p. 268), it is pointed out that the average rate of mortality, namely, 8·7 per-cent, closely agrees with that of 9·4 mentioned above.

The question of free limits for travel and residence was discussed by Mr. Thiselton in his "Plea for Uniformity in Life Assurance Practice" in 1893 (*J.I.A.*, xxxi, 29), and I think we have now certainly approached more nearly to uniformity in this respect.

The general free limits may be said to include:—

- Most portions of the world (Asia excepted) not between 33° North and 30° South; the more civilized parts of Africa North of 30°; and Egypt as far South as the Second Cataract; Asia Minor; Japan; and Siberia (the East Coast and the more accessible parts);
- All Australia (the portion North of 20°, or North of the Tropic of Capricorn, sometimes excepted);
- Fiji and some other islands of the South Pacific Ocean;
- The Canary Islands;
- South Africa (including Cape Colony, Natal, the Orange River Colony, and the Transvaal), though in these parts there certainly exist some divergencies in practice, the free limits being sometimes South of 23°, 25°, 28°, or 30°, or in one or two cases extending as far North as the Zambesi River, so far as British Possessions are concerned;
- 6 out of 32 offices do not except Asia from the general free limits, whilst nearly one-half of those who referred to the point would make a restriction in respect of the Northern (Tropical) half of Australia.

The replies received from the various companies with reference to climate extras do not lend themselves to any convenient form of summary, owing to the very wide differences existing in practice. A detailed analysis, however, would only be wearisome. After trying several methods, I selected the following form of partial summary, which will, I hope, serve its purpose sufficiently to indicate the salient features of the replies.

The columns in the appended Table of "lowest" and "highest" extras are supplemented by another giving what may be termed a

"general" extra premium. The latter is the extra which appears to be charged more often than any other. To exhibit the tendency of the companies charging different premiums each "general" rate is followed by a positive or negative sign indicating to which side, whether to greater or smaller extras, the balance of practice tends; thus, a general extra premium 20*s.*—, means that the most frequently charged rate is 20*s.* %, but that in the case of the companies whose charge is different the majority charge less than 20*s.* %, so that the average rate may be considered as 20*s.* or a little less, and *vice versa*, where the other quotations are more often in excess.

It is necessary, however, first to divide the companies according to their custom in limiting the extra premium. Whilst the majority of them charge the climate extra for so long as the risk may be incurred, a substantial minority limit the total amount to be received. The system of limitation of the payment of extra premium is probably somewhat on the increase. It will easily be imagined that it originated amongst companies transacting a "Home" business, whose foreign risks were confined to assurances upon the lives of persons proceeding abroad on government or mercantile business, and intending eventually to return to this country.

Removal of Climate Extras on Return.

Out of 37 companies:—

29 remove the extra without question; the practice of
2 is not definitely fixed; and
4, only, require evidence or a statement of satisfactory health;
2 maintain the extra premium throughout.

On the point, however, of

Limiting the payment of Climate Extras generally—

22 offices (out of 38) require payment so long as the life remains abroad; and in the case of those who incorporate the climate extra in the permanent premium (upon which profits are in some instances based) the payment (full or as reduced by bonus) is thus permanent; some are willing to consider the *reduction* of the extra charge after the first few years upon satisfactory evidence of continued health; others make an *additional* charge for the first years for places like the West Indies and Central and South America in respect of unacclimatized lives. The latter practice is not so prevalent now as formerly;

16 offices have some system of limitation in force. With these, time is sometimes the basis of the condition (*i.e.*, the payment is limited to the first 5 or 10 years of the duration of the policy), otherwise the limitation is to a number of full annual payments (5, 10, or 15, after allowance for any refunds), or to a total percentage of the sum assured, 10 or 15 %.

Climate Extras.

Locality	Unlimited in Duration			Limited in Duration			Remarks
	Lowest Extra	Highest Extra	General Extra	Lowest Extra	Highest Extra	General Extra	
Asia—							
India	10*s.*	30*s.*	20*s.* –	10*s.*	40*s.*	20*s.* –	
Burmah	10*s.*	30*s.*	20*s.* –	10*s.*	40*s.*	20*s.* –	
Persia	nil*	40*s.*†	20*s.* –	10*s.*	25*s.*	20*s.* –	* N. of 33° † Gulf of Persia
Malay Peninsula and Siam	10*s.*	30*s.*	20*s.* +	10*s.*	40*s.*	20*s.* +	1 office discourages
East India Islands—							
Dutch	20*s.*	40*s.*	20*s.* +	10*s.*	40*s.*	20*s.* +	
British or American	20*s.*	40*s.*	20*s.* +	10*s.*	40*s.*	20*s.* +	
China—							
Treaty Ports	nil*	30*s.*	20*s.* –	nil	40*s.*	20*s.* –	* N. of Shanghai, or 33° N.
Remainder	nil†	40*s.*	20*s.* –	20*s.*	40*s.*	20*s.* +	† N. of 33° N.
Africa—							
Northern Coast	nil	10*s.**	nil	nil	nil	nil	* 1 office only
Egypt, Nile Valley	nil*	20*s.*†	...	nil*	20*s.*†	...	* N. of 1st Cataract; or 2nd Cataract; or N. of 18° † S. to Khartoum
West Coast	£5. 5*s.*	£10. 10*s.*	£5. 5*s.* +	£5. 5*s.*	£10. 10*s.*	£5. 5*s.* +	18 offices discourage
Congo	£5. 5*s.*	£10. 10*s.*	£5. 5*s.* +	£5. 5*s.*	...	...	18 " "
South West Coast, 8°–20° S.	£3. 3*s.*	£7. 7*s.*	£5. 5*s.* –	40*s.*	£5. 5*s.*	£3. 3*s.* –	12 " "
Rhodesia—							
N. of R. Zambesi	10*s.*	£5. 5*s.*	20*s.* +	10*s.*	£5. 5*s.*	20*s.* +	4 " "
S. " "	nil	20*s.*	nil to 20*s.**	nil	20*s.*	nil to 20*s.**	1 office discourages * According to locality
Delagoa Bay	nil	£5. 5*s.*	40*s.* –	nil	£3. 3*s.*	40*s.* +	4 offices discourage
Zanzibar	30*s.*	£3. 3*s.*	£3. 3*s.* –	20*s.*	£3. 3*s.*	20*s.* or £3. 3*s.*	7 " "
British Central Africa	20*s.*	£5 5*s.*	40*s.* +	40*s.*	£5. 5*s.*	£3. 3*s.* +	7 " "
Mauritius	nil	40*s.*	20*s.* –	10*s.*	20*s.*	20*s.* –	
America—							
United States (portions S. of 33° N.L. bordering on Gulf of Mexico)	nil	40*s.*	20*s.* –	nil	40*s.*	10*s.* +	
Mexico—							
Tableland	nil	40*s.*	20*s.* –	nil	40*s.*	20*s.* –	
Coast and low-lying country	10*s.*	40*s.*	20*s.* +	30*s.*	40*s.*	30*s.* or 40*s.*	1 office discourages
Central America	20*s.*	40*s.*	40*s.* –	20*s.*	£3. 3*s.*	40*s.* –	
Panama	20*s.*	£3. 3*s.*	40*s.* +	30*s.*	£3. 3*s.*	£3. 3*s.* –	8 offices discourage
West Indies	10*s.*	50*s.*	20*s.* +	20*s.*	30*s.*	20*s.* +	1 office discourages
Guiana	20*s.*	40*s.*	20*s.* +	20*s.*	30*s.*	20*s.* +	1 " "
Brazil	nil*	45*s.*	20*s.* +	10*s.*	30*s.*	20*s.* +	* S. of free limit; 1 office discourages
Argentina	nil*	10*s.*	nil +	nil*	10*s.*	nil +	* All; or S. of 30°
Chili	nil	10*s.*	nil +	nil	10*s.*	nil +	
Peru	nil	20*s.*	20*s.* –	nil	20*s.*	10*s.* +	

2 M 2

The statement of extra premiums is accordingly subdivided as to the permanence or limitation of the payments, and it will be noted that this difference of practice is not necessarily reflected in the columns indicating the general extra of the class. In only a few instances is any difference of rate indicated.

Some companies publish separate prospectuses applicable to their larger foreign branches or agencies, in which the premiums are often based upon the results of their own experience, and for endowment assurances the extra charge generally varies from one-half (or more often about three-quarters) to within a shilling or two of the whole-life extra, according to the term of the policy. Amongst the companies which quote a general extra rate applicable to whole-life policies at all ages the same charge is generally made for endowment assurances, but there is a distinct and increasing tendency to discriminate in favour of the latter class by a reduced extra as above.

As regards **Double Endowment Assurances.**

Out of a total of 37 offices :—

5 offices waive the extra (or extras of not more than 20*s.* %) altogether, and
10 charge a smaller extra, whilst
3 give each case special consideration, and
7 charge the usual extra rate on the death benefit;
12 do not grant this form of policy, or have no definite rule.

According to the paper by Monsieur Fleury, Actuary of "Le Phénix" Assurance Company of Paris, previously mentioned, it is usual on the Continent for extra premiums to be charged, not on the full sum assured but on the amount at risk, *i.e.*, after deducting the reserve (or surrender) value.

Inferior Eligibility.

Finally, we come to extra premiums charged on account of the inferiority of the life proposed for assurance.

Here, to all intents and purposes, we have little available experience, though one should not forget to mention the general table relating to Diseased Lives (D^{MF}) included in the Mortality Experience of 1869; the "Attempt to Measure the Extra Risk arising from a Consumptive Family History", by Mr. H. W. Manly (*J.I.A.*, xxx, 97); the "Remarks on Consumption in relation to Life Assurance", by Dr. T. Glover Lyon (*idem*, p. 120); the letter concerning "Consumptive Family History" from Mr. T. B. Macaulay (*idem*, p. 337); and also the papers submitted at the meetings of the Life Assurance Medical

Officers' Association, and published in their periodical transactions, which are full of interest to the actuary in his daily business.

Each office is, however, accustomed to deal in its own way with this class of risks, and no assimilation of practice is possible, or perhaps even desirable, since difference of opinion is bound to exist in the medical profession itself as to the comparative importance of various characteristics encountered in the consideration of lives for assurance, and this will be reflected in the advice tendered to the office, and in the terms upon which acceptance is granted.

Medical opinion, however, must necessarily vary with the accumulation of general knowledge as to cause and effect of disease; and, if assurance offices from their records can assist the medical officer in his estimates by affording him a reference to past experience, they would also be assisting themselves in attaining more scientific methods of assessing under-average lives for their purposes.

Methods of dealing at Valuation with Policies subject to Extra Premiums.

Not the least important portion of our subject is the question of the reserves to be made for policies subject to extra premiums on any account.

The following Table summarizes the present procedure:

Nature of Special Reserve	Offices making Special Reserves in respect of Policies subject to extra risk on account of			
	Climate	Occupation (constant monetary addition)	Inferior Eligibility	War Risks covered by Annual Premium
No special reserve	2	3	4	2
Special reserve of—				
½ annual extra	18	17	6	9
⅔ " "	1	1	...	1
1 " "	10	10	2	5
"Proportion of annual extra for unexpired risk"	7	6	2	4
Valuation of policy according to equivalent rated-up age	2	3	26	2
Reserve of proportion of total extra premiums paid in quinquennium	1	1	...	3
Other special reserves in existence (or under consideration)	...	...	...	5
Special schemes (*e.g.*, postponement of profits)	...	...	1	3
Total number of Offices	41	41	41	34

Contingent Debts in lieu of charging an extra premium.

17 offices do not permit this system;
18 offices allow it in suitable cases, of whom
3 allow the fixed debt only,
7 allow the decreasing debt only, and
8 allow either.

Most of the companies in valuation make the ordinary reserve without reference to the debt; two, however, value the policy strictly according to its terms at the equivalent "rated-up" age.

Bonuses.

The bonuses in 3 companies which incorporate all extras in the permanent premium are increased in amount or effect (according to the system of distribution) by reason of the increased premium;
In the remaining (38) offices, all other climate and occupation (monetary) extras do not affect the policy's share of profits; though
7 of the latter, in the case of a policy rated up by addition to the age, adopt the increased age as the basis of participation.

Surrender-Values.

As regards Surrender-Values,
Of the 26 offices who make reserves in respect of inferior eligibility at the rated-up ages,
10 only grant surrender-values on this basis, whilst
3 allow the mean between the ordinary and the "rated up" value;
1 company makes some allowance in respect of constant annual extras paid for climate or occupation; and
Another deals with certain climate extra cases on the basis of the special system employed for the corresponding reserve.

Extra Risk Tables.

After the foregoing summary of the history and practice of life assurance Offices in connection with extra risks, I now proceed, by means of a series of tables, to discuss the question from a theoretical point of view, in order to ascertain how far (or in some cases it might be said how little) theory accords with practice.

For the completion of these tables I should first like to acknowledge my indebtedness to Mr. W. Penman, F.I.A., of the "Atlas" Assurance Company (lately a colleague in the "Northern"), who not only kindly undertook their preparation, but also checked the subsidiary values and made many valuable suggestions in connection with their use; indeed, without his aid this portion of the paper could not have been submitted to the Institute.

The time at my disposal, however, was unfortunately so limited that it was impossible to attempt any adequate consideration of the many problems presented to us. I may

have an opportunity in the future of returning to the subject, but, meantime, I submit the results for what they are worth, in the hope that a more able criticism than I can bring to bear upon them may find grounds for an instructive discussion and for reference to many of the points which I have had to leave untouched. I shall confine myself here to an explanation of the tables and an indication of some of the inferences to which they apparently lead.

The selection of the $O^{M(5)}$ Table as a standard is doubtless open to criticism, but this Table is certainly now being recognized as a valuation basis, and, as a mortality standard for comparison with such very general results as have been deduced from the limited experiences of extra risks available, it affords great convenience, owing to the graduation by Makeham's formula which was so fortunately found to be practicable.

Table I consists simply of the various $O^{M(5)}$ 3% functions, including policy-values for quinquennial ages and terms, namely, a_x, A_x, π_x, q_x, and $100\,{}_nV_x$, thereafter taken as the standard table.

Table II involves an addition to the age of 10 years, and similar functions are given on this basis, thus $a^2_x = a_{x+10}$, $A^2_x = A_{x+10}$, &c., and ${}_nV^2_x = {}_nV_{x+10}$, where the ordinary symbols are the values in Table I, and further columns are added, giving the differences between these and the standard (Table I) values. For simplicity the differences are designated by ΔA_x, $\Delta\pi_x$, Δq_x, but it must be remembered that the symbol Δ always has reference to the similar function of the standard table, and not to any differences of the successive functions themselves, so that, for example, $\Delta q_x = q^2_x - q_x$. A supplementary table is furnished for the differences of the policy values, namely, $\Delta 100_n V_x$.

Table III is the well-known table involving a constant extra mortality (μ) of ·01, and in accordance with Mr. Makeham's suggestion—carried out by Messrs. White and Whittall in *J.I.A.*, xxiv, 385—the functions have been calculated from the $O^{M(5)}$ 4% Annuities.

> $a^3_x = a^{4\%}_x$ from which, by the 3% conversion table, we obtain—A^3_x and π^3_x; q^3_x is calculated from the relation $a^{4\%}_x = v^{3\%}.p^3_x.\mathbf{a}^{4\%}_{x+1}$, as explained in Messrs. White and Whittall's Paper; and ${}_nV_x = 1 - \dfrac{\mathbf{a}^{4\%}_{x+n}}{\mathbf{a}^{4\%}_x}$.

The practice of discontinuing payment of an extra premium on the actual cessation of the risk for which it was imposed, and

of reverting to the ordinary premium for the original entry age, involves the assumption of equal reserves under the different tables of mortality: *i.e.*, that the ratio of the annuities due is constant for all ages, which is not a general experience. If, however, the extra mortality is constant up to a point and then ceases, the reserves for a *participating* life policy, under which the bonuses have been left to accumulate, would nearly meet the condition of equality.

There is not much to be gained, therefore, by endeavouring to show that the existing practice is justified; it is a matter of expediency, and we can only say that for a class of risks which fulfilled the condition, and which left no subsequent ill-effects upon health, the reduction would probably be as equitable as any other method that did not involve a special calculation for the reduced premium in each case. Again, if it were customary to hold increased reserves during the currency of an extra risk, and, moreover, the ill-effects upon health were partially continued thereafter, then, provided the partial reduction permissible in the reserve were sufficient to counterbalance any excess premium required by the new conditions, we might still find it feasible to reduce the premium to the ordinary rate for the age at entry.

The discussion of the latter scheme would appertain to the mortality described under Table IV (*see* page 486), and under present circumstances is rendered impracticable by the absence of any suitable experience upon the point, whilst the former will be referred to under the heading of Policy Values; it remains merely to see what modification of conditions is involved by the assumed cessation of all extra risk.

The following Table, which I call IIIb, is based upon the increase of ·01 in μ_x, like Table III, but only until age 55, when the mortality reverts to that of the $O^{M(5)}$ Table.

Table IIIb.

Age x	a_x	A_x	π_x until age 55	ΔA_x	$\Delta\pi_x$ until age 55	n	100_nV_{20}	$\Delta 100\,_nV_{20}$ negative	100_nV_{30}	$\Delta 100\,_nV_{30}$ negative
20	18·894	·42056	·02207	·09432	·00797	5	3·652	·613	5·530	·586
25	18·275	·43859	·02396	·08358	·00793	10	8·010	1·149	11·977	·977
30	17·543	·45991	·02638	·07192	·00792	15.	13·176	1·540	19·391	1·092
35	16·687	·48485	·02952	·05947	·00796	20	19·208	1·719	27·783	·823
40	15·707	·51339	·03361	·04614	·00806	25	26·159	1·607	37·162	...
45	14·612	·54528	·03896	·03195	·00824	30	34·053	1·093	...	...
50	13·428	·57977	·04606	·01672	·00853	35	42·918	...	...	...
55	12·204	·61543	·04661	·00000	·00000	...	...	...	...	...

The column of q_x would be similar to that under Table III as the discounting function is $\frac{D_{x+1}}{D_x}$ at 4 %.

The annual extra premium is only payable during the currency of the extra risk, and the ordinary $O^{M(5)}$ premium for the age at entry commences with the payment at age 55. It will be seen that the extra premium increases from age 30, but for all intents and purposes may be considered constant at 16*s*. %, larger throughout than under Table III, whilst, as would be anticipated, the differences of A_x are smaller at the outset and diminish more rapidly.

Any equality of rates in the annuity-values is out of the question from our assumptions, and the specimen values for ages 20 and 30 at entry indicate that the policy reserves, like those under Table III, are less during the currency of the extra risk. Even including a uniform bonus of 30*s*. % per annum, the above differences in reserve values would not be fully utilized. If we may be allowed to draw any inference from so few figures it will be that, *if such a table be found to fulfil the conditions of practice*, the method of assuming the extra risk to be met out of the current premiums and of keeping the same reserves as for ordinary policies makes adequate provision for the liability.

There is one other point that may be mentioned. Table IIIB will afford an example in connection with the postponed bonus schemes for military and naval risks. The annual loading for a simple reversionary bonus of 1 % per annum to commence at age 55, calculated from the Table, can be compared with that for the ordinary 1 % bonus under Table I.

Annual Bonus loadings payable during the currency of a policy of 100.

Age at Entry	Bonus postponed to Age 55 Table IIIB	Immediate Bonus Table I	Difference
20	·083	·459	·376
30	·141	·524	·383
40	·253	·595	·342

Where ordinary participating rates are charged under the scheme, the difference in the above loadings will approximately represent the extra premium allowed for, assuming retirement to take place at age 55, whilst any excess of the ordinary rate of

bonus declared would involve an additional contribution from profits to meet the extra risk. If, however, the value of such bonus be reserved though profits are not allotted, it would be equivalent in principle to the practice of those offices which reserve the total annual extra premiums paid in respect of war risks, to accumulate in time of peace until war may unfortunately necessitate their use in settlement of claims.

The functions of Table IIIB were calculated in the following manner:

$a'_x = a^{4\%}_{x\overline{54-x}|} + \frac{D^{4\%}_{55}}{D^{4\%}_x}\mathbf{a}^{3\%}_{55}$, from which A'_x was obtained by entering the 3 % conversion table.

$\pi^{3\%}_x \mathbf{a}'_x + \Delta\pi_x . \mathbf{a}'_{x\overline{55-x}|} = A'_x$, whence we obtain $\Delta\pi_x$, the annual extra limited to $(55-x)$ payments—

$${}_nV'_x = A'_{x+n} - \pi^{3\%}_x \mathbf{a}'_{x+n} - \Delta\pi_x(\mathbf{a}^{4\%}_{x+n\overline{55-x-n}|}).$$

The accented symbols relate to the Table functions, the unaccented to the $O^{M(5)}$ 3 % and 4 % Tables.

Table IV involves a constant percentage (of about 80 %) extra mortality, *i.e.*, mortality in a constant ratio to the standard.

This idea was suggested by Messrs. Hardy and Rothery in the *Journal* (vol. xxvii, p. 178), when publishing the "Barbados Mutual" Experience, though they subsequently (p. 180) introduced a modification of Makeham's constants for the purposes of comparison, which did not involve the fixed percentage.

The advantage of being able to base a limited experience upon a standard table by a comparison of the ratios of actual to expected deaths is obvious. We can make use of larger groupings (*e.g.*, 10 or 15 year periods) to obtain the expected deaths, and thus secure a better comparison by the larger numbers involved, whilst a varying percentage will be sufficiently indicated by studying the successive groups. The method facilitates several comparisons with other standard tables at the same time so that it can at once be seen whether the percentage is more constant in relation to one table than another. Assuming we have been able to find a suitable basis, we can by this means obtain without further trouble a series of annuities, premiums, &c., which will meet the requirements of the limited experience.

The discrepancies in the observed rates in advanced life will have no great effect upon values at ordinary ages (particularly in

these days of endowment assurances, when mortality over age 65 can be disregarded), and if that portion of our experience is somewhat limited, we may still find it possible to assume the constant ratio without greatly disturbing our results.

The system adopted by the American Offices in their classification of the special classes of risk previously mentioned involved the comparison of the actual with the expected deaths for grouped ages at entry in intervals of 13 years, and the results, expressed as a percentage of the expected deaths, will I think be a sufficient indication of the use to which this method could be adapted.

Of our home experiences, we can refer to Mr. Spencer Thomson's investigation of the "Standard" East Indian Mortality (1885 to 1900) showing an average extra mortality of 30% over the H^M, and though the deviations from this average are certainly large in particular groups, they are of a character to suggest that they may be caused by a limited number of observations. Moreover, they indicate the fact, commented upon by many writers, that the rate of extra mortality for tropical climates increases with age, and cannot therefore be represented by a constant addition throughout to μ_x as in Table III.

Mr. Winter referred particularly to this feature in his paper on the "British Empire" Indian Experience (*Transactions* of Berlin Congress, vol. ii, p. 68), and found that the mortality applicable to European lives was represented by the $O^M q_{x+5} + \cdot 003$. He also showed that this would indicate a percentage extra of about 50% on the $O^{M(5)}$ Table.

Using ordinary symbols as before for the $O^{M(5)}$ 3% Table, and accenting them for the special risk table under discussion, the well-known formula

$$\mu_x = -\lambda_\epsilon s - \lambda_\epsilon c \,.\, \lambda_\epsilon g \,.\, c^x$$

becomes $$\mu'_x = (1+b)\mu_x = -(1+b)\lambda_\epsilon s - (1+b)\lambda_\epsilon c \,.\, \lambda_\epsilon g \,.\, c^x,$$

whence $$\log_\epsilon l'_x = -\int (1+b)\mu_x dx = h + (1+b)x \,.\, \lambda_\epsilon s + (1+b) \,.\, \lambda_\epsilon g \,.\, c^x$$

where h is the constant introduced by integration.

$$\therefore \qquad l'_x = h' \,.\, s^{x(1+b)} g^{(1+b)c^x}$$

and $$l'_{x+t} = h' \,.\, s^{(x+t)(1+b)} g^{(1+b)c^{x+t}}$$

$$\therefore \qquad {}_tp'_x = s^{(1+b)t} g^{(1+b)c^x(c^t-1)}.$$

Substituting G for g^{c^t-1}, and introducing v^t,

$$v^t\,.\,{}_tp'_x = v^t s^{(1+b)t}\,.\,\mathrm{G}^{(1+b)c^x}.$$

If we find an age w such that $c^x.(1+b)=c^w$, we have—

$$v^t{}_tp'_x = (vs^b)^t s^t \mathrm{G}^{c^w} = (vs^b)^t{}_tp_w = \text{say } v'^t{}_tp_w,$$

so that, if we substitute age w for x in the table, and assume a rate of interest i' where $(1+i')s^b=1+i$ (or $vs^b=v'$), we shall have the means of ascertaining the annuity-values under our special table without the necessity of constructing it for the original values of q'_x.

If $i=\cdot03$ and $i'=\cdot035$, we find $b=\cdot8215$.

Again $$c^x(1+b)=c^w$$

$\therefore$ $$(w-x)\lambda c = \lambda(1+b)$$

or $$w-x=\frac{\lambda(1+b)}{\lambda c}=6\cdot678,$$

which is the increase of age corresponding to an increase of the rate of interest from 3 to $3\frac{1}{2}$ % necessary to produce annuity-values based on a constant extra percentage mortality of 82·15.

The annuities obtained by rating up the age 7 years on the $\mathrm{O}^{\mathrm{M}(5)}$ $3\frac{1}{2}$ % table will very nearly involve an extra mortality of 80 %, whilst if we assumed $3\frac{1}{4}$ %, the addition to age would be 4 years, and the extra mortality about 40 %.

Table IV has been calculated upon the former basis— $a^4_x = a^{3\frac{1}{2}\%}_{x+7}$, from which were obtained A^4_x and π^4_x by means of the 3% conversion table, q^4_x was calculated as before from the consecutive annuity-values, and ${}_n\mathrm{V}^4_x = 1-\dfrac{\mathbf{a}^4_{x+n}}{\mathbf{a}^4_x}$.

It is rather difficult, at present, to find an example to which to apply this Table. None of the recent climate experiences published have, as far as I know, been accompanied by corresponding tables of annuities and premiums. The following, however, is an approximation, and illustrates also a method of the partial application of the principle which may be very useful.

In his paper on extra risk and the mortality of liquor sellers (*Transactions*, Act. Soc. Edin., vol. iv, p. 117), Mr. Low gives a table of premiums deduced from the results of the Associated Scottish Life Offices' Investigation; and the Committee's Report previously mentioned (*J.I.A.* xxxiii, 257) contains a comparison of the mortality (q_x) with the H^{M} rates.

From the following extract:

Annual Mortality per-cent (q_x).

Age	H^M	Inn-keepers and Hotel-keepers	Ratio of (3) to (2)
(1)	(2)	(3)	(4)
25	·66	1·25	1·89
30	·77	1·60	2·08
35	·88	1·90	2·16
40	1·03	2·21	2·15
45	1·22	2·51	2·06
50	1·60	2·72	1·70
55	2·10	3·14	1·50
60	2·97	3·76	1·27

approximately double the mortality seems to have been experienced up to age 45, after which the ratio decreases. If we assume that the double mortality extends to 60 and then entirely disappears, our formula must be limited to this extent.

At 3 % interest, annuities involving double mortality will be found from the formula to be represented by an addition to age of about 8 years, and $3\frac{5}{8}$ % interest.

Consequently, using accented symbols for the special risk table,

$$a'_x = a^{3\frac{5}{8}\%}_{\overline{x+8.59-x|}} + \frac{D^{3\frac{5}{8}\%}_{68}}{D^{3\frac{5}{8}\%}_{x+8}} \mathbf{a}^{3\%}_{60}.$$

Taking the values at $3\frac{1}{2}$ % and 4 % (except $\mathbf{a}^{3\%}_{60}$ which is constant) we have—

Entry age	20	30	40
$a'(3\frac{1}{2}\%)$	19·186	17·168	14·694
$a'(4\%)$	17·796	16·094	13·954

whence, by differencing,

$a'(3\frac{5}{8}\%)$	18·838	16·900	14·509

and from the 3% conversion table

π	2·128	2·674	3·536
whilst	2·042	2·662	3·387

are the corresponding premiums of the experience.

The results indicated by this method are, I think, decidedly encouraging, and the sign of the differences might well be anticipated from the assumptions made as to the mortality ratio.

Inferior Eligibilty.

In discussing inferior eligibility I am following in the footsteps of Mr. Joseph Burn, who dealt ably with this important question at the last Actuarial Congress, but who has not yet submitted to the Institute the anticipated paper upon the subject.

I quote the following from the *Transactions* of the Berlin Congress vol. i, p. 207.

"It is a comparatively simple matter to equate the suggested "surcharge to the extra mortality which the medical adviser "anticipated, but it is extremely doubtful whether the curve "representing such mortality rates would be recognized by the "doctor as an interpretation of his opinion."

It is a usual custom to ask the doctor who examines an applicant for assurance to state (if he does not deem the life eligible at ordinary rates) what addition to the age he considers necessary, in order to render the life equivalent to the average at the advanced age. To answer this question he is often furnished with a table of expectations of life. I assume that from his general examination and the history of the applicant he forms an opinion as to the amount (if any) by which the average expectation may be shortened; and that he then ascertains from the table the age corresponding to such reduced expectation, at which he accordingly advises that the life should be rated.

We interpret this opinion to mean that the life should be charged the premium for the rated-up age, and, if the inferiority is derived from tendency to disease, such as gout, or to early degeneration of the circulatory system, we are probably correct in treating the life throughout as of the advanced age. But do we stop here if the rating arises from other causes such as a consumptive taint? Do we not sometimes in the latter case take more from the opinion than is intended? We very often rate the life accordingly, and continue to treat him in this way throughout the duration of the policy. The medical examiner, having given his opinion, probably hears nothing further of the case. It would doubtless be of interest to him to know the actual terms of acceptance by the office, and whether the policy had been issued, or the proposer had refused to complete on these terms, and had secured a more favourable decision elsewhere. But the examiner would probably be surprised, if he lived long enough, to know the alarming extra mortality to which we might still be consigning an old friend who had been assured with us for

perhaps 50 years, and had reached the mature age of 75, because acceptance had been recommended with 10 years' addition in days gone by, on account of an extra risk to which the life would be subject during the earlier years of the policy.

I am not by any means the first to desire a better system of discrimination in connection with under-average lives. Mr. J. Chisholm, in his paper on the Assessment of Life Risks, in April 1886 (*J.I.A.*, xxv, 408), deplored the want of uniformity in our methods, and recommended more co-operation between the doctor and the actuary. We must not, however, lightly condemn one method before we can find a better; and, considering that we have to deal with general medical practitioners, who cannot be expected to possess the technical assurance experience of our chief medical officers, I do not see in what other form we could ask for their opinion.

But I think we might obtain useful information if we supplemented the question by others. We might ask a little more as to the nature of the extra mortality which the examiner anticipated; as to when, for instance, it would be likely to accrue, and whether it would eventually disappear, so that the life, having once passed the danger zone, might thereafter be regarded as normal. And we might in particular see that the examiner was acquainted with the form of assurance desired, asking him to add to his recommendation (based, we will assume, on the eligibility for a whole-life assurance) whether any modification in this respect would alter his opinion.

The chief medical officer can doubtless advise upon these points from the papers submitted to him, but the information obtained *ad hoc*, though not emanating from a life assurance expert, would probably assist him in arriving at a truer appreciation of the examiner's opinion. The facts thus elicited, in relation particularly to the form of policy, afford ample scope for the combined skill of both the medical and actuarial professions.

We may say, then, that second only in importance to the rate of premium to be charged comes the mode in which the extra risk is to be treated in our books; and, though it may not be considered a matter of grave concern that some individual risks should be treated in valuation in a particular way, in order to facilitate their inclusion in the average which they are not sufficient to disturb, I think we may devote a short time to consider whether our methods cannot be brought into a more

scientific relation with the risks which we undertake. The consideration is perhaps more important in connection with offices which allot the profits upon the contribution method, but it must affect the question of surrender-values in all offices.

When considering a curve to represent the early mortality risk (which would be very different from those we are in the habit of studying), my attention was drawn to Mr. Burn's paper read before the Insurance Institute of Toronto in April 1904, and he very willingly gave me permission to follow up the idea, which I have done, though perhaps in a different fashion.

The curve adopted for Table V is essentially an "early risk" curve, and to exhibit this in graphic form I have laid it out in a diagram, with the rates of mortality corresponding to the other Tables (I to IV) of this paper.

It will be seen that the rate actually decreases from age 41 to 52, when it again increases, and coincides with the $O^{M(5)}$ mortality at 60—practically at 55. The form of the curve is not due to mere chance, it is intended to give effect to the opinion that a life subject to a large extra risk (*e.g.*, light weight and a family susceptibility to consumption) would sustain a benefit from increasing age, as years passed and disease did not develop, which could be represented by an increased chance of living still longer. Thus, the probability of surviving five years on the $O^{M(5)}$ Table, compared with the present assumptions would be:

Age attained	Probability of living 5 Years	
	$O^{M(5)}$ Table	Early Risk Table
20	·967	·965
25	·965	·949
30	·961	·916
35	·956	·896
40	·948	·893
45	·935	·900
50	·916	·905

I do not think these probabilities are inherently impossible, and I hope it may not be considered unprofitable to devote a small space to the consideration of the results to which they lead.

Table V has been prepared on the same lines as the others, except that a junction with the $O^{M(5)}$ Table is assumed between ages 55 and 60, so that it is not necessary to tabulate the ordinary

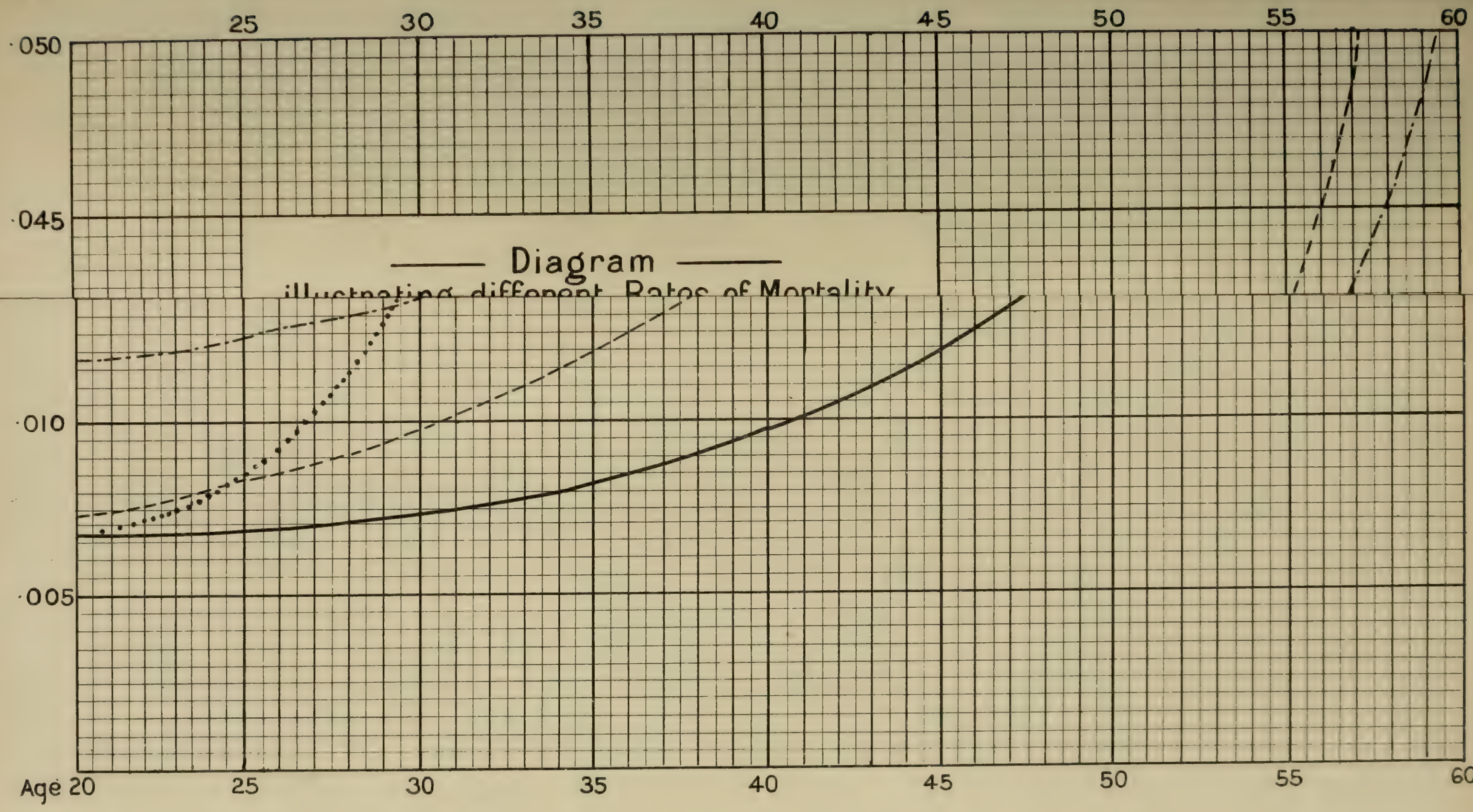

Diagram
illustrating different Rates of Mortality
·050
·045
·010
·005
Age 20
25
30
35
40
45
50
55
60

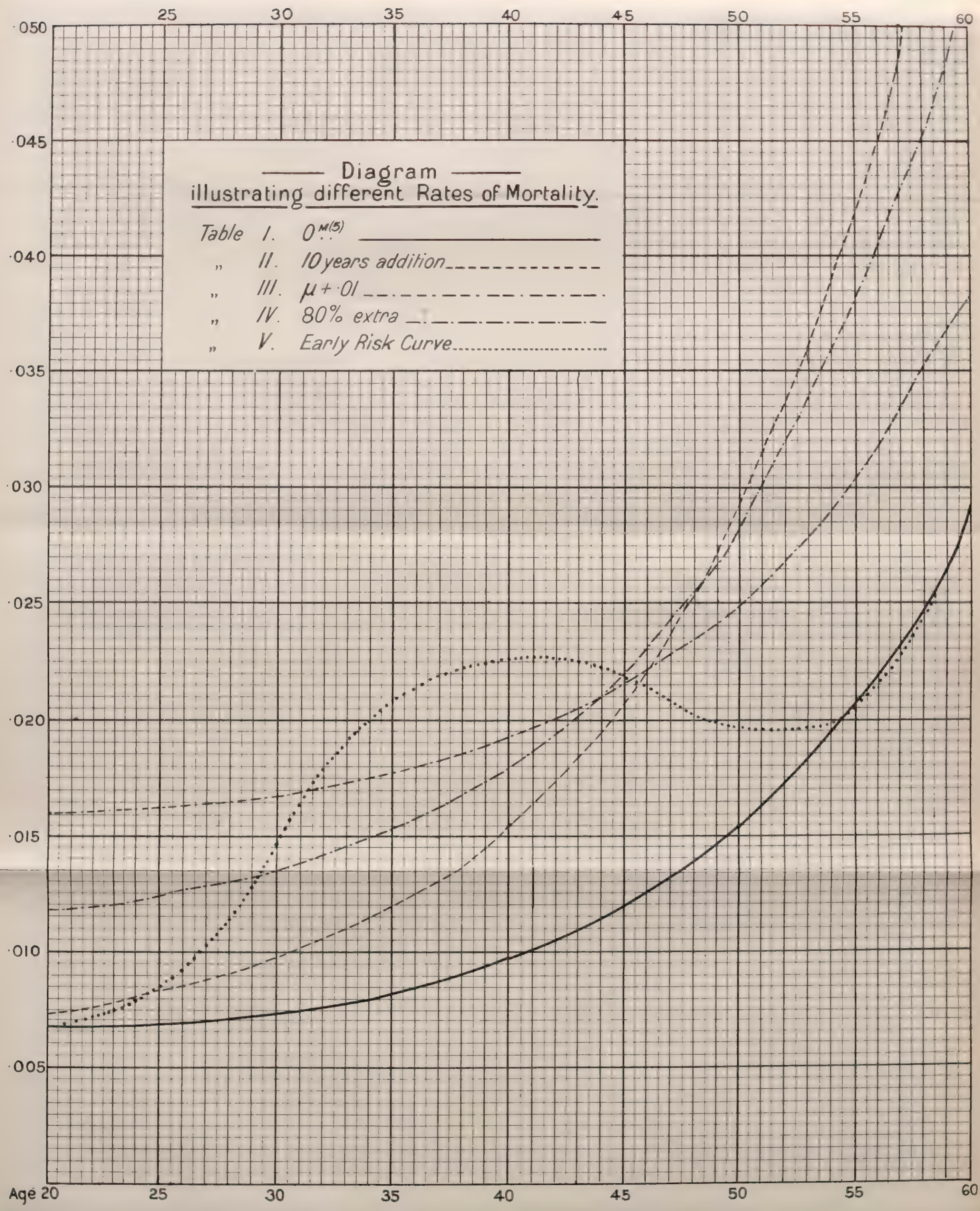
Diagram
illustrating different Rates of Mortality.
Table I. $O^{M(5)}$
" II. 10 years addition
" III. $\mu + \cdot 01$
" IV. 80% extra
" V. Early Risk Curve
·050
·045
·040
·035
·030
·025
·020
·015
·010
·005
Age 20
25
30
35
40
45
50
55
60

functions at or above these ages. Seeing that the mortality curves intersect at age 55 the annuity-values under my table would be slightly less between these ages; the differences however are very small, and the values have not been inserted.

PREMIUMS.

We can now take the Tables generally, and I would draw attention, first, to the Premiums.

The extra assumed (10 years, ·01, 80 %, and the early risk mortality) is, perhaps, rather large, but the Tables will serve the purpose of indicating the general effect. It will be noticed that the monetary additions to the whole life uniform premium at age 30 are very similar in Tables II, III and V. For a policy of 100 they are :

Age	Table II	Table III	Table IV	Table V
20	·436	·753	·646	·420
30	·709	·724	·806	·657

Tables III and IV give heavier additions at age 20; in the former they decrease with age, but in the latter they increase rapidly, though the equivalent addition to age decreases from between 13 and 14 years at age 20, to 8 years at 55. Table IV certainly involves heavy extras, and I should have liked, had time permitted, to substitute a constant 40 % extra mortality. It may, however, be assumed that the monetary differences would be reduced by about one-half on the latter basis : the equivalent increase to age at 20 would be 8 years, and at 55 about 5 years only.

The largest difference in Table V is at age 30; but if a select and an under-average life be supposed to start with similar mortality, our present form of comparison would be taken after 5 years; say at age 25, when the difference is ·549, and, as at age 20, is almost equal to 10 years' addition, so that Table V does not involve any modification in the monetary addition that is not unusual at this age for consumptive risks.

There will also be found appended to this paper :

(*A*) A Table of Whole-Life Premiums limited to 15 payments.

(*B*) A Table of Specimen Endowment Assurance Premiums for 15, 25 and 35 years.

(*C*) A Table of Double Endowment Assurance Premiums.

The limited-payment premiums include at the younger ages a larger extra, but in Tables II, IIIb and IV, the differences are smaller than the whole-life extra at ages 40 and 50. In Tables II and IV, this is accounted for by the fact that the greatest extra mortality will fall when the policy will be fully paid, and the strain on the reserves will consequently be least. In Table III the differences are larger throughout. In Table IIIb the extra will cease at age 55, when the premium will be reduced to the Table I rate, and the reduction of the strain in the early years owing to the form of policy serves to reduce the extra payable during that period at ages 40 and 50.

For endowment assurances the differences under all except Table V are less than the whole-life extras, though for Table IIIb (which coincides with Table III for ages at maturity 55 or less), the extras are not much smaller for the temporary risk. Table II indicates the advantage of accepting that class of risk under an endowment assurance policy.

Under Table V, at the entry age 20, no effective reduction in extra could be entertained: the "early risk" cannot be accepted on more favourable terms under an endowment assurance policy.

Double endowment assurance premiums as exhibited in the Table have a peculiar interest, owing to the growing popularity of the form of policy, and to the fact that some companies are accustomed to waive climatic and other extras under this table. It will be seen that for policies maturing at ages below 60 under Table II the premiums are lower, but that the premium for age 40 maturing at 65 is larger than under the normal table, whilst for all other tables except Table V the premiums are higher. This is especially noteworthy under Table IIIb. If a life aged 40 who is proceeding abroad effects a double endowment assurance payable at 65, and we assume that he will cease incurring extra risk at 55, this would seem to be a policy specially necessitating an extra premium, as the mortality would improve just when (from the company's point of view) the reverse would be advantageous; even under Tables III and IV a small increase is in all cases apparent, and for the longer terms in Table III an extra should

certainly be charged. The premium under Table V at age 20 indicates that this "early risk" is acceptable under those conditions. Mr. Levine's Note (*J.I.A.*, xxxiv, 514) confirms some of the above remarks.

Policy-Values.

The differences of the ordinary policy-values indicated in the tables relate to non-participating assurances, but the columns of ΔA_x afford the means of correcting the figures for any stated amount of bonus under participating policies.

Tables II and IV always give larger values than the standard, those of Table IV coming between the other two.

On the other hand Table III, involving a constant extra only, gives smaller values throughout.

When we take the bonus into consideration the differences in Table II are considerably enhanced, whilst those of Table IV are increased but not to such extent; with a simple bonus of 30*s*. % per annum, the differences of the latter would be under two-thirds of those by Table II.

In the case of Table III, a bonus of as much as 40*s*. % would not increase the total reserves beyond the standard amounts.

Taking Table IV again, for the moment, we know from theory that extra mortality in a constant ratio to the standard, whatever be the amount of that ratio, will always give increased values, but if we remember the method of construction of this Table—a constant and an addition to the age—we know, also, that the constant extra invariably reduces policy-values, as is exemplified in Table III, and the net result will consequently be less than that involved by the full addition to age. Hence, if climatic exposure can be exemplified by the conditions of this Table, we have a ready means of valuing policies subject to such extra risks,—simply by our ordinary method of increased ages, the addition being perhaps a little less than is involved in the premiums. Comparing Table I, with five years' addition, and Table IV, it will be seen that for non-profit assurances this is barely sufficient. The resulting values are less except for the first 5 years up to the age at entry 30, and a little longer for younger ages. On the other hand an addition of 7 years will always give larger values. The same relation holds for participating policies, when we take a uniform bonus of 30*s*. % per annum into account; and we may come to the simple conclusion that an

addition of 6 years to both kinds of assurance will practically meet our requirements. Should the extra risk definitely cease we could consider the reduction or cancelment of the additional reserve according as some of the ill-effects might be anticipated to remain or not.

The "rated-up" age reserve here enters into a new sphere of usefulness, and, in contrast with the reserve of a proportion only of the extra premium, merits serious attention. On reference to the table on page 481 it will be seen that two offices already adopt this method, whilst another reserves a proportion of the total extras paid in the quinquennium.

The $O^{M(5)}$ Table includes in its observations, policies issued without restriction upon lives not intending to proceed abroad; but where extra risk is being incurred from the outset—and particularly where the extra premium payable is limited—we may well consider whether our ordinary reserve without addition is sufficient.

Under Table V, for age at entry 20, the values, larger at first, become less after 15 years, and continue smaller throughout. The increased premium continues payable, though the extra risk has vanished. When the reserve for a bonus of 30*s*. % per annum is added, the total required is in excess until after age 40.

Under Table IIIb the policy-values have already been considered when the construction of the table was under discussion.

It may be said, therefore, that "rated-up age" valuations for all extra risks afford reserves at which—in spite of criticisms of the assumptions involved—no one could find reason to cavil on the score of safety. It would appear that extra reserves are necessary in those cases (Tables II and IV) where the extra risk increases with the age. On the other hand, for a constant extra mortality over the whole of life, or a short period, the ordinary valuation—supplemented by a proportion of the additional premium to cover the unexpired portion of the year—would seem ample. It remains to be decided to which category an extra risk belongs, before we can be satisfied that the ordinary policy reserve is sufficient.

Mortality.

The differences of q_x which represent the extra mortality are perhaps sufficiently well indicated in the columns themselves,

and by the preceding diagram; but the following table of the percentages is interesting:

Age	Ratios of the Rates of Mortality			
	$\frac{q^2_x}{q_x}$	$\frac{q^3_x}{q_x}$	$\frac{q^4_x}{q_x}$	$\frac{q^5_x}{q_x}$
20	1·15	2·46	1·82	1·00
25	1·21	2·39	1·82	1·23
30	1·31	2·28	1·83	1·93
35	1·43	2·14	1·83	2·49
40	1·58	1·97	1·84	2·30
45	1·74	1·79	1·84	1·82
50	1·89	1·61	1·84	1·28
55	2·03	1·45	1·84	·99
60	2·13	1·32	1·84	1·00
65	2·20	1·22	1·83	1·00
70	2·23	1·14	1·82	1·00
75	2·22	1·09	1·80	1·00

The accuracy of the approximate method employed in the calculation of Table IV will be apparent. A table in the above form could be utilized to ascertain whether a limited experience corresponded with any of these assumptions, by comparison of the percentages of the actual to expected deaths with these series of ratios.

When we make the ordinary reserves for policies at increased premiums, the mortality for which we provide is quite different from the above.

From the well-known formula

$$(\pi_x + {}_nV_x)(1+i) = q_{x+n} + p_{x+n} \cdot {}_{n+1}V_x$$
$$= {}_{n+1}V_x + q_{x+n}(1 - {}_{n+1}V_x)$$

we have by substitution of the increased mortality functions

$$\pi'_x,\ q'_x,$$

$$(\pi'_x + {}_nV_x)(1+i) = {}_{n+1}V_x + q'_{x+n}(1 - {}_{n+1}V_x)$$

$$\therefore \qquad (\pi'_x - \pi_x)(1+i) = (q'_{x+n} - q_{x+n})(1 - {}_{n+1}V_x),$$

so that the extra mortality is merely a function of the amount of extra premium paid, and it will be sufficient only to indicate the results under Table II, for which, taking age at entry 30, we have for a policy of 100, $\pi^2_{30} - \pi_{30} = {\cdot}709$,

and

x	$q'_x - q_x$	x	$q'_x - q_x$
30	·00739	60	·01396
35	·00789	65	·01670
40	·00853	70	·02052
45	·00937	75	·02588
50	·01048	80	·03340
55	·01195	85	·04387

The progression in the above series is quite different from that in the Δq_x columns of Tables II, III, or IV; the mortality is dependent solely on the monetary addition, irrespective of the nature of the risk for which it was imposed.

A valuation according to the "rated-up" age would assume the values of q_x indicated in Table II, but if an office make all its reserves according to this method the mortality experienced in the early years may be considerably more than the calculations provide. Some of the rated-up lives aged x will probably live beyond the limit corresponding to the increased age, and hence, in order to reduce the annuity-value to that for the life t years older, it will be necessary that a larger number of the others should die early than amongst a similar group of lives whose true age is $x+t$. The reverse would be the case in the older years.

It will be noticed from Table III that the premium to compensate a constant extra mortality involves a monetary addition to the ordinary rate which diminishes with age, so that a fixed extra premium would pay for an increasing extra mortality. This is the point emphasized in the above table where we have assumed a fixed addition of ·709 to the premium, though in the latter case it results from the method of *equal* reserves.

Consequently, to cover an equal extra risk, a policy which has been in force some years does not require so large an additional premium as one of similar amount just issued. This is another way of saying that the extra premium should be based upon the amount at risk (*i.e.* the sum assured and bonus *less* the reserve).

The mortality under Table V was discussed when considering its construction.

The Contingent Debt System.

Certain questions of mortality and reserves are considerably affected by this system when adopted, and we can best deal with the points together.

The principle is not new. Mr. Samuel Younger mentioned it as a suggestion of Mr. Morrice Black's in October 1862 (*J.I.A.*, x, 268); but the method in which he applied it brought down upon him the criticisms of Messrs. Brabrook, Smith and Gray (pp. 349–57), and their denunciations did not stop at the method, but involved the system. Nor was it received with universal favour when Mr. Sunderland re-opened the subject in 1891 (*J.I.A.*, xxix, 419). Times change, and modern business tends to necessitate that a surcharge should be made in the least discouraging form. The contingent debt appeals to the natural instinct of a proposer to rely upon his own opinion that an extra premium is unjustified, and the system is now becoming fairly general, as will be seen from the former portion of this paper. Is it, after all, less scientific than our other modes of assessing extra rating? The method of charging an extra corresponding to an increased age is perfectly justifiable in so far as the increased premium (however arrived at) meets the risk. It is in the manner of dealing with it subsequently that science occasionally has to sacrifice itself to expediency.

It may be held that the decreasing debt is not applicable to whole-life policies in substitution for a larger premium, when the extra risk is likely to increase or even to remain throughout, but others are of opinion that, provided the debt be properly calculated as an equivalent of the total extra risk, there is no reason why it should not so apply. The office receives its full consideration in this shape, and it becomes a question of reserves accumulated while the extra has least force to meet the mortality when the force is greatest. This is a difference of view, which probably no amount of argument on either side will alter; but a debt adopted in the instances where the danger will probably disappear after a well-marked interval, seems if adequate in amount to be a feasible way of imposing the extra charge. It has the advantage that the extra is only required (or the deduction suffered) during the period before the life becomes normal, and it automatically removes at this epoch a constant source of dissatisfaction amongst the older lives who have been accepted on special terms.

The unscientific element in the system, as usually applied, is that in the early years of assurance, when probably the extra risk is very small, the deduction from the sum assured is at its maximum. A remedy (suggested in America where the Lien

system is quite usual) is to make the deduction fixed at the outset, and increasing after a few years to decrease again thereafter. I am afraid the latter plan would not be relished by the assuring public, and the alternative seems to be the calculation of the debt by means of a table such as No. V, which shall give effect to the precise nature of the extra mortality. The latter, however, is a counsel of perfection, and could only very approximately be attained in practice.

The fixed debt differs considerably from the decreasing system which we have just been discussing, and in some circumstances may seem better adapted for use. We shall, however, return to the point when considering its effect upon the assumed mortality.

The debt system meets, I think, another class of risk—the uncertain life—where there is no actual taint, but the examiner hesitates to recommend acceptance at ordinary rates, because of an apparent want of vigour in a young life which may lead to early deterioration, or, on the other hand, may entirely disappear as he grows older. If the applicant be alive twenty-five years hence, the uncertainty has probably vanished, and from that cause there should be no reason further to surcharge him.

There is, however, one disadvantage, an important one when we view the true principle of assurance in affording protection to dependants. The debt system involves a large diminution in the early years, during which protection is most needed, and is consequently unsuitable for purposes of family provision, or for assurances in connection with loans.

To illustrate the effect of fixed and diminishing debts upon mortality and reserves, I have calculated

(i) The fixed debts ceasing at 55 under Table IIIB in lieu of the additional premiums, namely :

·797 at age 20 for 35 years
·792 at age 30 for 25 years.

The formula is

$$\Delta\pi_x . \mathbf{a}^{4\%}_{x.\overline{55-x}|} = X\left(A^{3b}_x - \frac{D^{4\%}_{55}}{D^{4\%}_x} . A^{3\%}_{55}\right)$$

and the resulting debt (X) is ·45 for age 20 and ·41 for 30.

(ii) The fixed and decreasing debts at ages 20 and 30 under Table V, also ceasing at 55.

The formulæ were :

(*a*) $$(\pi^5_x - \pi_x)\mathbf{a}^5_x = \mathrm{X}\frac{\mathrm{M}^5_x - \mathrm{M}^5_{55}}{\mathrm{D}^5_x}$$

where X = the fixed debt, namely, ·33 at age 20, and ·42 at 30, and,

(*b*) $$\pi_x = \pi^5_x(1 - \overline{55 - x}\mathrm{X}') + \mathrm{X}'\frac{\mathrm{R}^5_{x+1} - \mathrm{R}^5_{56}}{\mathrm{N}^5_x},$$

whence

$$\pi^5_x - \pi_x = \mathrm{X}'\left\{\frac{(55-x)\mathrm{M}^5_x - \mathrm{R}^5_{x+1} + \mathrm{R}^5_{56}}{\mathrm{N}^5_x}\right\}$$

π_x is the $\mathrm{O}^{\mathrm{M}(5)}$ 3 % premium, and $(55-x)\mathrm{X}'$ = the commencing debt, which decreases annually by X′, namely,

·6062 diminishing by ·01732 at age 20,

and

·6966 „ ·027864 „ 30.

(iii) The exact reserves according to Tables IIIB and V respectively for policies subject to these debts, the formulæ being :

Table IIIB—

$${}_n\mathrm{V}'_x = \mathrm{A}^{3b}_{x+n}(1-\mathrm{X}) + \mathrm{X}\frac{\mathrm{D}^{4\%}_{55}}{\mathrm{D}^{4\%}_{x+n}}\cdot\mathrm{A}^{3\%}_{55} - \pi^{3\%}_x\,.\,\mathbf{a}^{3b}_{x+n}.$$

Table V—

(*a*) Similar to the preceding for the fixed debt, using Table V symbols throughout, except for π_x $\mathrm{O}^{\mathrm{M}(5)}$ 3 %.

(*b*) $${}_n\mathrm{V}_x'' = \mathrm{A}^5_{x+n}(1 - \overline{55 - x - n}\mathrm{X}) + \mathrm{X}\frac{\mathrm{R}^5_{x+n+1} - \mathrm{R}^5_{56}}{\mathrm{D}^5_{x+n}} - \pi^{3\%}_x\mathbf{a}^5_{x+n}$$

for the decreasing debt.

(iv) The rates of mortality which are provided for on the assumption that the reserves for these policies are made at the actual ages on the $\mathrm{O}^{\mathrm{M}(5)}$ 3 % Table, neglecting the debt entirely.

Here the formula becomes

$$q'_{x+n}(1-X-{}_{n+1}V_x)=q_{x+n}(1-{}_{n+1}V_x)$$

where q'_{x+n} = the rate of mortality to be ascertained,

q_{x+n} = the standard ($O^{M(5)}$) mortality rate,

and X = the balance of debt remaining at age $x+n$.

Exact Reserve-Values for Policies of 100 *(nominal).*

Duration n	Table IIIB		Table V			
	Age at Entry 20	Age at Entry 30	Age at Entry 20		Age at Entry 30	
	Fixed debt	Fixed debt	Fixed debt	Decreasing debt	Fixed debt	Decreasing debt
5	3·128	4·925	5·293	6·219	5·078	7·173
10	6·938	10·850	10·646	12·635	10·268	13·970
15	11·572	17·971	15·157	18·305	16·802	20·874
20	17·193	26·574	19·637	23·471	25·580	28·503
25	24·024	...	25·254	28·891	...	...
30	32·401	...	32·850	35·269	...	...

Under Table IIIB these exact reserves are less throughout than the values allowing for the temporary increased premium indicated on page 484, but for age at entry 20 in Table V the exact fixed debt reserve is only less for 20 years, and afterwards becomes larger than the ordinary policy-value.

The decreasing debt reserves are greater than for the fixed debt, and with the exception of those at age 20 for 5 years in force they are larger than the ordinary policy-values by Table V. This excess will continue throughout the duration of the policy, as only the $O^{M(5)}$ 3 % premium is payable, instead of the larger premium of Table V.

In comparison with our standard $O^{M(5)}$ reserves the fixed debt values under Table IIIB are less while the debt lasts; but they vary again under Table V, being larger at the commencement for age 20 at entry, and subsequently less until age 55; the decreasing debt at 20 at entry gives larger values till the policy becomes normal.

The question of the sufficiency of the standard reserve consequently becomes very complicated; so much depends upon the nature of the extra mortality that it is intended to cover.

Unless we can lay down definite rules upon this point, an exact valuation becomes theoretically, as well as practically, impossible. Such a valuation of the Table V debt policy by the $O^{M(5)}$ Table would not give an intelligible result. We should be assuming the life perfectly normal, which we know is not warranted; whilst, if it is to be a "rated-up" valuation, we experience the same trouble as in the case of ordinary policies in making our reserves fit the conditions of the extra mortality, but with the added difficulty here that we are called upon first to decide to what "rating-up" our policy debt is equivalent.

The use of symbols may render the matter easier to discuss. We will assume that the policy is effected at true age x; that for a policy without debt we should charge a premium corresponding to age y $(=x+v)$; and that our valuation is at the end of n years.

(α) If we make an exact valuation:—

(i) On the standard table at age $x+n$, we are not keeping all the ordinary reserve ${}_nV_x$, since we take credit for an estimated value of the debt remaining.

(ii) If we value at age $y+n$ with net premium π_x, we keep a rated-up reserve less an assumed value of the remaining debt, leaving us when the debt ceases with the rated-up reserve corresponding to the net premium π_x only, *i.e.*, $A_{y+n}-\pi_x\mathbf{a}_{y+n}$.

(iii) If we substitute π_y for π_x throughout, we are taking credit for a larger net premium than we receive; and though the reserve may be ample during the early years, it will depend upon the amount of the continuing extra risk whether it remains sufficient.

(β) If, however, we neglect the debt, in valuing by method (i) we place an estimated value of the debt against the extra risk, and when the debt vanishes so does our extra reserve.

By method (ii) we maintain a larger reserve against the sum assured throughout, and during the currency of the debt we also keep the assumed value of the balance as an additional safeguard.

By method (iii), however, we are taking credit for a larger net premium than we receive; that is, we consistently reduce the reserve according to the last method by the value of the difference of premium $(\pi_y - \pi_x)$, and after cessation of the debt the same question arises as in α (iii), namely whether

$$ {}_nV_y \gtreqless A'_{x+n} - \pi_x \mathbf{a}'_{x+n} $$

(using accented symbols for the values corresponding to the mortality after age $x+n$), or, let us say

$$ {}_nV_y \gtreqless A_{x+n} + \Delta A_{x+n} - \pi_x(\mathbf{a}_{x+n} + \Delta \mathbf{a}_{x+n}), $$

according as

$$ {}_nV_y - {}_nV_x \gtreqless \Delta A_{x+n} + \pi_n(-\Delta \mathbf{a}_{x+n}). $$

Method (β) (i) is that usually employed, and the following Table indicates the mortality for which it provides—

Extra Rates of Mortality provided by Contingent Debts under Equal Reserve System.

Age attained	Table IIIb Fixed Debt		Table V			
			Fixed Debt	Decreasing Debt	Fixed Debt	Decreasing Debt
	Age at Entry 20	Age at Entry 30	Age at Entry 20		Age at Entry 30	
25	·00620	...	·00372	·00836	...	...
30	·00747	...	·00439	·00696	...	...
35	·00958	·00662	·00547	·00586	·00705	·01265
40	·01336	·00894	·00732	·00491	·00956	·00932
45	·02079	·01324	·01062	·00389	·01425	·00668
50	·03762	·02192	·01707	·00245	·02386	·00386

These figures do not bear comparison with the columns Δq_x in Tables IIIb and V. The progression of the differences is precisely opposite, according as a fixed or a decreasing debt is involved, and one cannot but remark that neither seems very likely to be experienced in connection with assurance risks.

With the fixed debt the mortality, after increasing rapidly, becomes abruptly reduced to normal when the deduction ceases; and if this cannot be anticipated, extra provision is necessary for the further liability.

As the period during which the decreasing debt is operative comes to a close, provision by this method for the extra risk gradually vanishes. If the latter vanish also, no question arises; but if for example the risk is one that could properly have been assessed at a "rated-up" age, then, no matter what has been past relief, provision for present liability necessitates the strengthening of the ordinary reserve. How is this additional amount to be obtained? When the nature of the extra risk demands it, we must during the period of less strain accumulate a sufficient amount for the purpose. Thus, even the full ordinary reserve at the true age is not under these circumstances sufficient for the contingent debt policy.

The paper is already very long, and with these remarks I leave an interesting subject which will repay closer attention; but I hope that those members of the Institute who are versed in the practical application of the debt system may furnish us in discussion with the views derived from their experience.

Conclusion.

In conclusion, I would venture to suggest for consideration by the Council of the Institute, whether it would not be feasible to obtain more information upon the important subject of extra risks than is afforded by the limited statistics at our disposal.

Competition is an excellent thing, but it takes something more than competition to account for the differences in the quotations of some extra premiums at the present time. It is want of experience of the class of risk, and in endeavouring to supply this information the Institute would be conferring an inestimable benefit upon, not only the assurance offices, but the medical profession and the country generally. There are, I am aware, very great difficulties in the way, owing to the absence of any recognized classification of impaired risks, but some such system could I think be devised upon lines broad enough to enable the companies to classify their own experience.

If we could adopt some common form of card on which to record particulars of extra risks accepted, and keep these cards in the office marked up with reference to changes during the currency of the policy and the mode of its termination, we should gradually accumulate amongst individual companies a mass of information which in the aggregate would become exceedingly useful; and, when the time had come to warrant its collection by the Institute

and Faculty, the statistics would be already available, so that considerations such as the trouble of searching registers and the preparation of the particulars would not arise to prevent the realization of the purpose.

The advantage of such an experience could not be over-estimated, whilst the difficulties have already been overcome in America. The Life Offices' Association started a system of collecting the particulars of climate risks, but it has not, I believe, met with the anticipated support. The initiation of a system involving at present merely the preparation of such particulars might have better results.

Table I.

Normal Table $O^{M(5)}$ 3 %.

x	a_x	A_x	π_x	q_x	x
20	22·132	·32624	·01410	·00652	20
25	21·145	·35501	·01603	·00689	25
30	20·013	·38799	·01846	·00747	30
35	18·728	·42538	·02156	·00837	35
40	17·291	·46725	·02555	·00978	40
45	15·709	·51333	·03072	·01200	45
50	14·002	·56305	·03753	·01545	50
55	12·204	·61543	·04661	·02083	55
60	10·364	·66902	·05887	·02921	60
65	8·542	·72207	·07567	·04221	65
70	6·808	·77259	·09896	·06219	70
75	5·224	·81875	·13155	·09267	75
80	3·843	·85895	·17736	·13850	80
85	2·697	·89234	·24134	·20569	85
90	1·794	·91863	·32879	·30075	90
95	1·122	·93819	·44211	·42473	95
100	·551	·95488	·61580	·57143	100

$100_n V_x$.

n	$x=20$	$x=25$	$x=30$	$x=35$	$x=40$	$x=45$	$x=50$	$x=55$	n
5	4·265	5·112	6·116	7·283	8·650	10·216	11·984	13·936	5
10	9·159	10·916	12·954	15·303	17·982	20·976	24·250	27·735	10
15	14·716	17·404	20·483	23·955	27·811	31·989	36·395	40·867	15
20	20·927	24·548	28·606	33·069	37·872	42·893	47·954	52·863	20
25	27·766	32·256	37·162	42·396	47·832	53·271	58·512	63·322	25
30	35·146	40·375	45·920	51·632	57·313	62·751	67·718	72·001	30
35	42·918	48·684	54·590	60·421	65·972	71·016	75·357	...	35
40	50·873	56·912	62·842	68·451	73·523	77·874	...	...	40
45	58·749	64·742	70·380	75·451	79·788	...	...	...	45
50	66·246	71·895	76·953	81·260	...	...	...	...	50

TABLE II.

10 *Years' addition to Age.*

x	$\overset{2}{a}_x$	$\overset{2}{A}_x$	$\overset{2}{\pi}_x$	$\overset{2}{q}_x$	ΔA_x	$\Delta \pi_x$	Δq_x	x
20	20·013	·38799	·01846	·00747	·06175	·00436	·00095	20
25	18·728	·42538	·02156	·00837	·07037	·00553	·00148	25
30	17·291	·46725	·02555	·00978	·07926	·00709	·00231	30
35	15·709	·51333	·03072	·01200	·08795	·00916	·00363	35
40	14·002	·56305	·03753	·01545	·09580	·01198	·00567	40
45	12·204	·61543	·04661	·02083	·10210	·01589	·00883	45
50	10·364	·66902	·05887	·02921	·10597	·02134	·01376	50
55	8·542	·72207	·07567	·04221	·10664	·02906	·02138	55
60	6·808	·77259	·09896	·06219	·10357	·04009	·03298	60
65	5·224	·81875	·13155	·09267	·09668	·05588	·05046	65
70	3·843	·85895	·17736	·13850	·08636	·07840	·07631	70
75	2·697	·89234	·24134	·20569	·07359	·10979	·11302	75
80	1·794	·91863	·32879	·30075	·05968	·15143	·16225	80
85	1·122	·93819	·44211	·42473	·04585	·20077	·21904	85
90	·551	·95488	·61580	·57143	·03625	·28701	·27068	90
95	...	...	...	...	...	...	...	95
100	...	...	...	...	...	...	...	100

$100 {}_n V^2_x$.

n	$x=20$	$x=25$	$x=30$	$x=35$	$x=40$	$x=45$	$x=50$	$x=55$	n
5	6·116	7·283	8·650	10·216	11·984	13·936	16·033	18·172	5
10	12·954	15·303	17·982	20·976	24·250	27·735	31·292	34·773	10
15	20·483	23·955	27·811	31·989	36·395	40·867	45·230	49·246	15
20	28·606	33·069	37·872	42·893	47·954	52·863	57·383	61·256	20
25	37·162	42·396	47·832	53·271	58·512	63·322	67·467	70·720	25
30	45·920	51·632	57·313	62·751	67·718	72·001	75·414	77·761	30
35	54·590	60·421	65·972	71·016	75·357	78·839	81·328	...	35
40	62·842	68·451	73·523	77·874	81·375	83·929	...	...	40
45	70·380	75·451	79·788	83·278	85·856	...	...	...	45
50	76·953	81·260	84·725	87·301	...	...	...	...	50

Difference between Policy-Values based on above Table and those by Table I ($O^{M(5)}$ 3 %).

$\Delta 100 {}_n V_x$.

n	$x=20$	$x=25$	$x=30$	$x=35$	$x=40$	$x=45$	$x=50$	$x=55$	n
5	1·851	2·171	2·534	2·933	3·334	3·720	4·049	4·236	5
10	3·795	4·387	5·028	5·673	6·268	6·759	7·042	7·038	10
15	5·767	6·551	7·328	8·034	8·584	8·878	8·835	8·379	15
20	7·679	8·521	9·266	9·824	10·082	9·970	9·429	8·393	20
25	9·396	10·140	10·670	10·875	10·680	10·051	8·955	7·398	25
30	10·774	11·257	11·393	11·119	10·405	9·250	7·696	5·760	30
35	11·672	11·737	11·382	10·595	9·385	7·823	5·971	...	35
40	11·969	11·539	10·681	9·423	7·852	6·055	...	...	40
45	11·631	10·709	9·408	7·827	6·068	...	...	...	45
50	10·707	9·365	7·772	6·041	...	...	...	...	50

TABLE III. *Constant addition of* ·01 *to* μ_x.

x	a_x^3	A_x^3	π_x^3	q_x^3	ΔA_x	$\Delta\pi_x$	Δq_x	x
20	18·705	·42606	·02163	·01606	·09982	·00753	·00954	20
25	18·037	·44553	·02341	·01644	·09052	·00738	·00955	25
30	17·242	·46868	·02570	·01699	·08069	·00724	·00952	30
35	16·307	·49592	·02866	·01791	·07054	·00710	·00954	35
40	15·224	·52745	·03251	·01929	·06020	·00696	·00951	40
45	13·992	·56334	·03758	·02150	·05001	·00686	·00950	45
50	12·621	·60327	·04429	·02492	·04022	·00676	·00947	50
55	11·133	·64661	·05330	·03027	·03118	·00669	·00944	55
60	9·568	·69220	·06550	·03854	·02318	·00663	·00933	60
65	7·979	·73848	·08225	·05141	·01641	·00658	·00920	65
70	6·429	·78362	·10549	·07120	·01103	·00653	·00901	70
75	4·984	·82571	·13799	·10139	·00696	·00644	·00872	75
80	3·700	·86311	·18363	·14676	·00416	·00627	·00826	80
85	2·618	·89462	·24730	·21333	·00228	·00596	·00764	85
90	1·752	·91984	·33422	·30747	·00121	·00543	·00672	90
95	1·102	·93877	·44672	·43027	·00058	·00461	·00554	95
100	·544	·95503	·61869	·57555	·00015	·00289	·00412	100

$100_nV_x^3$.

n	$x=20$	$x=25$	$x=30$	$x=35$	$x=40$	$x=45$	$x=50$	$x=55$	n
5	3·390	4·177	5·125	6·257	7·594	9·145	10·924	12·900	5
10	7·426	9·088	11·061	13·376	16·044	19·070	22·415	25·995	10
15	12·171	14·776	17·815	21·297	25·216	29·510	34·080	38·771	15
20	17·666	21·248	25·331	29·895	34·863	40·108	45·459	50·680	20
25	23·918	28·450	33·488	38·938	44·656	50·447	56·068	61·263	25
30	30·876	36·266	42·068	48·119	54·210	60·086	65·494	70·181	30
35	38·427	44·488	50·778	57·075	63·117	68·650	73·438	...	35
40	46·370	52·834	59·275	65·424	71·031	75·867	...	...	40
45	54·433	60·976	67·197	72·813	77·700	...	...	...	45
50	62·299	68·567	74·235	79·095	...	...	...	...	50

Differences between Policy-Values based on above Table and those by Table I ($O^{M(5)}$ 3 %). $-\Delta 100_nV_x$.

n	$x=20$	$x=25$	$x=30$	$x=35$	$x=40$	$x=45$	$x=50$	$x=55$	n
5	·875	·935	·991	1·026	1·056	1·071	1·060	1·036	5
10	1·733	1·828	1·893	1·927	1·938	1·906	1·835	1·740	10
15	2·545	2·628	2·668	2·658	2·595	2·479	2·315	2·096	15
20	3·261	3·300	3·275	3·174	3·009	2·785	2·495	2·183	20
25	3·848	3·806	3·674	3·458	3·176	2·824	2·444	2·059	25
30	4·270	4·109	3·852	3·513	3·103	2·665	2·224	1·820	30
35	4·491	4·196	3·812	3·346	2·855	2·366	1·919	...	35
40	4·503	4·078	3·567	3·027	2·492	2·007	...	...	40
45	4·316	3·766	3·183	2·608	2·088	...	...	...	45
50	3·947	3·328	2·718	2·165	...	...	...	...	50

The differences in the policy-values are negative.

TABLE IV.

½ % addition to rate of interest, 7 years' addition to Age; representing a mortality of 80 % above the normal.

x	a_x^4	A_x^4	π_x^4	q_x^4	ΔA_x	$\Delta\pi_x$	Δq_x	x
20	19·127	·41378	·02056	·01188	·08754	·00646	·00536	20
25	18·123	·44302	·02317	·01256	·08801	·00714	·00567	25
30	16·969	·47663	·02652	·01365	·08864	·00806	·00618	30
35	15·664	·51464	·03088	·01533	·08926	·00932	·00696	35
40	14·213	·55690	·03660	·01796	·08965	·01105	·00818	40
45	12·639	·60275	·04419	·02204	·08942	·01347	·01004	45
50	10·975	·65121	·05438	·02846	·08816	·01685	·01301	50
55	9·270	·70087	·06825	·03841	·08544	·02164	·01758	55
60	7·586	·74993	·08734	·05376	·08091	·02847	·02455	60
65	5·990	·79641	·11393	·07736	·07434	·03826	·03515	65
70	4·545	·83849	·15123	·11315	·06590	·05227	·05096	70
75	3·298	·87482	·20354	·16645	·05607	·07199	·07378	75
80	2·276	·90459	·27612	·24367	·04564	·09876	·10517	80
85	1·481	·92774	·37387	·35104	·03540	·13253	·14535	85
90	·910	·94437	·49456	·48525	·02574	·16577	·18450	90
95	·000	·97087	·97087	1·00000	·03268	·52876	·57527	95

$100{}_nV_x^4$.

n	$x=20$	$x=25$	$x=30$	$x=35$	$x=40$	$x=45$	$x=50$	$x=55$	n
5	4·989	6·034	7·266	8·702	10·350	12·203	14·237	16·395	5
10	10·723	12·862	15·336	18·152	21·290	24·703	28·298	31·936	10
15	17·210	20·445	24·099	28·140	32·496	37·048	41·626	46·010	15
20	24·414	28·679	33·361	38·370	43·564	48·749	53·697	58·149	20
25	32·237	37·382	42·848	48·475	54·054	59·347	64·107	68·100	25
30	40·506	46·297	52·218	58·052	63·555	68·487	72·642	75·838	30
35	48·977	55·102	61·100	66·726	71·749	75·980	79·278	...	35
40	57·342	63·447	69·144	74·207	78·466	81·806	...	...	40
45	65·271	71·006	76·081	80·340	83·689	...	...	...	45
50	72·453	77·524	81·769	85·109	...	...	...	...	50

$\Delta 100{}_nV_x$.

n	$x=20$	$x=25$	$x=30$	$x=35$	$x=40$	$x=45$	$x=50$	$x=55$	n
5	·724	·922	1·150	1·419	1·700	1·987	2.253	2·459	5
10	1·564	1·946	2·382	2·849	3·308	3·727	4·048	4·201	10
15	2·494	3·041	3 616	4·185	4·685	5·059	5·231	5·143	15
20	3·487	4·131	4·755	5·301	5·692	5·856	5·743	5·286	20
25	4·471	5·126	5·686	6·079	6·222	6·076	5·595	4·778	25
30	5·360	5·922	6·298	6·420	6·242	5·736	4·924	3·837	30
35	6·059	6·418	6·510	6·305	5·777	4·964	3·921	...	35
40	6·469	6·535	6·302	5·756	4·943	3·932	...	...	40
45	6·522	6·264	5·701	4·889	3·901	...	...	...	45
50	6·207	5·629	4·816	3·849	...	...	...	...	50

Table V.

Hypothetical Table.

(Early Risk or Consumption Curve.)

x	a^5_x	A^5_x	π^5_x	q^5_x	ΔA_x	$\Delta \pi_x$	Δq_x	x
20	20·083	·38593	·01830	·00652	·05969	·00420	·00000	20
25	18·744	·42493	·02152	·00850	·06992	·00549	·00161	25
30	17·464	·46221	·02503	·01440	·07422	·00657	·00693	30
35	16·581	·48793	·02776	·02080	·06255	·00620	·01243	35
40	15·892	·50800	·03007	·02250	·04075	·00452	·01272	40
45	15·066	·53206	·03312	·02180	·01873	·00240	·00980	45
50	13·866	·56701	·03814	·01980	·00396	·00061	·00435	50

100 $_nV^5_x$.

n	$x=20$	$x=25$	$x=30$	$x=35$	$x=40$	$x=45$	$x=50$	n
5	6·350	6·483	4·784	3·918	4·889	7·471	11·057	5
10	12·421	10·957	8·515	8·616	11·994	17·702	23·557	10
15	16·611	14·446	12·988	15·443	21·726	29·267	35·810	15
20	19·878	18·629	19·488	24·793	32·726	40·605	47·481	20
25	23·796	24·708	28·391	35·362	43·509	51·404	58·134	25
30	29·488	33·033	38·454	45·722	53·780	61·262	...	30
35	37·285	42·444	48·319	55·591	63·156	...	...	35
40	46·099	51·670	57·716	64·599	...	...	...	40
45	54·738	60·457	66·293	...	...	...	...	45
50	62·968	68·478	...	...	...	...	...	50

Differences between Policy-Values based on above Table and those by Table I ($O^{M(5)}$ 3 %).

Δ100 $_nV_x$.

n	$x=20$	$x=25$	$x=30$	$x=35$	$x=40$	$x=45$	$x=50$	n
5	2·085	1·371	$\underline{1·332}$	$\underline{3·365}$	$\underline{3·761}$	$\underline{2·745}$	$\underline{·927}$	5
10	3·262	·041	$\underline{4·439}$	$\underline{6·687}$	$\underline{5·988}$	$\underline{3·274}$	$\underline{·693}$	10
15	1·895	$\underline{2·958}$	$\underline{7·495}$	$\underline{8·512}$	$\underline{6·085}$	$\underline{2·722}$	$\underline{·585}$	15
20	$\underline{1·049}$	$\underline{5·919}$	$\underline{9·118}$	$\underline{8·276}$	$\underline{5·146}$	$\underline{2·288}$	$\underline{·473}$	20
25	$\underline{3·970}$	$\underline{7·548}$	$\underline{8·771}$	$\underline{7·034}$	$\underline{4·323}$	$\underline{1·867}$	$\underline{·378}$	25
30	$\underline{5·658}$	$\underline{7·342}$	$\underline{7·466}$	$\underline{5·910}$	$\underline{3·533}$	$\underline{1·489}$	...	30
35	$\underline{5·633}$	$\underline{6·240}$	$\underline{6·271}$	$\underline{4·830}$	$\underline{2·816}$	...	...	35
40	$\underline{4·774}$	$\underline{5·242}$	$\underline{5·126}$	$\underline{3·852}$	...	...	...	40
45	$\underline{4·011}$	$\underline{4·285}$	$\underline{4·087}$	...	...	...	...	45
50	$\underline{3·278}$	$\underline{3·417}$	...	...	...	...	.	50

The differences underlined are negative.

Table A.

Whole-Life Assurances. Annual Premiums limited to 15.

Age at Entry	Table I	Table II	Δ	Table III	Δ	Table IIIB	Δ	Table IV	Δ	Table V	Δ	Age at Entry
20	·02773	·03327	·00554	·03845	·01072	·03796	·01024	·03644	·00871	·03320	·00547	20
30	·03327	·04097	·00770	·04267	·00940	·04188	·00861	·04269	·00942	·04238	·00911	30
40	·04097	·05204	·01107	·04906	·00809	·04775	·00678	·05191	·01094	·04740	·00643	40
50	·05204	·06981	·01777	·05905	·00701	·06014	·00810*	·06666	·01462	·05287	·00083	50

Table B.

Endowment Assurances. Annual Premiums.

Age at Entry	Maturity Age	Table I	Table II	Δ	Table III	Δ	Table IIIB	Δ	Table IV	Δ	Table V	Δ	Age at Entry
20	55	·02123	·02317	·00194	·02782	·00659	·02782	·00659	·02567	·00444	·02489	·00366	20
30		·03222	·03527	·00305	·03825	·00603	·03825	·00603	·03707	·00485	·03893	·00671	30
40		·05855	·06331	·00476	·06389	·00534	·06389	·00534	·06409	·00554	·06418	·00563	40
30	65	·02317	·02776	·00459	·02979	·00662	·03044	·00727	·02925	·00608	·02986	·00669	30
40		·03527	·04268	·00741	·04138	·00611	·04252	·00725	·04282	·00755	·04023	·00496	40
50		·06331	·07523	·01192	·06875	·00544	·07106	·00775*	·07323	·00992	·06412	·00081	50

* The extra premiums underlined are payable only until the anniversary of the policy preceding age 55.

Table C.

Annual Premiums for Double Endowment Assurances.

Age at Entry	Term of Years	Table I	Table II	Table III	Table IIIB	Table IV	Table V	Age at Entry
20	15	·10481	·10470	·10607	·10607	·10528	·10319	20
30		·10470	·10426	·10599	·10599	·10512	·10569	30
40		·10426	·10388	·10584	·10584	·10482	·10774	40
20	25	·05430	·05412	·05714	·05714	·05558	·05372	20
30		·05412	·05387	·05712	·05712	·05546	·05833	30
40		·05387	·05511	·05738	·05824	·05634	·05823	40

SCHEDULE.

....*Life Office.*

Extra Premiums charged to Females, and to those engaged in various Occupations.

N.B.—Please distinguish between cases—

(i) *Where no Extra is charged* ... "*Nil.*"
(ii) *Where no Extra rate is fixed* ... "__________"
(iii) *Not Entertained* ... ... "*Discouraged.*"

Female Extra:—

Rate charged generally, if any, ________ single,
or ________ annual till age ________

Do. if recently married ________.

Occupation Extras.

Bakers (journeymen)	
Butchers (who slaughter)	
Motor Driver (Chauffeur or Engineer) ...	
Hospital Nurse (i) no fever cases ...	
(ii) with fever cases ...	
Liquor Trade—	
Publicans	
Licensed Grocers	
Brewers (so engaged)	
First Class Hotel Proprietors ...	
Military Officers (to include War risk)...	
(i) at Home, or within free limits	
(ii) in India...	
Mining Engineers and Colliery Managers (who go underground):	
Miners—	
(i) Coal	
(ii) Lead and Tin	
(iii) Gold and Silver	
Painters, Plumbers, and Glaziers ...	
Railway Risks—	
Guards	
Porters	
Engine Drivers and Stokers ...	
Stonemasons	

	Within free limits.	Beyond free limits.
Sea Risks—		
Officers of R.N.		
Officers of 1st Class Merchantmen		
Engineers		
Stewards		
Pilots		
Fishermen		
Other Seafaring men		
Are "occupation" extras removed on retirement?		

Climate Extras

For Persons incurring extra risk at date of issue of Policies.

N.B.—Please distinguish between cases—

(i) *Where no Extra is charged* ... "*Nil.*"

(ii) *Where no Extra rate is fixed* ... "__________"

(iii) *Not Entertained* ... ... "*Discouraged.*"

The Extras quoted are assumed to be for Whole-Life Uniform Premium Policies.

Is any difference made for—
- (i) Endowment Assurances? ...
- (ii) Limited Premium Life Policies either
  - (1) before fully paid? ...
  - (2) after fully paid? ...
- (iii) Short Term Policies taken out to cover the special risk? ...
- (iv) Double Endowment Assurances?

Is the extra premium limited in extent—
- *e.g.* (i) to so many years after issue of policy? ... ...
- or (ii) to so much extra premium per-cent paid and retained? ...
  (*i.e.*, after allowing for refunds, if any)

Is any modification made in respect of acclimatized lives in certain districts?
- *e.g.* (i) West Indies ... ...
- (ii) Central and South America ...
- (iii) India and East Indies... ...

If so, please state how "acclimatization" is defined? ... ...

Is the extra premium removed *in toto* on return to free limits without evidence of health? ... ...

Countries.

ASIA—

Do the free limits apply, or is the Continent excepted therefrom? ...
- India ... ...
- Burmah ... ...
- Persia ... ...
- Malay Peninsula and Siam ...
- East India Islands (Dutch) ...
- Do. (British or American)
- China (i) Treaty Ports ... ...
  - (ii) Remainder ... ...

AFRICA—
- Territories on Northern Coast ...
- Egypt (Nile Valley) ... ...
- West Coast ... ...
- Congo State ... ...
- South-West Coast 8°–20° S. ...
- Rhodesia (i) N. of R. Zambesi ...
  - (ii) S. of R. Zambesi ...
- Delagoa Bay ... ...
- Zanzibar and East Coast ... ...
- British Central Africa ... ...
- Mauritius ... ...

Climate Extras—*Continued.*

Countries—*Continued.*

America—

United States (portion S. of 33° N. L. of States bordering on Gulf of Mexico) ... ...
Mexico Tableland ... ...
Coast and low lying country
Central America ... ...
Panama ... ...
West Indies and Trinidad ...
Guiana ... ...
Brazil ... ...
Argentine Republic ... ...
Chili and Peru ... ...

Australia—

Is any restriction from free limits made as regards the Northern (Tropical) portion? ... ...

Methods of Dealing at Valuation with Policies subject to Extra Premiums.

How are the Special Reserves made in respect of policies subject to extra premium on account of—
- (i) Climate? ... ...
- (ii) Occupation (if by constant monetary addition)? ...
- (iii) Inferior eligibility (*i.e.*, where life is "rated up")? ...
- (iv) War Risk when covered—
  - (*a*) by constant annual extra payable till retirement, or
  - (*b*) otherwise (*i.e.*, any system *not* involving a large extra in war time)? ... ...

Is the system of Contingent Debts in lieu of increased premiums permitted? and what variation is made in valuation of such cases? ... ...

Is the Debt "fixed" or "diminishing"?

Is any extra share of profits (Reversionary or Cash) obtained by policies subject to extra premiums for—
- (i) Climate? ... ...
- (ii) Occupation? ... ..
- (iii) Inferior eligibility? ... ...

Is any special allowance made in quoting Surrender Values for these policies, viz.—
- (i) Climate? ... ...
- (ii) Occupation? ... ...
- (iii) Inferior eligibility? ... ...

(N.B.—If policies are treated for Valuation and Surrender at their "rated-up" age please state this.)

Abstract of the Discussion.

Mr. J. R. Hart said that, judging from the amount of literature on the subject, the question of extra premiums would appear to be an exceptionally popular one. On page 482 of the paper, there was a reference to "the foregoing summary of the history and practice of life assurance offices in connection with extra risks." But if the paper was regarded as a summary of the history of the subject, it seemed to be wanting in many important respects. There was no mention whatever of one of the most valuable contributions upon the subject, Mr. J. J. McLauchlan's interesting paper on "The Mortality in certain hazardous Occupations", read before the Actuarial Society of Edinburgh in 1898, although another paper by that gentleman on "Naval and Military Risks" was mentioned. Possibly Mr. McLauchlan's essay had escaped the author's attention, or he would have noticed that in it the comparative mortality table on page 463 was not only exhibited but exhaustively analyzed; and the only way in which the demonstration could be supplemented would be by analyzing in a similar way more recent statistics, if available. That, however, had not been done.

A review of the many papers on occupation statistics forced one to the conclusion that, with the exception of one or two features, population returns were of practically no value to the actuary in his consideration of the question of extra premiums. The real effect of occupation on mortality was masked by three important factors at least: (1) The selection exercised on entering the occupation—that requiring physical strength would attract the vigorous, and sedentary occupations the more unhealthy; (2) the withdrawal of invalid lives from these vigorous occupations; and (3) the effect of medical selection upon the occupation risk, as demonstrated by Mr. Barrand in his paper read before the Paris Congress (*Transactions of Third International Congress*, p. 446). But most offices obtain the bulk of their business from the middle and commercial classes, and with one or two exceptions—naval and military risks and those attaching to the liquor trade—were not seriously concerned with occupations that were distinctly hazardous. In regard to sea risk, however, some discrimination should be used. The author of the paper had contented himself with mentioning the "Marine and General" experience submitted in 1887 (*J.I.A.*, xxvi, 413), and he had made no reference whatever to Mr. McLauchlan's valuable analysis of that risk in the paper to which he had referred, and in which very important comparisons were instituted. It was shown, first, how the rate of mortality among those engaged in steamships was about half that among those in sailing vessels: secondly, how the risk varied with the tonnage of the vessel; thirdly, how the risk of accidents was less among officers than men; and, fourthly, how masters of large steamers experienced a mortality less than that of the H^M Table.

On the subject of female mortality, a large number of papers had been contributed from time to time. The author had mentioned the results of investigations made as long ago as 1854, 1861, and 1869, but he could find no references to Mr. Chatham's paper analyzing the results of the 1893 investigation (*Transactions of the Faculty of Actuaries*, vol. i, pp. 116, 154), nor to Mr. Baker's paper read before the Berlin Congress (*Transactions of Fifth International Congress*, vol. i, p. 591). In the former it was shown that a charge of 5*s*. up to age 50 was justified. The general conclusion appeared to be that the excess mortality of female lives was due, first, to medical examination being less effective than with men; secondly, to the risks of childbirth; and thirdly, to the greater proportion of insurances on the lives of females being in connection with loan transactions. He noticed that on page 473 the author said: "The conditions under which assurances upon female lives are effected have materially altered within the last fifteen years. Endowment assurances have become very popular amongst females engaged in business." No figures were, however, given in support of that statement.

Passing on to the question of climate extras, there were well-settled countries like India, where the extra mortality was now pretty well known, and newer countries, where not only were statistics wanting, but there was so much difference between the circumstances surrounding individual risks, that there would probably continue to be considerable divergence in office practice. Taking Africa as an example, some of the statistics related to the pioneers of travel, some to missionaries, and others to commercial men. The evidence went to show that changes in the risk were taking place, and that in the towns real improvements in sanitation and the general surroundings of health had been effected. What offices had suffered from was the excessive risk in the early years of residence. That counteracted, as Mr. Winter pointed out in his paper on "Life Assurance in India" (*Transactions of Fifth International Congress*, vol. ii, p. 61), the effect of medical selection, and moreover it seemed to point to the fact that the offices ought to charge not an average but a diminishing extra.

The second part of the paper, which was no doubt more valuable, seemed to be a consideration of the relationship between various ways of regarding excess mortality, and to show, as various writers had previously pointed out, that the usual method, of leaving the medical officer to recommend so many years' addition to the age, was often unsatisfactory. Sometimes, however, that system would seem to meet the case. When, for example, the extra mortality was in the form of what was called in the paper "The Early Risk Curve", the resulting addition to the premium seemed to be equivalent to the commonly adopted ten years' addition to age. That was shown by a comparison of the extra premiums in the table on page 493, which were given for ages 20 and 30 only, and might be continued for the higher ages. As regarded the Early Risk Curve, he was puzzled to know how its form was arrived at.

Except for the statement that it was not due to mere chance, there appeared to be no explanation of it. The curve seemed, however, to be identical with that exhibited in Mr. Burn's paper read before the Berlin Congress (*Transactions of Fifth International Congress*, vol. i, p. 205).

Mr. C. R. V. COUTTS proposed merely to refer to one or two of the more general points involved in the paper. It had always seemed to him a curious anomaly that, although so much organized effort on the part of the Institute had been applied to investigating normal mortality, there had been apparently little organized co-operation applied to extra mortality. Mr. Hart had pointed out that there was a great mass of literature dealing with extra mortality, but it was nearly always private investigation, the experience of one office or another, of comparatively little value compared with an investigation comprising the experience of, say, a dozen of the chief offices. The author had referred to the Life Offices' Association scheme of registering extra premiums charged, and he suggested that it had not been quite a success. That was not surprising. If one office fixed an extra, and sent it in to the Life Offices' Association, another office might accept it on that authority, and that might be the explanation why people had hesitated to send in the extras charged—they had not felt inclined to lead the others astray. In most cases an office had to quote an extra premium where they had no adequate data on which to assess it. That, he thought, explained the extraordinary divergence shown in the table of "Climate Extras", where one office, for example, assessed Delagoa Bay at no extra at all, and another assessed it at five guineas per cent.

Turning to under-average lives, or as the author called it, inferior eligibility, it seemed to him that this was a far more complex problem to deal with than assessing extra risks of climate or occupation. In the case of under-average lives, as in the case of climate extras, there had been no organized effort to discover the lessons of the vast amount of data which were latent in the office records for the past fifty years. The data were there, waiting for somebody to collect and analyze them. It was a comparatively simple matter to ascertain that a life was an under-average one, but the main problem was how to *measure* the extra risk, and ascertain the character and the distribution of that extra risk. He was afraid that that difficulty was avoided by throwing it on to the doctor. As the author had pointed out, it was a common practice to take a table of expectations of life to the doctor, and then put the case before him and ask how many years he would add to the age. That did not seem to be the most satisfactory or most scientific way of assessing an extra risk. The doctor was put rather in the same position with regard to medical extras as the actuary was when he was asked to fix a climate extra premium without any statistics bearing on the risk. The doctor had to fix some extra, and perhaps suggested an addition of five, seven, or ten years; but in most cases he was guided by tradition and not by any scientific basis.

One of the commonest cases of extra risk was probably the case of tubercular family history. There was a tradition that such cases should be met by an extra of five or seven years, but was there any real ground for so assessing that extra risk? It had arisen largely as a matter of competition among offices and tradition. There were no statistics, as far as he was aware, that showed that the mortality of healthy lives with a tubercular family history was equivalent to that of normal lives five years older. Was it not the business of the actuary to attempt to supply the necessary statistics to the doctor to enable him to assess those risks?

The author, in the last section of the paper, alluded to the methods of valuation to be adopted in the case of extra risks, but seeing that so little was known about the nature of those risks, it was not of very much importance at the present time to attempt to get out tables to value them by. Of course it was easy to criticize the existing state of affairs; the really important problem was to discover some solution, and he thought there was the germ of a solution in the author's suggestion, towards the end of the paper, that offices should adopt a standard form of card for recording the data for use afterwards, the cards to be collected by the Institute, and the results calculated from them.

He thought the value of statistical and research work was becoming more recognized year by year amongst individual offices, and several companies were setting up, in an embryo form, a statistical department, but he could not help feeling that the work of such small statistical departments was largely wasted, because of the paucity of the data on which they worked. He had heard of several offices, for example, taking out their own experience of Indian risks, and those who had done so would he was sure agree that if they had combined their experience on uniform lines, and then got out a complete experience for the combined offices, the value of that experience would be much greater than the value of the individual experiences, each got out on different lines.

The suggestion he himself wished to make was that a permanent Statistical and Research Department should be set up by the Institute itself. The principle had been already adopted in a spasmodic way in the investigations into normal mortality made for the periods ending in 1863 and 1893. The task of the Institute now was to put their statistical work on a permanent basis, and there would be no lack of work to do. The suggested department could collect the data from the offices, analyze them, and form a sort of "general staff" for the Institute, to undertake all the more theoretical problems that had to be dealt with. Another advantage would be that it would afford a sphere for those actuaries for whom the business of life assurance had comparatively little attraction. That suggestion rather assumed that the offices would support the scheme, and it would be necessary for them to support it. They supported the efforts made to take out the experience of normal lives, and it had been always a very honourable tradition of the offices to contribute to the common stock, and he felt sanguine

that if the attempt were made it would meet with a response, at any rate from the majority of offices concerned.

Mr. W. H. HODGSON said that the question of assessment of extra premiums and the method of dealing with them at a valuation was one about which there appeared to be a great divergence of practice. The difference in the rates was doubtless due more particularly to the absence of available statistics, and he hoped that the paper would lead to the collection of such data as might be of advantage to life offices. With reference to the paper itself, he noticed there was a remark that extra premiums for certain occupations might be waived under endowment assurances. He would suggest that if the mortality was at all likely to exceed the normal in the early years of assurance, endowment assurances ought to be rated up, though perhaps not to such a great extent as whole-life policies. Later on in the paper the author furnished a statement as to the existing practice of offices with regard to the assurance of female lives. The author was unfortunate in that he had not been able to obtain returns from more offices, so that the results might have been compared with those obtained by the Actuaries' Club in 1890 (*J.I.A.*, xxix, 75). However, from the information in the paper it would appear that offices were not now so leniently inclined towards the assurance of female lives as they were sixteen years ago; but he did not himself think that this could be actually the case.

The most interesting portion of the paper, he considered, was the Schedule of Climate Extras, and that schedule would be of practical use in daily work in giving some idea of the extra premiums now being charged by offices. The unanimity shown in the practice of twenty-six offices, which valued policies on rated-up lives according to the rated-up ages, was satisfactory. That method was certainly safe, but seemed somewhat stringent, when it was remembered that in many cases the additional risk wore off as the age increased. With regard to the valuation of postponed bonus policies, it would be interesting to know if these were generally dealt with as non-profit policies, or whether the offices were accustomed to reserve the value of the bonuses that might have been allotted. He entirely concurred in the suggestion of the author, that local medical referees should be supplied with information as to the nature of the insurance, because this should have a considerable influence on the form in which their opinion of the life was given.

Mr. J. BURN was particularly interested to notice the remarks in reference to the medical examination, namely, where it was pointed out that the medical examiner was asked to furnish his opinion as to the future vitality of the life. For some reason or other, it had become the custom for the medical examiner to give that information (often very valuable) either in the form of a certain number of years' rating-up, or by reference to the expectation of life. He had tried to find out from several medical men exactly what they meant by that rating-up, but they generally had a very

vague idea as to its effect. The information might be extremely valuable in its original form, but when translated by the medical examiner into what he imagined to be actuarial language a great deal of its value was certainly lost.

The author had stated that in his opinion the manner in which under-average lives were treated was a matter of "expediency." Perhaps some actuaries would have to acknowledge that it was rather lack of energy than expediency which prevented them from attempting to find out what really should be done in order to arrive at some more scientific basis for the treatment of under-average lives. He thought it could be shown by sound reasoning, that there was practically no extra mortality (such as the actuary was in the habit of dealing with) which assumed the form provided for by rating-up a certain number of years. That method assumed a constantly increasing extra strain, but such a strain, he believed, was practically unknown. On the other hand it could be shown that the extra strain to the office was generally of a temporary nature, which reached a maximum at some time or other and eventually disappeared. He thought it was quite reasonable to suppose that if there were a large number of under-average lives of any one particular class under consideration, then, however careful might have been the original medical examination, it would be found that in a certain number of years those persons whose eventual history had justified the doctor's fears would have died out, and a class of lives would be left who would on the average be similar in all respects to those remaining from a class which were considered good lives at the time of selection; in other words, that the curve representing the extra mortality would again reach the normal, which of course was exactly opposite to what was arranged for by rating-up a life.

The author had given a curve in his diagrams which he referred to as an Early Risk Curve. He (Mr. Burn) thought perhaps that he himself was responsible for that somewhat remarkable looking curve, but he might say that the one which he originally produced was not quite so exaggerated as that of the author, and moreover was not an imaginary one at all; it was actually obtained from the records of a particular class given in the American Experience of under-average lives. He was surprised to find that it did produce a curve which was practically of the form already referred to. The author remarked, "It may be said, therefore, that rated-up age valuations for all extra risks afford reserves at which—in spite of criticisms of the assumptions involved—no one could find reason to cavil on the score of safety." He presumed that the author in making that statement did not refer to the granting of surrender-values, at least if those surrender-values were based on the reserve-values of the policies, for if that were so, it was evident there would be very considerable danger. The author also recommended that some of those extra risks could be provided for by deferring the vesting of the bonus. That method, he believed, might be made of considerable value. The

idea, of course, was that the office should hold the full normal reserves for sums assured and bonus, but grant no bonus in the event of death within a certain number of years. The result of that was that a considerable increase in the strain could be provided for, out of money saved by refusing to grant the bonus. After the maximum was arrived at, the bonus could be allowed to vest, not in one lump, which of course would involve the assumption of a sudden fall of the rate of mortality provided for, but in stages, in which way even such a difficult curve as that shown might be made practically to coincide with the normal curve.

MR. JAMES BACON said that he had occasionally to consider as to rates of extra premium, and had experienced the difficulty that other speakers had appeared to meet with as to obtaining any reliable information. The table of extras which the author had supplied would undoubtedly be very valuable in such cases. There was evidence of very great divergence of opinion as to the correct extra to be charged in different cases, and they were thankful for some kind of guidance. There was, however, no evidence that the rates quoted were correct. They were merely the rates that were usually charged, and there appeared to be no evidence at all as to whether they were even safe rates. The author had made it abundantly clear that on the subject of extras practically nothing was known, and it appeared to him to be somewhat regrettable, if data were available, that the Institute should not proceed to collate it. There were difficulties in the way of getting data, no doubt, but in the case of the American Specialized Experience, and in the case of the British Offices Experience the offices were willing to supply data, and if they were asked, not for the rates of extra charged in different cases, as to which there might be some hesitancy, but for the actual experience of lives subject to extra risks over a limited period, there would he thought be enough public spirit amongst the offices to supply it.

MR. E. A. RUSHER, whilst expressing his concurrence in what had been said with reference to the collection of statistics on the subject of extra premiums both for occupation and residence and for under-average lives, desired also to thank the author for the very valuable table he had given on "Climate Extras", showing the methods of different companies in dealing with extra premiums for residence. The table was one that should give food for thought, when it was found that for the same risk the premium per-cent charged by some offices was five guineas, by others ten guineas, while some made no charge at all. The author had given a record of some of the experiences dealing with the lives of persons connected with the sale of intoxicating liquors, and he was rather surprised he had omitted one by Mr. McDonald dealing with a very large number of lives engaged in that occupation. From a table he gives on page 527 of vol. i of the Transactions of the Berlin Congress, it would be found that premiums on whole-life policies showed an increase over the normal of from £1. 7*s.* 8*d.* per-cent at the younger ages to about £3. 15*s.* per-cent at the higher ages.

Mr. W. PENMAN said that he had hoped that one or other of the speakers would have referred to the subject of double endowment assurances. It was stated in the paper that in such cases five offices waived the climate extra altogether, except when the ordinary rate was a very heavy one, and tables of premiums were given later on in the paper from which it appeared that the practice of the offices in that respect was not borne out by theory, although under Table VI, which appeared to be a more probable hypothesis than Table III, the extra premiums were not very large. He had been looking through the prospectuses of companies engaged in that class of business and he found that five of them issued double endowment policies with profits. It would be rather interesting to ascertain whether those were the same companies that waived extra premiums, because it seemed to him that if a company was going to dispense with climate extras under a double endowment policy it was in a better position if it issued a with-profit policy, especially if the bonus were only payable at death. In one of the five companies the bonus was only payable at death, while in the others it was payable at death or maturity. The loss that might arise from light mortality at the end of the term was very considerably reduced by a bonus payable at death, and a with-profit policy to a certain extent met the difficulty that arose when a life assured under a double-endowment policy ceased to incur extra mortality during the few years preceding maturity.

Mr. H. W. MANLY said that the paper would be a means of ascertaining the views of the profession on the many points raised, and perhaps eventually would lead up to that time, which some of the speakers had looked forward to so anxiously, when an attempt would be made to collect some of the experiences under the various forms of extra risk. The tables in the paper were admittedly hypothetical, and the author seemed to regret that he was unable to get the same reserve value when he used the hypothetical tables as when he used the normal table. That might be got over in a very simple way. He did not know whether the author was acquainted with that instructive and valuable paper written by the late Mr. Meikle which was published some thirty-six years ago on "Policy Life Lines" (*J.I.A.*, xxiii, 385), in which he showed that it was possible to construct any number of mortality tables with an increased rate of mortality or a decreased rate of mortality, which would give exactly the same policy-values as the table upon which the author had based his original calculations. He thought himself it was very likely that one of the tables which Mr. Meikle produced, or tables produced in the same way, would be found very suitable for some of the risks. The only difficulty seemed to be that the reserves for the reversionary bonuses would not be the same, but the reserves for the sums assured could be made to agree throughout the whole of the table. If there was any case where it was desirable to have a select experience at different ages it was in the consideration of extra premiums. Young men of from nineteen to twenty-five years of age went out to tropical climates, full of life and energy, and

were not so careful to follow the strict rules of hygiene which more experienced residents had found to be absolutely necessary. Those young men were subject in the early years to a very heavy risk, and should be charged accordingly. But when a man went out at the age of, say, thirty-five, it would be found very often that his risk was not so great as in the case of the young man.

He did not agree entirely with Mr. Burn that in the case of a constitutional defect, such as a consumptive family history, the time ever arrived when all the abnormal risk died out and the experience became absolutely normal. Although the experience would approach very closely to the normal curve towards the end of life, such lives were never so good as first-class lives who were without taint. He took exception to the author's early risk curve, because it assumed that after a certain age the mortality grew lighter. He thought that the curve only represented the case where, during the period of heavy mortality the lives had been subject to accidents, and that after a certain age the risk of accidents grew less, and at age 55 ceased altogether. He preferred to extend the curve from the highest point, and with a very slight but steady increase in the mortality run the line nearly up to the normal line at age 60 or 65. At age 20 a proposer with a history of consumption in his family was examined and found to be perfectly sound; but, having a constitution susceptible of contracting a tubercular disease, the risk for the next 10 or 15 years was very heavy, and he did not object to the author's mortality curve, as he (Mr. Manly) had altered it, for a life starting at that age, 20. But when a proposer was examined at age 30, and was found perfectly sound, his rate of mortality did not start near the top of the curve, but started again on the normal curve, and as he had already passed the years when the risk from tubercular disease was heavy, the curve representing the mortality he was subject to would be very much depressed as compared with the curve for age 20; and the curve for each increasing age at entry would more and more approach the normal line. The curve which he gave in his paper on "An Attempt to ascertain the Extra Risk arising from a Consumptive Family History" (*J.I.A.*, xxx, 97) curiously represented the average of those curves, when the numbers remaining at each age were taken into consideration. It was only by making select tables of the experience of each class of risk that the true value of the risk could be obtained.

Mr. J. Burn said Mr. Manly had stated that he did not think it possible that the curve could return to the normal curve ages, but after having obtained all the information he (Mr. Burn) could from various medical men, he gathered that in their opinion the amount of phthisis which was included in the total normal mortality at older ages was really very much larger than was generally supposed. Moreover he believed that the risk of phthisis which they could foresee, and for which they really intended to rate a life up, was only a temporary matter, and did not come into the phthisis that occurred at extreme old age.

Mr. MANLY said that if a single class were taken and considered by itself, he did not think that the experience of that class would ever actually run into the normal curve, but that was a matter for investigation hereafter, when a large experience had been collected together.

Mr. S. G. WARNER, in concluding the discussion, said that the subject of extra premiums must always be one of considerable difficulty, owing to the paucity of available facts. That operated in two important ways. Not only was there the paucity of general facts in many cases of climatic or foreign extra, but there was the still more important consideration that actuaries could seldom hope, whatever the general experience might be, to get enough cases to form anything like an average for their own practice. Taking as an illustrative parallel the case of British life business, there had been built up, as the result of research operating upon a large mass of facts, an experience of mortality of normal lives which could be absolutely relied upon; and there was also the confident assurance that in practice an actuary would obtain a sufficient number of such lives to form a corresponding average for his own company. That was a double source of security, and there existed a corresponding double difficulty with regard to rated-up cases. It might be possible to have a fairly stable experience established by a consensus of statistics contributed by those who had had experience of lives at risk in a given quarter of the world, but what guarantee was there that in an individual office there would be more than perhaps 20 or 30 cases in the course of ten years? There would thus be nothing in the individual experience properly corresponding to the general experience. That was a difficulty peculiar to the subject, and one which should be taken into consideration. It went far to account for the wider varieties of practice revealed in the valuable tables given in the paper with regard to climatic and foreign extras. It indicated that those extras would probably be always more or less arbitrary, heavily loaded on the side of safety by those who could not expect to get enough of them to constitute an average.

Coming to the second section of the paper, one governed by very different considerations, the question of "inferior eligibility", as it had been called, he had to confess to being more or less out of accord with a good deal that had been said that evening. Old-fashioned as it might appear, he was a believer in the method of rating-up by adding, more or less arbitrarily, a number of years to the age of the life. He might say something, by the way, with regard to the suggestions that had been thrown out with reference to the advisability of collecting experience. About that he supposed there would be only one opinion. As one of the results of the recent Congress, an attempt has been made to organize an international investigation of the kind, and the Institute, among other bodies, was approached upon the subject. But in that particular case, the investigations required were so exceedingly elaborate, difficult and laborious, that after

considering it carefully, the Council with great regret were compelled to say that they felt it to be impracticable so far as British Offices were concerned, and he was afraid that would be the general opinion, and that that particular scheme would fall to the ground. That was not to say, of course, that a scheme conceived on somewhat different lines might not be successful, and should such a thing prove to be at all practicable, all would agree that there could be no doubt about its beneficial effect.

Reverting, however, to the principle of dealing with under-average lives by what was generally known as "rating-up", that was probably now, as had been said, a somewhat arbitrary process. The doctor was asked to say how long he thought a man was likely to live. The life, for some reason, was considered to be below the average; another way of expressing which was that it would not probably endure for the normal average period applicable to the age at entry. The doctor, with the assistance of a table of "Expectations of Life", and on the principle of average, added what he considered the requisite number of years to the age. The life was accepted on that basis, and treated on that basis throughout. Now the point was, whether that was scientific. It seemed to him to be really more so than some more plausible schemes, for the reason that it was built upon the lines of that law of average which lay at the root of the entire business. It was quite possible that in many cases the extra risk incurred rested largely upon the earlier years of life and tended to lessen as life advanced, and that, apart from the principle of average, such a life in its later years might not require any extra premium at all. But, surely, by an extension of the same reasoning, one might quite as plausibly say that the ordinary assurer, who outlived his expectation of life, might expect to be relieved from premiums, and that an endeavour should be made in some artificial manner so to arrange the ordinary premiums in the early stages of life as to give relief under such circumstances.

Reference had been made in the paper to the dissatisfaction felt by those who had passed the expectation of life and still found themselves rated up; but he supposed it was not an uncommon experience that similar dissatisfaction was sometimes felt by people who similarly survived, and had not been rated-up at all. Might it not be arguable that the importation of the "expectation of life" into the problem was itself unscientific; that the law of average applied equally to the normal life and the rated-up life; and should so apply? Any alternative system could only work equitably by increasing the penalty in earlier years of life, and so diminishing the protection of life assurance in the very years when it was most required. For those reasons it appeared to him that the existing system of rating-up had the great merit of simplicity; the scientific soundness of being based upon the law of average which underlay all insurance business; and the practical and beneficent effect of distributing the burden in comparatively small proportions equally over the whole of life, instead of fixing it

with greatest weight upon the years when, should death happen, it would be most keenly felt by those for whom the provision had been made.

THE PRESIDENT, in asking the members to accord a hearty vote of thanks to the author for the very full and interesting paper which he had prepared under considerable pressure, and submitted to the Institute, said that he thought it put on record many valuable facts, not the least important being the table showing the actual practice of a number of companies in regard to certain parts of the world where assured lives were living. He would like to call attention to a paragraph in the paper which showed how the practice on the Continent differed from what it was in this country. He thought it was almost a universal rule here, in the case of extra premiums for foreign residence, to assess the charge on the sum assured. He remembered some years ago, in the case of a war, some actuaries advocated that the companies should assess the very heavy war extra on the sum assured, less the reserve value, but he did not think such a plan was carried out in practice now in regard to extra premiums for foreign residence. It would be observed from Mr. Lutt's paper that M. Fleury, in his paper read at the recent International Actuarial Congress, pointed out that it was usual on the Continent for extra premiums to be charged not on the full sum assured but on the amount at risk. He therefore thought the author, when he had sufficient leisure, might take up that point in an addendum to his paper, and show how it would affect the case when an extra premium was assessed on the actual sum at risk.

The vote of thanks was carried unanimously.

MR. H. E. W. LUTT, in reply, said that, with reference to Mr. Hart's remarks, he must confess that in the space of time during which the paper was prepared, one or two previous papers had escaped his notice. With regard to Mr. McLauchlan's paper, he thought if Mr. Hart would refer to page 482 where he spoke about the history and practice of life assurance offices, he would see that the population statistics of Mr. McLauchlan's paper hardly came within that category. It was certainly a most valuable contribution, but Mr. Hart admitted, in his later remarks, that population returns were practically of no value for assurance purposes. Mr. Hart had also referred to the want of acknowledgment of Mr. Burn's curve. He (the author) had stated in the paper that his attention had been drawn to Mr. Burn's paper, read before the Insurance Institute of Toronto in April, 1904, and that Mr. Burn had very willingly given him permission to follow up the matter, which he had done, though perhaps in a different fashion. His attention was, however, drawn to Mr. Burn's curve at a time subsequent to the conception of the original idea which was merely to obtain some kind of "early risk" curve. This was based on no actual statistics, but was merely experimental, and perhaps was an interpretation of the ideas of one or two medical gentlemen whom he consulted, and who were of opinion that such a progression of mortality was possible

With regard to the valuation of policies in which the bonus was postponed, he confessed he had no practical experience, and he would be much interested to know whether Mr. Burn's suggestions were followed in making provision for the bonuses, although none were allotted, or whether the policies were simply valued as carrying so much annual extra premium to cover current extra risks which were not taken further into account. Mr. Burn had referred to the "rated-up" reserves for all extra risks mentioned in the paper as the method to which no exception could be taken on the score of safety; that was intended to refer to reserves only, and not to surrender-values. He thanked Mr. Rusher for drawing his attention to Mr. McDonald's paper; it had escaped his notice. With the permission of the Editor of the *Journal* he should probably be able to include a reference to it.

Mr. Manly had mentioned Mr. Meikle's paper on "Policy Life Lines." He had not forgotten that paper, and he thought Mr. Manly would see that he had referred to the mortality which the assumption of ordinary reserves involved. Such mortality was rather different from that which had been assumed in arriving at the extra premium to be charged. The mortality shown by a valuation at ordinary reserves was really dependent upon the amount of extra premium paid, and for a constant extra premium showed an increasing extra mortality. In the paper, there was a table giving the difference in mortality on the assumption of 10 years' rating-up under the $O^{M(5)}$ Table at age 30 at entry. With regard to Mr. Manly's remarks on select lives, Table 5 was only supposed to deal with ages at entry 20 to 30. The remaining figures were included in order to give an idea of the assumed mortality in connection with contingent debts, seeing that the assumption of a decreasing contingent debt and the ordinary policy reserve involved a mortality which started considerably in excess.

The PRESIDENT said there was present that evening Mr. H. N. Sheppard, who had come over from the United States, and would have the pleasure of showing the members the standardized forms of policies, adopted in accordance with the recent Law in the State of New York. No doubt the members had all read in the last few days that the Royal Commission in Canada had recommended legislation on the lines of the New York legislation.

Mr. H. N. SHEPPARD said that Section 101 of Chapter 324 of the Laws of New York, which became effective on 27 April 1906, read as follows: "Standard forms of policies.—On and after the first day of January, nineteen hundred and seven, all policies of insurance, other than industrial policies, issued or delivered within this State by any domestic life insurance corporation, shall be in the forms hereby prescribed and not otherwise save as hereinafter provided. There shall be four standard forms of policies, to wit: (1) An ordinary life policy, (2) a limited payment life policy, (3) an endowment policy, and (4) a term policy." Then followed the first

drafts of the four standard policies, and then it went on: "The standard forms above provided, or any one or more of them, may be altered or amended by the direction of the Superintendent of Insurance at any time prior to the first day of October, nineteen hundred and six, by direction filed in his office stating the reasons therefor, and if any such alteration or amendment is made the Superintendent shall promulgate the amended form or forms on said date." In accordance with that the New York actuaries got together and a Committee was formed, and they drew up a brief in which they suggested quite a large number of amendments, as the first drafts were very rough and the English in a good many cases very bad. Most of these suggestions were accepted, and some changes were made by the State Insurance Department itself. On October 1st the standard forms were automatically fixed, and now not a single word might be changed, except by a Special Act of the Legislature. The Section went on to say: "Whenever any domestic life insurance corporation shall desire to issue or deliver within this State any kind of policy other than ordinary life, limited payment life, endowment and term policies, it shall submit a proposed form of policy to the Superintendent of Insurance, who shall thereupon fix a day for a hearing upon said application and cause notice thereof to be given to every domestic life insurance corporation; and the Superintendent may, after such hearing, approve the said form with or without modifications thereof as may seem to him expedient and establish the same as a standard form of policy which any domestic life insurance corporation shall be entitled to use in addition to the forms hereby prescribed." It would be noticed that the words "domestic life insurance corporation" were used, which meant effectively that perhaps for the first time in history a sovereign State had passed laws which discriminated against its own citizens in favour of citizens of other States, because by law an insurance company that was incorporated outside the State of New York might deliver in New York State policies that New York insurance companies themselves could not deliver. No mention was made of annuities, but the actuaries thought that it would be a good thing to have those standardized at the same time, and so there were various forms for annuities.

Mr. Sheppard distributed various forms of policy, and mentioned that up to the time of his sailing for Europe, in addition to the four standard forms there had been standardized a Single Premium Life Policy, a Single Premium Endowment, Ordinary Joint Life, and an Ordinary Life Sub-standard. The only difference between the sub-standard and the standard was the addition of a Clause regulating what was to be done in case the age had been incorrectly stated, because the policy was only meant for rated-up ages. He believed the idea later on was to have a sub-standard policy standardized which would allow a decreasing limit. In addition to those he had previously mentioned, there were also policies for a Limited Payment Life Sub-standard and an Endowment Sub-standard, and policies for Early Renewal Term, Joint Life

Annuity, Deferred Annuity, and various forms of Survivor's Annuity or Pure Endowment. He should like to take the standard policy and go through one or two points in it which illustrated the present law. The policies were drawn up on the general lines of policies issued in the States in which loan-values and paid-up assurances were guaranteed.

All the American Rate Books, which were only issued to agents, had all the guarantees given in them, and as a rule an increasing percentage of the reserve was taken, and using that with net figures the paid-up policy was calculated as an extended assurance, which meant simply that the reserve allowed was used to carry the face value of the policy for a term of years, and in the case of an endowment, if the reserve allowed was sufficient to carry to the end of the endowment term, any balance was converted into the corresponding amount payable if the assured was alive at the end of the endowment term. The first important thing was participation—"The portion of surplus accruing from this policy shall be ascertained and distributed annually and not otherwise"; that was in accordance with the law which prohibited New York companies issuing Deferred Dividend Policies, as a result, of course, of the investigation. Where the company was a non-participating company that of course was struck out and replaced by words stating that the policy did not share in the surplus of the company. Under the new law no domestic company might undertake both participating and non-participating business, although any company incorporated outside the State of New York might do so, and that was another instance of discrimination against its own citizens. The options of surrender or lapse were clearly expressed, and he believed met with the approval of most actuaries. Some of them objected to the fact that the continued assurance was automatic. There was a difference of opinion in the United States as to whether an automatic paid-up insurance or an automatic extended insurance ought to be given, some believing that there was a selection against the company where an automatic extended assurance was given, and some companies had protected themselves by deducting from the sum assured two or three premiums in case death took place within the first few years of extended assurance.

The President asked whether the surrender-values were compulsorily calculated on any basis or was it open to the company to select its own basis?

Mr. SHEPPARD said the only limitation was that at least 80 per-cent of the reserve must be given, but where the reserve was less than a certain sum that did not hold and a reduction was made instead. Usually companies gave from 80 per-cent to 100 per-cent, graduating up to about the tenth, fifteenth or twentieth year, according to their own ideas. The options of settlement were very elaborate, especially in the case of an endowment policy. It was not known whose idea that was, and as a matter of fact it would lead to a lot of calculation. Fortunately there was no compulsion to give any extra tables. In every case the tables

giving the amount of each instalment corresponding to the annual instalment, and the amount of the instalments for continuous annuities-certain, in which the purchase money was used for the payment of an annuity for so many years certain and thereafter so long as the beneficiary was alive—were printed in full by all companies. He should like to mention that he had also with him Sections A and B of the Convention Act of the State of New York; so that anyone interested in that Act might see what an enormous amount of information had to be given. Last winter, for the first time, in consequence of the new law, some of the companies had had the statement of income and disbursements, assets and liabilities, and miscellaneous questions, printed in order to save time. The Schedules were very elaborate and very difficult to copy by hand, especially those in which statements were made of the dividends paid during the year. He supposed that many had read about the Gain and Loss Exhibit; that was more elaborate than ever, and was now incorporated in Part A of the New York Statement. Other States only demanded as a rule Part A, but Louisiana also asked for Part B, which required very elaborate statements of all the securities of the company, and that had been interpreted to mean bonds and stocks, showing the date of purchase and sale, and so on.

The President expressed the thanks of the Institute to Mr. Sheppard for his remarks, and the proceedings terminated.

Notes on Summation Formulas of Graduation, with certain new Formulas for consideration. By George King, F.I.A., F.F.A., *Consulting Actuary.*

The main object of my paper on the Error introduced into Mortality Tables by Summation Formulas of Graduation, published at page 54 of the present volume of the *Journal,* has already been to a large extent fulfilled; and the admirable papers which it has called forth from Messrs. Lidstone and Spencer are a liberal reward for my labours. That object was, to show that the theoretical error in question is so small as to be quite insignificant; and thereby to bring summation formulas up again for discussion, with a view to ascertain which of them are the best as means of graduation pure and simple, and how they can be used most effectively.

In the concluding sentences of the paper I spoke of a graduation by summation, to be followed, in order to gain smoothness, by one by the graphic method; and in the reply to

the discussion I casually threw out the idea of correcting the theoretical error by means of the fourth central difference; but these suggestions were only concessions to the misgivings of weaker brethren, and were not put forward seriously as being in my own opinion either necessary or desirable. I am convinced that we have now summation formulas which give as smooth graduations as need be desired, and the results of which conform with close accuracy to the original facts; and, emphasizing what has already been said, my own view is that the theoretical error they introduce is so minute as to be unworthy of consideration for a moment.

The scope of the paper was intentionally strictly limited. It was merely to ascertain the magnitude of the theoretical error which these formulas do introduce into a curve already perfectly smooth, and to examine the matter with regard to the several functions which might be selected for graduation purposes. A numerical example was essential to the enquiry, and, almost of necessity, a Makeham curve was selected, because, where is another to be found which is smooth enough? But only as a matter of convenience and accuracy were the differential coefficients calculated by Makeham's constants; and perhaps it would have been well if this point had been stated more emphatically, because, as matters stand, there may be the appearance that my investigations apply only to Makeham curves. At any rate, that seems to be the view taken by Mr. Spencer in the opening sentence of his paper, and again on page 379, where he adopts my example, instead of the principles which it was intended to illustrate; and where he implies, but I think mistakenly, that these principles may lead sometimes to erroneous results.

It would do equally well if the differential coefficients could be found otherwise than by Makeham's constants. For instance, if we difference the function u_x centrally, and stop at sixth differences, we shall have

$$\frac{d^4u_0}{dx^4}=d_0-\frac{1}{6}f_0;\quad \frac{d^6u_0}{dx^6}=f_0;$$

where d_0 and f_0 are respectively the fourth and the sixth central difference. These formulas might have been employed, except for the arithmetical difficulty. But that difficulty is almost insuperable. Even if we take the $O^{M(5)}$ values of q_x derived from Mr. Hardy's seven figure values of colog p_x given on page 153 of the "Account of Principles and Methods, &c.", we shall find that, through

cutting the function down even not farther than to seven figures, the fourth and sixth differences are so small, and run so irregularly, as to be useless; and that we should have to carry our calculations out to nine or ten places to be of any good. Therefore recourse was had to Makeham's constants.

This once more shows how elusive are the theoretical errors of summation formulas. They are so minute as to necessitate nine or ten decimal places for their discovery and measurement.

In the paper, in order to keep the examples homogeneous, a table following Makeham's law throughout was employed; but, except for that reason, any other perfectly smooth table, could one be found, would have served equally well. This is explained on page 60 of the paper, where I say that, with a table that does not follow Makeham's law, the law can be applied to the short sections we require, of, say, twenty-five terms at the most, without altering the sequence of the values. This point should be remembered in reading the paragraph at the foot of page 379 of Mr. Spencer's paper. The constants of the graduated $O^{M(5)}$ table do not apply, even approximately, at the old ages of the ungraduated O^M. If for ages 80 to 98 we take the values of the O^M p_x given in the "Unadjusted Data", and from them form values of $\log l_x$ from 80 to 99, true to five decimal places; and if we calculate the constants by the aggregate method of the *Text-Book*, we shall have

$$\left.\begin{aligned}\log c &= \cdot 0539605\\ \log g &= \bar{1}\cdot 9999899\\ \log s &= \bar{1}\cdot 954178\end{aligned}\right\}\quad \begin{matrix}\text{whereas the}\\ O^{M(5)}\text{ values}\\ \text{are}\end{matrix}\quad \left\{\begin{aligned}\log c &= \cdot 0390000\\ \log g &= \bar{1}\cdot 9994980\\ \log s &= \bar{1}\cdot 9974425\end{aligned}\right.$$

The differential coefficients according to these new O^M constants are

Age	$\frac{d^4p_x}{dx^4}$	Age	$\frac{d^4p_x}{dx^4}$
80	$-\cdot 00000741$	88	$+\cdot 00000092$
81	$-\cdot 00000757$	89	$+\cdot 00000484$
82	$-\cdot 00000755$	90	$+\cdot 00000987$
83	$-\cdot 00000730$	91	$+\cdot 00001613$
84	$-\cdot 00000673$	92	$+\cdot 00002372$
85	$-\cdot 00000573$	93	$+\cdot 00003268$
86	$-\cdot 00000422$	94	$+\cdot 00004292$
87	$-\cdot 00000205$		...

On a graduation of the q_x of the O^M table, for the ages 80 to 94, by Mr. Spencer's 21-term formula, these differential

coefficients give an estimated error of 4·1 deaths in defect. Mr. Spencer shows an actual deviation of 42 deaths in defect, so that, by applying a correction on the lines suggested in my paper, we reduce Mr. Spencer's deviation, and do not increase it. Here we see a theoretical error of only 4·1 deaths, against an actual deviation of 42 deaths, and we have an illustration of the insignificance of these errors as compared with the changes we necessarily make in a table by graduating it.

After my paper was read I computed to the very end the graduation of the $O^M\ q_x$ by Mr. Spencer's formula, and found that the results were about as good as those at the younger ages, and I meant to go into the matter fully here, but Mr. Spencer has forestalled me. It is therefore sufficient to say that any doubt which may have been harboured as to the validity of Mr. Spencer's 21-term formula throughout the whole length of a mortality table completely vanishes, and that in my opinion it is the most powerful and the best formula which up to the time of its publication had been devised. To improve upon it will be very difficult, but nevertheless no one will object to other formulas being presented for consideration.

Before we proceed to the new summation formulas which it is the more particular object of this paper to submit to the profession, there is only one other preliminary point to be cleared out of the way. Equation 7 on page 58 of my former paper was deduced for finding the multiples of the several summations to be included in a Summation Formula of Graduation, constructed on the plan of the late Mr. Higham. Adopting his method, the initial term of the series to be summed was taken for origin, and it was the differences of that initial term which were involved. From this it seemed to follow that all the sets of summations must be applied to one section only of a series, or otherwise the differences could not be eliminated. If there were two sections, one consisting of $n+1$ terms, and the other of $n+m+1$, the initial terms would differ, and consequently their differences would differ, and could not be equated against each other, and therefore apparently could not be eliminated. That, however, is only in appearance, and the formula which was deduced for use in the case of the varied summations of one section only, is equally applicable if two sections differing in length be brought in. This becomes evident from the investigations of Mr. R. Todhunter, *J.I.A.*, xxxii, 378. He found an alternative expression to that of Mr. Higham, for the value of the central term of a series summed first in p's,

then in q's, then in r's, &c.; and in doing so he made use of the central term, u_0, of the original series, and the differences of that central term. Therefore, no matter what may be the lengths of the several sections, the central term and its differences remain the same, and the differences can be eliminated without difficulty. Moreover, when they are eliminated, the same equation is produced which I arrived at in the other way.

Mr. Todhunter proceeded by the method of differences, and the expression so evolved is complicated when carried to the fifth difference, but it is capable of being considerably reduced and simplified. We can, however, do still better by using differential coefficients instead of differences, and we thereby arrive at a very short expression which is perhaps worthy of being placed on record. In demonstrating it, I will adhere to the notation used in my former paper, so that the two papers may be read together without inconvenience. The only change in this respect will be the adoption of the alternative symbols u^{II}, u^{IV}, &c., for the differential coefficients, instead of $\frac{d^2u}{dx^2}$, $\frac{d^4u}{dx^4}$, &c. This will conduce to compactness in the printing.

Let there be a series consisting of an odd number of terms, of which u_0 is the central term; and let it be required to find S, the value of the great central term produced by summing the original series first in p's, then in q's, then in r's, and so on until the whole series is collected into the one term S. Also, let there be t summations in all; and let $p^2+q^2+r^2+\&c.=s_2$, and $p^4+q^4+r^4+\&c.=s_4$.

It is shown in my former paper, page 59, that when *

$$\gamma_h = u_{-h} + u_{+h},$$

$$\gamma_h = 2u_0 + h^2 u^{\mathrm{II}} + \frac{h^4}{12} \cdot u^{\mathrm{IV}} + , \&c.,$$

and here we shall stop at the fourth differential coefficient.

Adopting the notation of Mr. G. F. Hardy, in which $[p]$ represents a summation in p's, we have

$$[p]\,u_0 = pu_0 + \left(\Sigma_1^{\frac{p-1}{2}} h^2\right) u_0^{\mathrm{II}} + \frac{1}{12}\left(\Sigma_1^{\frac{p-1}{2}} h^4\right) u_0^{\mathrm{IV}}.$$

* Mr. Spencer writes $u_{\pm h}$ for γ_h, but it seems better to retain the original notation of Mr. Woolhouse.

Now, $$\Sigma_1^n h^2 = \frac{n(n+1)(2n+1)}{6},$$

and $$\Sigma_1^n h^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30};$$

and, when we write $\frac{p-1}{2}$ for n, these become

$$\Sigma_1^{\frac{p-1}{2}} h^2 = \frac{p(p^2-1)}{24}$$

and

$$\Sigma_1^{\frac{p-1}{2}} h^4 = \frac{p(p^2-1)(3p^2-7)}{480} = p \cdot \frac{3(p^4-1)-10(p^2-1)}{480}.$$

Therefore,

$$[p]u_0 = p\left\{u_0 + \frac{p^2-1}{24}u_0^{\mathrm{II}} + \frac{3(p^4-1)-10(p^2-1)}{5760}u_0^{\mathrm{IV}}\right\}.$$

Here it will conduce to brevity if for the moment we write $(p^2-1)=a_2$, $(p^4-1)=a_4$; $(q^2-1)=b_2$, $(q^4-1)=b_4$; and so on for r, &c.

This gives us

$$[p]u_0 = p\left\{u_0 + \frac{a_2}{24}u^{\mathrm{II}} + \frac{3a_4-10a_2}{5760}u_0^{\mathrm{IV}}\right\} = v_0, \text{ say.}$$

Similarly,

$$[q]v_0 = q\left\{v_0 + \frac{b_2}{24}v_0^{\mathrm{II}} + \frac{3b_4-10b_2}{5760}v_0^{\mathrm{IV}}\right\}$$

$$= pq\left\{u_0 + \frac{a_2}{24}u_0^{\mathrm{II}} + \frac{3a_4-10a_2}{5760}u_0^{\mathrm{IV}} + \frac{b_2}{24}u_0^{\mathrm{II}} + \frac{a_2}{24}\cdot\frac{b_2}{24}\cdot u_0^{\mathrm{IV}} + \frac{3b_4-10b_2}{5760}u_0^{\mathrm{IV}}\right\}.$$

That is,

$$[p][q]u_0 = pq\left\{u_0 + \frac{a_2+b_2}{24}u_0^{\mathrm{II}} + \frac{a_2b_2}{576}u_0^{\mathrm{IV}} + \frac{3(a_4+b_4)-10(a_2+b_2)}{5760}u_0^{\mathrm{IV}}\right\}.$$

By bringing in a third summation in r's, we shall have

$$[p][q][r]u_0 = pqr\left\{u_0 + \frac{a_2+b_2+c_2}{24}\cdot u_0^{\mathrm{II}} + \frac{a_2b_2+a_2c_2+b_2c_2}{576}\cdot u_0^{\mathrm{IV}} + \frac{3(a_4+b_4+c_4)-10(a_2+b_2+c_2)}{5760}u_0^{\mathrm{IV}}\right\}.$$

The form of the expression is sufficiently evident, but, if necessary, it could easily be proved by induction that it is general, and holds for all the t summations.

The coefficient of the first term which involves u_0^{IV} can be given more convenient shape, which will enable us to effect reductions, because

$$(a_2+b_2+c_2+\&c.)^2=(a_2^2+b_2^2+c_2^2+\&c.)$$
$$+2(a_2b_2+a_2c_2+b_2c_2+\&c.)$$

Therefore $$(a_2b_2+a_2c_2+b_2c_2+\&c.)=\frac{1}{2}\{(a_2+b_2+c_2+\&c.)^2$$
$$-(a_2^2+b_2^2+c_2^2+\&c.)\}$$

But $a_2^2=(p^2-1)^2=p^4-2p^2+1=(p^4-1)-2(p^2-1)=a_4-2a_2;$

and similarly for b_2^2, c_2^2, &c. Whence

$$(a_2b_2+a_2c_2+b_2c_2+\&c.)=\frac{1}{2}\{(a_2+b_2+c_2+\&c.)^2$$
$$-(a_4+b_4+c_4+\&c.)+2(a_2+b_2+c_2+\&c.)\}$$

Returning now to the original notation, we have

$$(a_2+b_2+c_2+\&c.)=(s_2-t)$$

and $$(a_4+b_4+c_4+\&c.)=(s_4-t)$$

And when we write pqr &c. $=$ P, we have

$$\{[p][q][r]\&c.\}u_0=\mathrm{P}\Big\{u_0+\frac{s_2-t}{24}u_0^{\text{II}}+\frac{\frac{1}{2}(s_2-t)^2-\frac{1}{2}(s_4-t)+(s_2-t)}{576}u_0^{\text{IV}}$$
$$+\frac{3(s_4-t)-10(s_2-t)}{5760}u_0^{\text{IV}}\Big\}$$

That is, after reduction,

$$\frac{\mathrm{S}}{\mathrm{P}}=u_0+\frac{s_2-t}{24}u_0^{\text{II}}+\frac{5(s_2-t)^2-2(s_4-t)}{5760}u_0^{\text{IV}} \quad . \quad . \quad \text{(i)}$$

which is the expression sought.

This expression can be used to form an equation by means of which to construct summation formulas. If we take a second set of summations, τ in number, we shall have, similarly,

$$\frac{\Sigma}{\Pi}=u_0+\frac{\sigma_2-\tau}{24}u_0^{\text{II}}+\frac{5(\sigma_2-\tau)^2-2(\sigma_4-\tau)}{5760}u_0^{\text{IV}}.$$

By means of these two equations we can now eliminate u^{II}; and, if we write (u) for the graduated value, we have

$$(u)=\frac{\frac{\mathrm{S}}{\mathrm{P}}(\sigma_2-\tau)-\frac{\Sigma}{\Pi}(s_2-t)}{(\sigma_2-\tau)-(s_2-t)}$$
$$+\frac{(s_2-t)\{5(\sigma_2-\tau)^2-2(\sigma_4-\tau)\}-(\sigma_2-\tau)\{5(s_2-t)^2-2(s_4-t)\}}{5760\{(\sigma_2-\tau)-(s_2-t)\}}u_0^{\mathrm{IV}}\ldots\text{(ii)}$$

When we neglect the term in equation (ii) which involves u^{IV}, we have equation 7, already referred to, of my former paper, but in the shape which was suggested by Mr. Lidstone in the discussion. The term which involves u^{IV} gives the error of the fourth differential coefficient when we stop at the second. It has the appearance of being a little intricate, but when applied numerically its value is easily calculated.

It is evident from equation (i) that from any three sets of summations we can construct a formula from which the fourth difference error, as well as the second, has been eliminated, and which is therefore practically free from theoretical error; and that an infinite number of such formulas could be formed. But only very few of them could be of any use in practice, because they would be too cumbrous to apply, and would not possess much graduating power. Moreover, to seek suitable formulas by means of the equation would be an almost hopeless quest, and only by mere chance would one be discovered. We can proceed in another way, and reach more satisfactory results.

Woolhouse's formula was arrived at by first fixing on certain interpolations which seemed to be promising, and from them deducing the expression

$$125(u)=25u_0+24\gamma_1+21\gamma_2+7\gamma_3+3\gamma_4+0\gamma_5-2\gamma_6-3\gamma_7.$$

Then, years afterwards, by an inverse process, the summation form was discovered, as given by Messrs. J. A. Higham and G. F. Hardy.

$$125(u)=[5]^3\{10[1]-3[3]\}u_0.$$

We can follow a similar course.

Here I must digress to explain that almost all that follows in this paper was blocked out, and all the formulas were devised, before I had read Dr. Karup's paper on Graduation, so that, although he and I have some ideas in common, and although his formula appears among mine, my work is independent of his, and I have derived but little inspiration from him. His paper is given in the Transactions of the Second Actuarial Congress,

pages 31 to 109; and, although I was Editor of the Transactions, this particular contribution did not come within my personal cognizance. At the time, my hands were otherwise very full, and the paper was in German, a language I do not understand. It was translated by the late Mr. A. G. Wiggins, who also corrected the proofs both German and English, and who saw the whole through the press. Hence, I had no knowledge of what Dr. Karup had done, and his work passed completely from my memory. This apology is due to Dr. Karup.

Mr. Woolhouse interpolated centrally in a quinquennial curve, and stopped at second differences. We might, for a fifth difference formula, do the same, but include fourth differences; but it is not likely that the result would give a very satisfactory graduation. Probably it would have all the roughness of Woolhouse's. I have not yet tried it, because better systems of interpolation are available. Instead of interpolating by means of single values of a function as did Woolhouse, we can employ groups of values. There are two systems, that of Dr. Sprague, *J.I.A.*, xxii, 270, and that used by the late Mr. G. W. Berridge, also published in the *Journal*, but reproduced in the *Text-Book*, Part II, ch. xxiv, Art. 20. Dr. Sprague's is the better, and will be taken up first. As given by him it is not directly applicable, but by a slight adaptation it can be made eminently suitable. In order to render what follows more intelligible, a brief sketch of the method will be useful.

If we have six consecutive quantities, y_0, y_1, y_2, y_3, y_4, and y_5, marking equidistant points on a curve, the problem is to interpolate by fifth differences, between the points y_2 and y_3, the central of the five spaces, in such a way as to obtain a smooth junction when the series is continued in each direction by interpolation in like manner between the points y_1 and y_2, and between the points y_3 and y_4. Dr. Sprague solves the problem by arranging that the two curves of the fifth order which meet at the point y_2 shall have at that point the same first differential coefficient and also the same second differential coefficient; that is, that they shall have at that point the same gradient, and the same radius of curvature; and, similarly, for the two curves of the fifth order which meet at the point y_3. He proceeds by writing the equation to the curve of interpolation between the points y_2 and y_3 in the form

$$y_{2+x} = y_2 + ax + bx^2 + cx^3 + dx^4 + ex^5.$$

He then finds the differential coefficients at the point y_2, by means of a curve of the fourth order passing through the points y_0, y_1, y_2, y_3, and y_4; and at the point y_3, by means of a similar curve passing through the points y_1, y_2, y_3, y_4, and y_5; and all these he expresses in terms of y_2 and the five differences of y_0; and thus, by solving five equations so formed, he finds the values of the constants a, b, c, &c., in terms of the five differences of y_0, and so arrives at the interpolation curve sought. Lastly, for practical calculation, he finds for the point y_2 the leading differences of the curve for the subdivided interval $\frac{1}{t}$. These are set forth as follows:—

$$\delta y_{2t} = \frac{\Delta y_0}{t} + \frac{3t+1}{2}\cdot\frac{\Delta^2 y_0}{t^2} + \frac{2t^2+3t+1}{6}\cdot\frac{\Delta^3 y_0}{t^3} - \frac{2t^3+t^2-2t-1}{24}\cdot\frac{\Delta^4 y_0}{t^4} + \frac{7t^2-12t+5}{24}\cdot\frac{\Delta^5 y_0}{t^5}$$

$$\delta^2 y_{2t} = \frac{\Delta^2 y_0}{t^2} + (t+1)\cdot\frac{\Delta^3 y_0}{t^3} - \frac{t^2-6t-7}{12}\cdot\frac{\Delta^4 y_0}{t^4} + \frac{7t^2-28t+25}{4}\cdot\frac{\Delta^5 y_0}{t^5}$$

$$\delta^3 y_{2t} = \frac{\Delta^3 y_0}{t^3} + \frac{t+3}{2}\cdot\frac{\Delta^4 y_0}{t^4} + \frac{7t^2-72t+125}{4}\cdot\frac{\Delta^5 y_0}{t^5}$$

$$\delta^4 y_{2t} = \frac{\Delta^4 y_0}{t^4} - (12t-50)\cdot\frac{\Delta^5 y_0}{t^5}$$

$$\delta^5 y_{2t} = 25\frac{\Delta^5 y_0}{t^5}$$

The interval to which the differences represented by the capital letter Δ refer is that between the original points y_0, y_1, &c., and, up to the stage of finding the subdivided differences, it was taken as the unit of distance. It may be any interval we please, and when we subdivide it into t minor intervals and make $\frac{1}{t}$ the unit of distance, then y_1 becomes y_t, y_2 becomes y_{2t}, &c., as in the above scheme of differences; and, $\Delta y_0 = y_t - y_0$; $\Delta^2 y_0 = y_{2t} - 2y_t + y_0$; &c.

To adapt this system of interpolation to summation formulas, let the function to be graduated be u; let the above function y be the finite integral of u; and let the integration constant be zero, so that $y_0=0$, and $y_x=\Sigma_0^{x-1}u$. Then $y_t=\Sigma_0^{t-1}u$; $y_{2t}=\Sigma_0^{2t-1}u$; and so on. Also $\Delta y_0=y_t-y_0=u_0+u_1+\ldots+u_{t-1}$; $\Delta^2 y_0=(u_t+u_{t+1}+\ldots+u_{2t-1})-(u_0+u_1+\ldots+u_{t-1})$; and so on; and $u_x=\delta y_x$.

It will be seen from the above scheme of subdivided differences that the function y has itself been eliminated, and that we have to deal only with its difference Δy_0, or $(u_0+u_1+\ldots+u_{t-1})$, and of course with the differences of higher orders. It will therefore now be convenient to treat Δy as the principal function, and to assign to it a symbol, w; and we shall have $w_0=(u_0+u_1+\ldots+u_{t-1})$, $w_1=(u_1+u_2+\ldots+u_t)$, and so on; and the scheme becomes

$$u_{2t}=\frac{w_0}{t}+\frac{3t+1}{2}\cdot\frac{\Delta w_0}{t^2}+\frac{2t^2+3t+1}{6}\cdot\frac{\Delta^2 w_0}{t^3}-\frac{2t^3+t^2-2t-1}{24}\cdot\frac{\Delta^3 w_0}{t^4}+\frac{7t^2-12t+5}{24}\cdot\frac{\Delta^4 w_0}{t^5}$$

$$\delta u_{2t}=\frac{\Delta w_0}{t^2}+(t+1)\cdot\frac{\Delta^2 w_0}{t^3}-\frac{t^2-6t-7}{12}\cdot\frac{\Delta^3 w_0}{t^4}+\frac{7t^2-28t+25}{4}\cdot\frac{\Delta^4 w_0}{t^5}$$

$$\delta^2 u_{2t}=\frac{\Delta^2 w_0}{t^3}+\frac{t+3}{2}\cdot\frac{\Delta^3 w_0}{t^4}+\frac{7t^2-72t+125}{4}\cdot\frac{\Delta^4 w_0}{t^5}$$

$$\delta^3 u_{2t}=\frac{\Delta^3 w_0}{t^4}-(12t-50)\cdot\frac{\Delta^4 w_0}{t^5}$$

$$\delta^4 u_{2t}=25\frac{\Delta^4 w_0}{t^5}$$

The only values of t which appear to be practically convenient are 5 and 3. A larger value than 5 produces a graduation formula much too long, and the value $t=4$ has been tried, but a workable expression does not seem to result.

When $t=5$

$$u_{10}=\frac{w_0}{5}+8\frac{\Delta w_0}{5^2}+11\frac{\Delta^2 w_0}{5^3}-11\frac{\Delta^3 w_0}{5^4}+5\frac{\Delta^4 w_0}{5^5}$$

$$\delta u_{10}=\frac{\Delta w_0}{5^2}+6\frac{\Delta^2 w_0}{5^3}+\frac{\Delta^3 w_0}{5^4}+15\frac{\Delta^4 w_0}{5^5}$$

$$\delta^2 u_{10}=\frac{\Delta^2 w_0}{5^3}+4\frac{\Delta^3 w_0}{5^4}-15\frac{\Delta^4 w_0}{5^5}$$

$$\delta^3 u_{10}=\frac{\Delta^3 w_0}{5^4}-10\frac{\Delta^4 w_0}{5^5}$$

$$\delta^4 u_{10}=25\frac{\Delta^4 w_0}{5^5}$$

Constructing u_{11}, u_{12}, u_{13}, and u_{14}, by means of these differences, we have

$$u_{10}=\frac{w_0}{5}+8\frac{\Delta w_0}{5^2}+11\frac{\Delta^2 w_0}{5^3}-11\frac{\Delta^3 w_0}{5^4}+5\frac{\Delta^4 w_0}{5^5}$$

$$u_{11}=\frac{w_0}{5}+9\frac{\Delta w_0}{5^2}+17\frac{\Delta^2 w_0}{5^3}-10\frac{\Delta^3 w_0}{5^4}+20\frac{\Delta^4 w_0}{5^5}$$

$$u_{12}=\frac{w_0}{5}+10\frac{\Delta w_0}{5^2}+24\frac{\Delta^2 w_0}{5^3}-5\frac{\Delta^3 w_0}{5^4}+20\frac{\Delta^4 w_0}{5^5}$$

$$u_{13}=\frac{w_0}{5}+11\frac{\Delta w_0}{5^2}+32\frac{\Delta^2 w_0}{5^3}+5\frac{\Delta^3 w_0}{5^4}-5\frac{\Delta^4 w_0}{5^5}$$

$$u_{14}=\frac{w_0}{5}+12\frac{\Delta w_0}{5^2}+41\frac{\Delta^2 w_0}{5^3}+21\frac{\Delta^3 w_0}{5^4}-40\frac{\Delta^4 w_0}{5^5}$$

Multiplying up by 5^4 we have

$$625u_{10}=125w_0+200\Delta w_0+55\Delta^2 w_0-11\Delta^3 w_0+\Delta^4 w_0$$

$$625u_{11}=125w_0+225\Delta w_0+85\Delta^2 w_0-10\Delta^3 w_0+4\Delta^4 w_0$$

$$625u_{12}=125w_0+250\Delta w_0+120\Delta^2 w_0-5\Delta^3 w_0+4\Delta^4 w_0$$

$$625u_{13}=125w_0+275\Delta w_0+160\Delta^2 w_0+5\Delta^3 w_0-\Delta^4 w_0$$

$$625u_{14}=125w_0+300\Delta w_0+205\Delta^2 w_0+21\Delta^3 w_0-8\Delta^4 w_0$$

Hence we can derive five values of the central term, u_{14}, the mean

of which to take for the final graduation. Thus, reversing the order of the equations,

$$625u_{14}=125w_0+300\Delta w_0+205\Delta^2 w_0+21\Delta^3 w_0-8\Delta^4 w_0$$
$$625u_{14}=125w_1+275\Delta w_1+160\Delta^2 w_1\ \ +5\Delta^3 w_1\ \ -\Delta^4 w_1$$
$$625u_{14}=125w_2+250\Delta w_2+120\Delta^2 w_2\ \ -5\Delta^3 w_2+4\Delta^4 w_2$$
$$625u_{14}=125w_3+225\Delta w_3\ \ +85\Delta^2 w_3-10\Delta^3 w_3+4\Delta^4 w_3$$
$$625u_{14}=125w_4+200\Delta w_4\ \ +55\Delta^2 w_4-11\Delta^3 w_4\ \ +\Delta^4 w_4$$

To make use of these equations, we prepare a statement of the values of w_0, w_1, &c., Δw_0, Δw_1, &c., and so on, in terms of u_0, u_1, &c.; and then in parallel columns we write, for each of the equations, the number of times u_0, u_1, &c., occurs; and, by summing the columns, we arrive at the graduation formula in its initial stage. So far, u_0 has been taken as the first term of the series, so that u_{14} has been the central term. Now it will be more convenient to call the first term u_{-14}, and to change correspondingly the numbering of the other terms, so that the central term becomes u_0. We thus have, when (u) is the graduated value,

$$\begin{aligned}5^5(u)=3125(u)=625u_0&+584\gamma_1+454\gamma_2+274\gamma_3+109\gamma_4\\&-0\gamma_5-61\gamma_6-71\gamma_7-46\gamma_8-16\gamma_9\\&+0\gamma_{10}+8\gamma_{11}+9\gamma_{12}+5\gamma_{13}+\gamma_{14}\end{aligned}$$

When we divide by 3125, we have a formula given by Dr. Karup in his paper, page 94, and called by him (6a), but he dismisses it as unworkable. Probably that is so by the method of calculation which he follows, but it can easily be thrown into the summation form with which we have been rendered familiar by Messrs. Higham and Hardy. It is then quite convenient to use, and gives a most excellent graduation, and one free from the fourth difference error. This last circumstance may be no great gain, but certainly it is not a disadvantage. The theoretical error is $+37u^{\text{VI}}$, which is inappreciable, and would scarcely affect the sixth place of decimals in q_x at any age.

To throw the expression into summation form we notice, from the coefficient, 5^5, of (u), that the summations cannot be other than in five's and three's, and that the governing summation must be $[5]^5$, which must enter into all the others. We also notice, because the expression contains twenty-nine terms, that there must be a summation of $[5]^7$. We therefore proceed to

express $[5]^7$, $[5]^6[3]$, $[5]^6$, $[5]^5[3]$ and $[5]^5$ in terms of u_0, γ_1, γ_2, &c. Keeping our eye on the extreme end of the expression, we observe $5\gamma_{13}+\gamma_{14}$. Now $[5]^7$ gives us $7\gamma_{13}+\gamma_{14}$, or $2\gamma_{13}$ in excess. We therefore correct by subtracting $2[5]^6[3]$, and are left with $14\gamma_{12}$, whereas we require $9\gamma_{12}$.

From the result already obtained we must therefore deduct $5[5]^6$, which puts γ_{12} right. So we proceed with γ_{11}, γ_{10}, &c., until the expression is exhausted, and we arrive at the final summation formula—

A. $5^5(u)=3125(u)=[5]^5\{[1]+10[3]-5[5]-2[5][3]+[5]^2\}v_0$

This formula has been built up by means of curves which touch each other at given points, and which have therefore received the descriptive and convenient designation of osculatory. It also depends on the function w, which consists of a group of values of the function u. We may, therefore, not inappropriately, speak of the formula as having been derived from osculatory grouped curves.

If in the scheme of sub-divided differences on page 540 we take t equal to 3, we have—

$$\begin{aligned}
u_6 &= \frac{w_0}{3}+5\cdot\frac{\Delta w_0}{3^2}+\frac{14}{3}\cdot\frac{\Delta^2 w_0}{3^3}-\frac{7}{3}\cdot\frac{\Delta^3 w_0}{3^4}+\frac{4}{3}\cdot\frac{\Delta^4 w_0}{3^5}\\
\delta u_6 &= \frac{\Delta w_0}{3^2}+4\frac{\Delta^2 w_0}{3^3}+\frac{4}{3}\cdot\frac{\Delta^3 w_0}{3^4}+\frac{\Delta^4 w_0}{3^5}\\
\delta^2 u_6 &= \frac{\Delta^2 w_0}{3^3}+3\frac{\Delta^3 w_0}{3^4}-7\frac{\Delta^4 w_0}{3^5}\\
\delta^3 u_6 &= \frac{\Delta^3 w_0}{3^4}+14\frac{\Delta^4 w_0}{3^5}\\
\delta^4 u_6 &= 25\frac{\Delta^4 w_0}{3^5}
\end{aligned}$$

Constructing u_7 and u_8 by means of these differences, we have—

$$\begin{aligned}
u_6 &= \frac{w_0}{3}+5\frac{\Delta w_0}{3^2}+\frac{14}{3}\cdot\frac{\Delta^2 w_0}{3^3}-\frac{7}{3}\cdot\frac{\Delta^3 w_0}{3^4}+\frac{4}{3}\cdot\frac{\Delta^4 w_0}{3^5}\\
u_7 &= \frac{w_0}{3}+6\frac{\Delta w_0}{3^2}+\frac{26}{3}\cdot\frac{\Delta^2 w_0}{3^3}-\frac{\Delta^3 w_0}{3^4}+\frac{7}{3}\cdot\frac{\Delta^4 w_0}{3^5}\\
u_8 &= \frac{w_0}{3}+7\frac{\Delta w_0}{3^2}+\frac{41}{3}\cdot\frac{\Delta^2 w_0}{3^3}+\frac{10}{3}\cdot\frac{\Delta^3 w_0}{3^4}-\frac{11}{3}\cdot\frac{\Delta^4 w_0}{3^5}
\end{aligned}$$

Multiplying up by 3^6 we have

$$\begin{aligned}
729u_6 &= 243w_0+405\Delta w_0+126\Delta^2 w_0-21\Delta^3 w_0+4\Delta^4 w_0\\
729u_7 &= 243w_0+486\Delta w_0+234\Delta^2 w_0-9\Delta^3 w_0+7\Delta^4 w_0\\
729u_8 &= 243w_0+567\Delta w_0+369\Delta^2 w_0+30\Delta^3 w_0-11\Delta^4 w_0
\end{aligned}$$

Hence we can derive three values of the central term, u_8, the mean of which to take for the final graduation. Thus, reversing the order of the equations,

$$729u_8 = 243w_0 + 567\Delta w_0 + 369\Delta^2 w_0 + 30\Delta^3 w_0 - 11\Delta^4 w_0$$
$$729u_8 = 243w_1 + 486\Delta w_1 + 234\Delta^2 w_1 - 9\Delta^3 w_1 + 7\Delta^4 w_1$$
$$729u_8 = 243w_2 + 405\Delta w_2 + 126\Delta^2 w_2 - 21\Delta^3 w_2 + 4\Delta^4 w_2$$

Proceeding as in the case of the quinary curves, we arrive at the graduated value

$$3^7(u) = 2187(u) = 729u_0 + 590\gamma_1 + 250\gamma_2 + 0\gamma_3 - 85\gamma_4 - 41\gamma_5 + 0\gamma_6 + 11\gamma_7 + 4\gamma_8$$

To pass to the summation formula we notice, from the factors which make up the coefficient, 2187, of (u), that all the summations must be in threes, and that, as the formula has seventeen terms, we must have $[3]^8$ as the highest summation. We therefore proceed to express $[3]^5$, $[3]^6$, $[3]^7$ and $[3]^8$ in terms of u_0, γ_1, γ_2, &c., and finally we arrive at the formula

B. $3^7(u) = 2187(u) = [3]^5\{81[1] + 3[3] - 21[3]^2 + 4[3]^3\}u_0$

We have here another formula free from fourth difference error, but it has not so great a graduating power as formula A. The theoretical error is $1\frac{20}{27}.u^{\mathrm{VI}}$, which is very small.

We have hitherto made use of osculatory curves of the fifth order, and have found formulas true to fifth differences, but that refinement is hardly necessary, and if we construct an osculatory curve of the third order we shall be able to form graduation formulas true to third differences, which will give excellent results.

If we limit ourselves to the third order, the osculatory curves will have the same gradient, but not necessarily the same radius of curvature, at the point of junction, because we leave out of account the second differential coefficient; but we can follow Dr. Sprague's method of constructing them.

We have four points, y_0, y_1, y_2, and y_3, and the problem is, to interpolate by third differences between the points y_1 and y_2 in such a way as to obtain a smooth junction when the series is continued in each direction, by interpolation in like manner between the points y_0 and y_1, and between the points y_2 and y_3. To solve the problem we arrange that the two curves of the third order which meet at the point y_1, shall have the same first differential coefficient at that point; and we find the value of that differential coefficient by means of a curve of the second order passing through the points y_0, y_1, and y_2: and similarly for the point y_2.

The required equation to the interpolation curve of the third order between the points y_1 and y_2 may be written,

$$y_{1+x}=y_1+ax+bx^2+cx^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

whence $$y_2=y_1+a+b+c$$

$$a+b+c=y_2-y_1=\Delta y_1=\Delta y_0+\Delta^2 y_0 \quad . \quad . \quad . \quad (2)$$

Differentiating equation (1),

$$\frac{dy_{1+x}}{dx}=a+2bx+3cx^2$$

whence $$\frac{dy_1}{dx}=a, \text{ and } \frac{dy_2}{dx}=a+2b+3c \quad . \quad . \quad . \quad . \quad (3)$$

The curve of the second order passing through the points y_0, y_1, and y_2 may be written

$$y_x=y_0+x\Delta y_0+\tfrac{1}{2}(x^2-x)\Delta^2 y_0$$

whence $$\frac{dy_x}{dx}=\Delta y_0+\tfrac{1}{2}(2x-1)\Delta^2 y_0$$

and $$\frac{dy_1}{dx}=\Delta y_0+\tfrac{1}{2}\Delta^2 y_0\,;$$

and therefore from equation (3)

$$a=\Delta y_0+\tfrac{1}{2}\Delta^2 y_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Similarly, from the curve of the second order passing through the points y_1, y_2, and y_3,

$$\frac{dy_2}{dx}=\Delta y_1+\tfrac{1}{2}\Delta^2 y_1$$

$$=\Delta y_0+\tfrac{3}{2}\Delta^2 y_0+\tfrac{1}{2}\Delta^3 y_0$$

and from equation (3)

$$a+2b+3c=\Delta y_0+\tfrac{3}{2}\Delta^2 y_0+\tfrac{1}{2}\Delta^3 y_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

By means of equations (2), (4), and (5) we find

$$a=\Delta y_0+\tfrac{1}{2}\Delta^2 y_0$$

$$b=\tfrac{1}{2}\Delta^2 y_0-\tfrac{1}{2}\Delta^3 y_0$$

$$c=\tfrac{1}{2}\Delta^3 y_0,$$

and after reduction, equation (1) becomes

$$y_{1+x}=y_1+x\Delta y_0+\frac{x+x^2}{2}\Delta^2 y_0-\frac{x^2-x^3}{2}\Delta^3 y_0$$

By differencing this last equation for the interval $\frac{1}{t}$, we have the scheme

$$\delta y_1 = \frac{\Delta y_0}{t} + \frac{t+1}{2} \cdot \frac{\Delta^2 y_0}{t^2} - \frac{t-1}{2} \cdot \frac{\Delta^3 y_0}{t^3}$$

$$\delta^2 y_1 = \frac{\Delta^2 y_0}{t^2} - (t-3)\frac{\Delta^3 y_0}{t^3}$$

$$\delta^3 y_1 = 3\frac{\Delta^3 y_0}{t^3}.$$

We adapt this curve to the present purposes, as we did the curve of the fifth order. As before, y is taken as the finite integral of the function u which is to be graduated, and, in the former notation, the above scheme of subdivided differences becomes

$$u_t = \frac{w_0}{t} + \frac{t+1}{2} \cdot \frac{\Delta w_0}{t^2} - \frac{t-1}{2} \cdot \frac{\Delta^2 w_0}{t^3}$$

$$\delta u_t = \frac{\Delta w_0}{t^2} - (t-3)\frac{\Delta^2 w_0}{t^3}$$

$$\delta^2 u_t = 3\frac{\Delta^2 w_0}{t^3}$$

When $t=5$,

$$u_5 = \frac{w_0}{5} + 3\frac{\Delta w_0}{5^2} - 2\frac{\Delta^2 w_0}{5^3}$$

$$u_6 = \frac{w_0}{5} + 4\frac{\Delta w_0}{5^2} - 4\frac{\Delta^2 w_0}{5^3}$$

$$u_7 = \frac{w_0}{5} + 5\frac{\Delta w_0}{5^2} - 3\frac{\Delta^2 w_0}{5^3}$$

$$u_8 = \frac{w_0}{5} + 6\frac{\Delta w_0}{5^2} + \frac{\Delta^2 w_0}{5^3}$$

$$u_9 = \frac{w_0}{5} + 7\frac{\Delta w_0}{5^2} + 8\frac{\Delta^2 w_0}{5^3}$$

Hence we derive five values of the central term u_9, from which to pass to the graduation formula:

$$125u_9 = 25w_0 + 35\Delta w_0 + 8\Delta^2 w_0$$

$$125u_9 = 25w_1 + 30\Delta w_1 + \Delta^2 w_1$$

$$125u_9 = 25w_2 + 25\Delta w_2 - 3\Delta^2 w_2$$

$$125u_9 = 25w_3 + 20\Delta w_3 - 4\Delta^2 w_3$$

$$125u_9 = 25w_4 + 15\Delta w_4 - 2\Delta^2 w_4$$

By means of these values we arrive at the graduation formula

$$625(u)=125u_0+114\gamma_1+87\gamma_2+53\gamma_3+21\gamma_4+0\gamma_5 \\ -8\gamma_6-9\gamma_7-6\gamma_8-2\gamma_9$$

This is the formula of Dr. Karup, his (5A), in the form which he gives to it when we divide by 625. He arrived at it much as we have done, only I venture to think that Dr. Sprague's way of constructing the osculatory curve, which has been followed above, is a little simpler and more direct than that adopted by Dr. Karup.

Dr. Karup applies the formula, as written above, by a method of summing special differences which he constructs. I have not tried that method, and therefore cannot express an opinion as to whether it is preferable to the ordinary summation method to which we are more accustomed, and which is very convenient and simple. When thrown into the usual summation form, the formula becomes

$$\text{C.} \quad 5^4(u)=625(u)=[5]^3\{5[3]+4[5]-2[5][3]\}u_0$$

There are nineteen terms and the theoretical error is

$$-7{\cdot}8u_0^{\text{IV}}-17{\cdot}5u_0^{\text{VI}}$$

If in the scheme of subdivided differences on page 546 we take $t=3$, we shall have

$$u_3=\frac{w_0}{3}+2\frac{\Delta w_0}{3^2}-\frac{\Delta^2 w_0}{3^3}$$

$$u_4=\frac{w_0}{3}+3\frac{\Delta w_0}{3^2}-\frac{\Delta^2 w_0}{3^3}$$

$$u_5=\frac{w_0}{3}+4\frac{\Delta w_0}{3^2}+2\frac{\Delta^2 w_0}{3^3}$$

Whence

$$27u_5=9w_0+12\Delta w_0+2\Delta^2 w_0$$

$$27u_5=9w_1\ +9\Delta w_1\ -\Delta^2 w_1$$

$$27u_5=9w_2\ +6\Delta w_2\ -\Delta^2 w_2$$

From these three values we obtain the formula

$$81(u)=27u_0+21\gamma_1+9\gamma_2+0\gamma_3-2\gamma_4-\gamma_5,$$

which may be written

$$\text{D.} \quad 3^4(u)=81(u)=[3]^3\{3[1]+3[3]-[3]^2\}u_0.$$

This is a formula of eleven terms only, and, so far as I know, is by far the best short formula which has yet been produced. For its length it has good graduating power, and the theoretical error is remarkably small, being only $-u_0^{\text{IV}}-{\cdot}7963u_0^{\text{VI}}$. It is a most useful auxiliary formula for completing the ends of the tables, as will be seen later on.

If we duplicate formula D, that is, if we graduate by it twice, we have interesting results. The double application of the formula gives us

$$6561(u) = 1783u_0 + 1516\gamma_1 + 891\gamma_2 + 276\gamma_3 - 69\gamma_4 - 138\gamma_5 - 78\gamma_6 - 18\gamma_7 + 4\gamma_8 + 4\gamma_9 + \gamma_{10}$$

which may be written

$$(\mathrm{D})^2. \quad 3^8(u) = 6561(u) = [3]^6\{9[1] + 18[3] + 3[3]^2 - 6[3]^3 + [3]^4\}u_0$$

This formula has twenty-one terms, and its theoretical error is exceedingly small for its length, being only $-2u_0^{\mathrm{IV}} - 1{\cdot}5926u_0^{\mathrm{VI}}$; and it gives a fairly good graduation. It is also very easily applied, the summations being all in three's, and it may sometimes, therefore, be very useful; for instance, in the case of Pension Funds, where a quick and simple process, producing a fairly smooth curve, is all that is required. Also as a subsidiary formula, to be used in connection with others more powerful, it may be of great assistance. It has the peculiarity that it can be split in two, and can be applied either directly by one process, or by graduating by formula D twice. Probably the latter course is the easier.

It will be observed that the sequence of the terms differs from that of all other third difference formulas so far published, and gives the appearance of a formula true to fifth differences. In the usual third difference formulas the terms nearest the centre are positive and those more remote are all negative, while formula $(\mathrm{D})^2$ has the terms at the centre and at both extremities positive, with negative intermediate terms. Any other third difference summation formula, if duplicated, would present similar features.

We may now very briefly deal with ordinary grouped curves, as distinguished from osculatory grouped curves. They will give graduation formulas corresponding to A, B, C, and D above, though not quite so good; but they will be better than those derived from single ordinary curves on Woolhouse's principle. If this part of the enquiry is not of immediate practical use, it may, nevertheless, possess theoretical interest.

By one of the fundamental principles of the calculus of finite differences, when we stop at fifth differences, and use the symbol δ to represent differences in respect of the interval unity, and Δ in respect of the interval t,

$$y_x = y_0 + x\delta y_0 + \frac{x^2-x}{2}\delta^2 y_0 + \frac{x^3-3x^2+2x}{6}\delta^3 y_0 + \frac{x^4-6x^3+11x^2-6x}{24}\delta^4 y_0 + \frac{x^5-10x^4+35x^3-50x^2+24x}{120}\delta^5 y_0$$

If we take y as the finite integral of u, and make zero the constant of integration, this becomes—

$$y_x = x u_0 + \frac{x^2-x}{2}\delta u_0 + \frac{x^3-3x^2+2x}{6}\delta^2 u_0 + \frac{x^4-6x^3+11x^2-6x}{24}\delta^3 u_0 + \frac{x^5-10x^4+35x^3-50x^2+24x}{120}\delta^4 u_0$$

Taking now the six values y_0, y_t, y_{2t}, &c. y_{5t}, and expressing them all in the above form, and then differencing successively for the interval t, we have—

$$\Delta y_0 = tu_0 + \frac{t^2-t}{2}\delta u_0 + \frac{t^3-3t^2+2t}{6}\delta^2 u_0 + \frac{t^4-6t^3+11t^2-6t}{24}\delta^3 u_0 + \frac{t^5-10t^4+35t^3-50t^2+24t}{120}\delta^4 u_0$$

$$\Delta^2 y_0 = t^2\delta u_0 + \frac{6t^3-6t^2}{6}\delta^2 u_0 + \frac{14t^4-36t^3+22t^2}{24}\delta^3 u_0 + \frac{30t^5-140t^4+210t^3-100t^2}{120}\delta^4 u_0$$

$$\Delta^3 y_0 = t^3\delta^2 u_0 + \frac{36t^4-36t^3}{24}\delta^3 u_0 + \frac{150t^5-360t^4+210t^3}{120}\delta^4 u_0$$

$$\Delta^4 u_0 = t^4\delta^3 u_0 + \frac{240t^5-240t^4}{120}\delta^4 u_0$$

$$\Delta^5 y_0 = t^5\delta^4 u_0.$$

Expressing now u_0 and its differences, δ, in terms of the differences, Δ, of y_0, we have—

$$\begin{aligned}
u_0 &= \frac{\Delta y_0}{t} - \frac{t-1}{2}\cdot\frac{\Delta^2 y_0}{t^2} + \frac{2t^2-3t+1}{6}\cdot\frac{\Delta^3 y_0}{t^3} - \frac{6t^3-11t^2+6t-1}{24}\cdot\frac{\Delta^4 y_0}{t^4} + \frac{24t^4-50t^3+35t^2-10t+1}{120}\cdot\frac{\Delta^5 y_0}{t^5} \\
\delta u_0 &= \frac{\Delta^2 y_0}{t^2} - (t-1)\cdot\frac{\Delta^3 y_0}{t^3} + \frac{11t^2-18t+7}{12}\cdot\frac{\Delta^4 y_0}{t^4} - \frac{10t^3-21t^2+14t-3}{12}\cdot\frac{\Delta^5 y_0}{t^5} \\
\delta^2 u_0 &= \frac{\Delta^3 y_0}{t^3} - \frac{3t-3}{2}\cdot\frac{\Delta^4 y_0}{t^4} + \frac{7t^2-12t+5}{4}\cdot\frac{\Delta^5 y_0}{t^5} \\
\delta^3 u_0 &= \frac{\Delta^4 y_0}{t^4} - 2(t-1)\frac{\Delta^5 y_0}{t^5} \\
\delta^4 u_0 &= \frac{\Delta^5 y_0}{t^5}
\end{aligned}$$

From the above equations the function y itself has disappeared, and Δy has become the main function. We may therefore write, as before, $\Delta y = w$, where w is the sum of t values of u, and we have—

$$\begin{aligned}
u_0 &= \frac{w_0}{t} - \frac{t-1}{2}\cdot\frac{\Delta w_0}{t^2} + \frac{2t^2-3t+1}{6}\cdot\frac{\Delta^2 w_0}{t^3} - \frac{6t^3-11t^2+6t-1}{24}\cdot\frac{\Delta^3 w_0}{t^4} + \frac{24t^4-50t^3+35t^2-10t+1}{120}\cdot\frac{\Delta^4 w_0}{t^5} \\
\delta u_0 &= \frac{\Delta w_0}{t^2} - (t-1)\frac{\Delta^2 w_0}{t^3} + \frac{11t^2-18t+7}{12}\cdot\frac{\Delta^3 w_0}{t^4} - \frac{10t^3-21t^2+14t-3}{12}\cdot\frac{\Delta^4 w_0}{t^5} \\
\delta^2 u_0 &= \frac{\Delta^2 w_0}{t^3} - \frac{3t-3}{2}\cdot\frac{\Delta^3 w_0}{t^4} + \frac{7t^2-12t+5}{4}\cdot\frac{\Delta^4 w_0}{t^5} \\
\delta^3 u_0 &= \frac{\Delta^3 w_0}{t^4} - 2(t-1)\frac{\Delta^4 w_0}{t^5} \\
\delta^4 u_0 &= \frac{\Delta^4 w_0}{t^5}.
\end{aligned}$$

For interpolation in the central interval y_{2t} to y_{3t}, we require t values of u, from u_{2t} inclusive, and these we obtain by calculating from the above scheme u_{2t} and its leading differences. We thus have—

$$\begin{aligned}
u_{2t} &= \frac{w_0}{t} + \frac{3t+1}{2}\cdot\frac{\Delta w_0}{t^2} + \frac{2t^2+3t+1}{6}\cdot\frac{\Delta^2 w_0}{t^3} - \frac{2t^3+t^2-2t-1}{24}\cdot\frac{\Delta^3 w_0}{t^4} + \frac{4t^4-5t^2+1}{120}\cdot\frac{\Delta^4 w_0}{t^5} \\
\delta u_{2t} &= \frac{\Delta w_0}{t^2} + (t+1)\frac{\Delta^2 w_0}{t^3} - \frac{t^2-6t-7}{12}\cdot\frac{\Delta^3 w_0}{t^4} - \frac{t^2-1}{4}\cdot\frac{\Delta^4 w_0}{t^5} \\
\delta^2 u_{2t} &= \frac{\Delta^2 w_0}{t^3} + \frac{t+3}{2}\cdot\frac{\Delta^3 w_0}{t^4} - \frac{t^2-5}{4}\cdot\frac{\Delta^4 w_0}{t^5} \\
\delta^3 v_{2t} &= \frac{\Delta^3 w_0}{t^4} + 2\frac{\Delta^4 w_0}{t^5} \\
\delta^4 u_{2t} &= \frac{\Delta^4 w_0}{t^5}.
\end{aligned}$$

When we take $t=5$, u_{2t} becomes u_{10}, and by constructing u_{11}, u_{12}, &c., by means of the differences, we have

$$u_{10}=\frac{w_0}{5}+8\frac{\Delta w_0}{5^2}+11\frac{\Delta^2 w_0}{5^3}-11\frac{\Delta^3 w_0}{5^4}+19{\cdot}8\frac{\Delta^4 w_0}{5^5}$$

$$u_{11}=\frac{w_0}{5}+9\frac{\Delta w_0}{5^2}+17\frac{\Delta^2 w_0}{5^3}-10\frac{\Delta^3 w_0}{5^4}+13{\cdot}8\frac{\Delta^4 w_0}{5^5}$$

$$u_{12}=\frac{w_0}{5}+10\frac{\Delta w_0}{5^2}+24\frac{\Delta^2 w_0}{5^3}-5\frac{\Delta^3 w_0}{5^4}+2{\cdot}8\frac{\Delta^4 w_0}{5^5}$$

$$u_{13}=\frac{w_0}{5}+11\frac{\Delta w_0}{5^2}+32\frac{\Delta^2 w_0}{5^3}+5\frac{\Delta^3 w_0}{5^4}-11{\cdot}2\frac{\Delta^4 w_0}{5^5}$$

$$u_{14}=\frac{w_0}{5}+12\frac{\Delta w_0}{5^2}+41\frac{\Delta^2 w_0}{5^3}+21\frac{\Delta^3 w_0}{5^4}-25{\cdot}2\frac{\Delta^4 w_0}{5^5}$$

Multiplying by 5^6, or 15625, we have

$$15625u_{10}=3125w_0+5000\Delta w_0+1375\Delta^2 w_0-275\Delta^3 w_0+99\Delta^4 w_0$$

$$15625u_{11}=3125w_0+5625\Delta w_0+2125\Delta^2 w_0-250\Delta^3 w_0+69\Delta^4 w_0$$

$$15625u_{12}=3125w_0+6250\Delta w_0+3000\Delta^2 w_0-125\Delta^3 w_0+14\Delta^4 w_0$$

$$15625u_{13}=3125w_0+6875\Delta w_0+4000\Delta^2 w_0+125\Delta^3 w_0-56\Delta^4 w_0$$

$$15625u_{14}=3125w_0+7500\Delta w_0+5125\Delta^2 w_0+525\Delta^3 w_0-126\Delta^4 w_0$$

Hence we get five equations for u_{14}, of which we take the mean for the graduated value.

$$15625u_{14}=3125w_0+7500\Delta w_0+5125\Delta^2 w_0+525\Delta^3 w_0-126\Delta^4 w_0$$

$$15625u_{14}=3125w_1+6875\Delta w_1+4000\Delta^2 w_1+125\Delta^3 w_1-56\Delta^4 w_1$$

$$15625u_{14}=3125w_2+6250\Delta w_2+3000\Delta^2 w_2-125\Delta^3 w_2+14\Delta^4 w_2$$

$$15625u_{14}=3125w_3+5625\Delta w_3+2125\Delta^2 w_3-250\Delta^3 w_3+69\Delta^4 w_3$$

$$15625u_{14}=3125w_4+5000\Delta w_4+1375\Delta^2 w_4-275\Delta^3 w_4+99\Delta^4 w_4$$

These are very similar equations to those derived by means of the osculatory curves, except that, to clear off fractions, we have been obliged here to multiply by 5^6 instead of by only 5^4. When this change is allowed for, it will be found that all the coefficients are identical with those of the osculatory curves, except the coefficients of $\Delta^4 w$. It is only in the fourth difference that the osculatory curves depart from the ordinary ones.

When we use these equations to form a graduation formula, we arrive at the expression

$$5^7(u)=78125(u)$$
$$=15625u_0+13860\gamma_1+10920\gamma_2+7280\gamma_3+3465\gamma_4$$
$$-0\gamma_5-1155\gamma_6-1560\gamma_7-1365\gamma_8-770\gamma_9$$
$$+0\gamma_{10}+126\gamma_{11}+182\gamma_{12}+168\gamma_{13}+99\gamma_{14}$$

which may be written

$$\text{A}^{\text{I}}.\quad 5^7(u)=78125(u)=[5]^6\{1085[1]-525[3]+99[5]\}u_0$$

This is a twenty-nine term formula, true to fifth differences, with a theoretical error of $+47{\cdot}6u_0^{\text{VI}}$. It is derived by means of ordinary grouped curves in the same way that formula A is derived by means of osculatory grouped curves; but obviously it is not so good, and it is much more troublesome to apply, on account of the magnitude of the coefficients of the subsidiary summations. It has been worked out, not with any expectation of its being useful in practice, but because of the instructive nature of the process.

When, in the scheme of differences for ordinary grouped curves, we take $t=3$, we arrive at the formula

$$3^7(u)=2187(u)=729u_0+560\gamma_1+280\gamma_2+0\gamma_3$$
$$-70\gamma_4+56\gamma_5-0\gamma_6$$
$$+8\gamma_7+7\gamma_8$$

which may be written

$$\text{B}^{\text{I}}.\quad 2187(u)=[3]^6\{84[1]-48[3]+7[3]^2\}u_0$$

This formula corresponds to formula B derived from osculatory grouped curves. It is true to fifth differences, has seventeen terms, and has an error of $+2\frac{2}{27}u_0^{\text{VI}}$.

To construct formulas by third differences from ordinary grouped curves, we have merely, from the scheme for fifth differences given on page 551, to drop the differences no longer required. The process is therefore easier than in the case of osculatory curves, where the scheme has to be entirely recast.

Taking ordinary grouped curves, with third differences, and making $t=5$, we have

$$5^4(u)=625(u)=125u_0+108\gamma_1+84\gamma_2+56\gamma_3+27\gamma_4+0\gamma_5$$
$$-6\gamma_6-8\gamma_7-7\gamma_8-4\gamma_9.$$

which may be written

$$\text{C}^{\text{I}}.\quad 625(u)=[5]^4\{13[1]-4[3]\}u_0.$$

This is a formula of nineteen terms, which corresponds to Dr. Karup's, formula C of this paper, formed in the same way, but from osculatory grouped curves. The error is $-9{\cdot}2u_0^{\text{IV}}-22\frac{1}{3}u_0^{\text{VI}}$.

Woolhouse's formula is made in a way similar to this one, but by only a single ordinary curve. It may be written $125(u)=[5]^3\{10[1]-3[3]\}u_0$. It thus appears that by grouping the curves we merely add one more summation to those of Woolhouse, and apply the necessary additional correction for the second difference.

When with ordinary grouped curves we take $t=3$, we have

$$3^5(u)=243(u)=81u_0+60\gamma_1+30\gamma_2+0\gamma_3-5\gamma_4-4\gamma_5$$

which may be written

$$\text{D}^{\text{I}}.\quad 243(u)=[3]^4\{15[1]-4[3]\}u_0.$$

Here we have a formula of eleven terms, corresponding to formula D which was formed in the same way, but from osculatory grouped curves. The subsidiary summations are curiously similar to those of Woolhouse. The error is $-1\frac{1}{9}u_0^{\text{IV}}-\frac{25}{27}u_0^{\text{VI}}$.

For convenience of reference it will be useful to recapitulate all the formulas which have been deduced.

Recapitulation of Formulas.

A. Osculatory Grouped Curves in Five's. Page 543.
Fifth Difference. 29 terms.
Theoretical Error $+37u_0^{\text{VI}}$.

$$5^5(u)=3125(u)=[5]^5\{[1]+10[3]-5[5]-2[5][3]+[5]^2\}u_0*$$

$$(u)={\cdot}2u_0+{\cdot}18688\gamma_1+{\cdot}14528\gamma_2+{\cdot}08768\gamma_3+{\cdot}03488\gamma_4$$
$$-{\cdot}01952\gamma_6-{\cdot}02272\gamma_7-{\cdot}01472\gamma_8-{\cdot}00512\gamma_9$$
$$+{\cdot}00256\gamma_{11}+{\cdot}00288\gamma_{12}+{\cdot}00160\gamma_{13}+{\cdot}00032\gamma_{14}$$

A^{I}. Ordinary Grouped Curves in Five's. Page 553.
Fifth Difference. 29 terms.
Theoretical Error $+47{\cdot}6u_0^{\text{VI}}$.

$$5^7(u)=78125(u)=[5]^6\{1085[1]-525[3]+99[5]\}u_0$$

$$(u)={\cdot}2u_0+{\cdot}177408\gamma_1+{\cdot}139776\gamma_2+{\cdot}093184\gamma_3+{\cdot}044352\gamma_4$$
$$-{\cdot}014784\gamma_6-{\cdot}019968\gamma_7-{\cdot}017472\gamma_8-{\cdot}009856\gamma_9$$
$$+{\cdot}0016128\gamma_{11}+{\cdot}0023296\gamma_{12}+{\cdot}0021504\gamma_{13}+{\cdot}0012672\gamma_{14}$$

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* *See* the shorter form, Postscript, p. 559.

B. Osculatory Grouped Curves in Three's. Page 544.
Fifth Difference. 17 terms.
Theoretical Error $+1\frac{20}{27}u_0^{\mathrm{VI}}$.

$$3^7(u)=2187(u)=[3]^5\{81[1]+3[3]-21[3]^2+4[3]^3\}u_0$$

$$(u)=\tfrac{1}{3}u_0+\cdot 2698\gamma_1+\cdot 1143\gamma_2-\cdot 0389\gamma_4-\cdot 0187\gamma_5 +\cdot 0050\gamma_7+\cdot 0018\gamma_8$$

$\mathrm{B^I}$. Ordinary Grouped Curves in Three's. Page 553.
Fifth Difference. 17 terms.
Theoretical Error $+2\frac{2}{27}u_0^{\mathrm{VI}}$.

$$3^7(u)=2187(u)=[3]^6\{84[1]-48[3]+7[3]^2\}u_0$$

$$(u)=\tfrac{1}{3}u_0+\cdot 2561\gamma_1+\cdot 1280\gamma_2-\cdot 0320\gamma_4-\cdot 0256\gamma_5 +\cdot 0037\gamma_7+\cdot 0032\gamma_8$$

C. Osculatory Grouped Curves in Five's. Page 547.
Third Difference. 19 terms.
Theoretical Error $-7\cdot 8u_0^{\mathrm{IV}}-17\cdot 5u_0^{\mathrm{VI}}$.

$$5^4(u)=625(u)=[5]^3\{5[3]+4[5]-2[5][3]\}u_0*$$

$$(u)=\cdot 2u_0+\cdot 1824\gamma_1+\cdot 1392\gamma_2+\cdot 0848\gamma_3+\cdot 0336\gamma_4 -\cdot 0128\gamma_6-\cdot 0144\gamma_7-\cdot 0096\gamma_8-\cdot 0032\gamma_9$$

$\mathrm{C^I}$. Ordinary Grouped Curves in Five's. Page 553.
Third Difference. 19 terms.
Theoretical Error $-9\cdot 2u_0^{\mathrm{IV}}-22\tfrac{1}{3}u_0^{\mathrm{VI}}$.

$$5^4(u)=625(u)=[5]^4\{13[1]-4[3]\}u_0$$

$$(u)=\cdot 2u_0+\cdot 1728\gamma_1+\cdot 1344\gamma_2+\cdot 0896\gamma_3+\cdot 0432\gamma_4 -\cdot 0096\gamma_6-\cdot 0128\gamma_7-\cdot 0112\gamma_8-\cdot 0064\gamma_9$$

D. Osculatory Grouped Curves in Three's. Page 547.*
Third Difference. 11 terms.
Theoretical Error $-u^{\mathrm{IV}}-\frac{215}{270}u_0^{\mathrm{VI}}$.

$$3^4(u)=81(u)=[3]^3\{3[1]+3[3]-[3]^2\}u_0$$

$$(u)=\tfrac{1}{3}u_0+\cdot 2593\gamma_1+\cdot 111\dot{1}\gamma_2-\cdot 0247\gamma_4-\cdot 0123\gamma_5$$

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* *See* the shorter form, Postscript, p. 559.

D$^{\mathrm{I}}$. Ordinary Grouped Curves in Three's. Page 554.
Third Difference. 11 terms.
Theoretical Error $-1\frac{1}{9}u_0^{\mathrm{IV}}-\frac{25}{27}u_0^{\mathrm{VI}}$.

$$3^5(u)=243(u)=[3]^4\{15[1]-4[3]\}u_0$$

$$(u)=\tfrac{1}{3}u_0+\cdot2469\gamma_1+\cdot1235\gamma_2-\cdot0206\gamma_4-\cdot0165\gamma_5$$

(D)^2. Formula D duplicated. 21 terms. Page 548.
Theoretical Error $-2u_0^{\mathrm{IV}}-1\frac{16}{27}u_0^{\mathrm{VI}}$.

$$3^8(u)=6561(u)=[3]^6\{9[1]+18[3]+3[3]^2-6[3]^3+[3]^4\}u_0$$

$$\begin{aligned}(u)=\cdot2718u_0+\cdot2311\gamma_1+\cdot1358\gamma_2+\cdot0421\gamma_3\\ -\cdot0105\gamma_4-\cdot0210\gamma_5-\cdot0119\gamma_6-\cdot0027\gamma_7\\ +\cdot0006\gamma_8+\cdot0006\gamma_9+\cdot0002\gamma_{10}\end{aligned}$$

It is not proposed on the present occasion to make any detailed enquiry into the effects of graduating by the various formulas submitted. That must be left to the future, or, better still, perhaps some other worker, who is not averse to taking trouble, will enter on the task. If so, I shall be very pleased to render all the assistance in my power. It may be well, however, to indicate what may be expected from those of the formulas which seem to be the most promising.

Three diagrams are appended. They are on the plan proposed first, I believe, by Dr. Karup, and used with great effect by Mr. Lidstone in his paper on page 348 of the present volume of the *Journal*, where he also discusses the principles on which the curves should be interpreted. Each diagram, to save space, is in two parts, the left and the right, which are distinct, and show separate curves; and, as the curves are symmetrical on the two sides of the ordinate axis, only half of each is given.

Diagram I gives the graphs of formulas A and A$^{\mathrm{I}}$, and it will be observed that formula A especially has a curve of graceful sweep, and we may therefore surmise that it will give a smooth graduation. It is derived by means of osculatory grouped curves, and possesses the advantage of being free from the fourth difference error, so that it will reproduce without alteration a mortality table already perfectly smooth. It affects only the

deviations from the true curve, and therefore, although it involves 29 terms, there can be no valid objection in theory to its use. The only objection is the practical one that 14 terms at each end of the table are left ungraduated, but that difficulty, as will be seen immediately, can be overcome by means of formula D. Moreover, formula A has very convenient coefficients for the subsidiary summations, so that it is easy to use. It may therefore become a strong competitor for actuarial favour.

The curve of formula A^I, derived by means of ordinary grouped curves, is very similar to that of A, but, where it differs, it is inferior. We should get a smooth graduation, but not quite so good as that by A. There is, however, the serious drawback that the coefficients of the subsidiary summations are intractable. No doubt on the arithmometer that does not much matter, but in this, as well as in other respects, formula A being decidedly better, we can scarcely expect A^I to come into use. I have not troubled to test it in actual graduation.

Formulas B and B^I are interesting as being short, and at the same time free from fourth difference error. But their shortness, 17 terms, is against them, because in so restricted a space the error cannot be eliminated except at the expense of angularity in their curves. This is so manifest that it has not been thought worth while to prepare a diagram for them. Formula B has been tested in a graduation, and has not come through the ordeal successfully. Both these formulas may therefore be dismissed as mere scientific curiosities.

In Diagram 2 are the graphs of formulas C and C^I. They both have good curves, but on the whole that of C is the better. It will be remembered that this, before it reaches its finished state, is the formula of Dr. Karup, but, so far, it has attracted very scant attention. Probably the explanation is that the way in which he applies it is unfamiliar; but here it has been given a form which renders it extremely easy to use; and a graduation by it could be effected as rapidly and simply as by any other formula that has been shown to be efficient. Mr. Spencer has done good service in comparing graduations by it with certain others, and I do not pursue the subject further here. More exhaustive investigation is, however, desirable.

Diagram 3 gives the graphs of formulas D, D^1, and $(D)^2$. The curve of D^1 has a better appearance than that of D, but yet formula D is probably to be preferred, principally because the arithmetical work in applying it is as simple and short as could

be desired. Formula $(D)^2$ has a good curve, and gives a smooth graduation. It can be used very conveniently by applying formula D twice, and that is what I have done.

The Government Female Annuitants (1883) Ultimate Table is a very good exercise ground for graduation methods, and partly for that reason, but also so that my results may be compared with those of Mr. Spencer, I have adopted it. I follow Dr. Sprague and Mr. Spencer, but not without protest, in graduating the table as far as age 20, and I have likewise assumed the average value ·01587 for the ungraduated q for ages 20 to 26 inclusive. But this portion of the table has no meaning whatever, as will be admitted when we remember that a difference of a single unit in the recorded deaths would alter by 25 per-cent the rate of mortality at these young ages. I do not think the original facts admit of legitimate use in adjustment beyond age 30 at the farthest. At the other end of the table I have taken the rate of mortality as ·66667 at age 100, and as unity at age 101, but I have not followed Mr. Spencer in making preliminary changes at ages 94 to 98. Therefore, quite apart from the merits of the formulas themselves, Mr. Spencer's graduations should be smoother than mine from, say, 85 onwards.

Table 1 gives the graduation by formula A. We have q_x, with its third differences; and we have the expected deaths, and the deviation and the accumulated deviation. The method of completing the ends of the table must be explained. At the beginning, formula A itself carries the work to age 34. Using the graduated values from that age, and the ungraduated values at the younger ages, formula D is then applied, which completes the table to age 25. Then, using the ungraduated values of q_x at ages 20 to 24 and the graduated values at ages 25 to 27, and starting with 100,000 as a radix, we form l_x for the section from 20 to 28. This gives us four graduated values of l_x for ages 25 to 28, from which we get three graduated differences, and we form a fourth with the aid of l_{20}, by means of the formula $\Delta^4 u_0 = \frac{1}{70}(u_8 - u_0 - 8\Delta u_0 - 28\Delta^2 u_0 - 56\Delta^3 u_0)$. By these four differences we interpolate the intervening values of l_x, and hence obtain q_x. It has been found convenient to repeat this process, and, as the final result, we have the values of q_x given in Table 1.

Similarly, at the other end of table, formula A graduates as far as age 87. By formula D we go on to age 96; and, taking $q_{101} = 1$, and using the graduated values of q for ages 93 to 96,

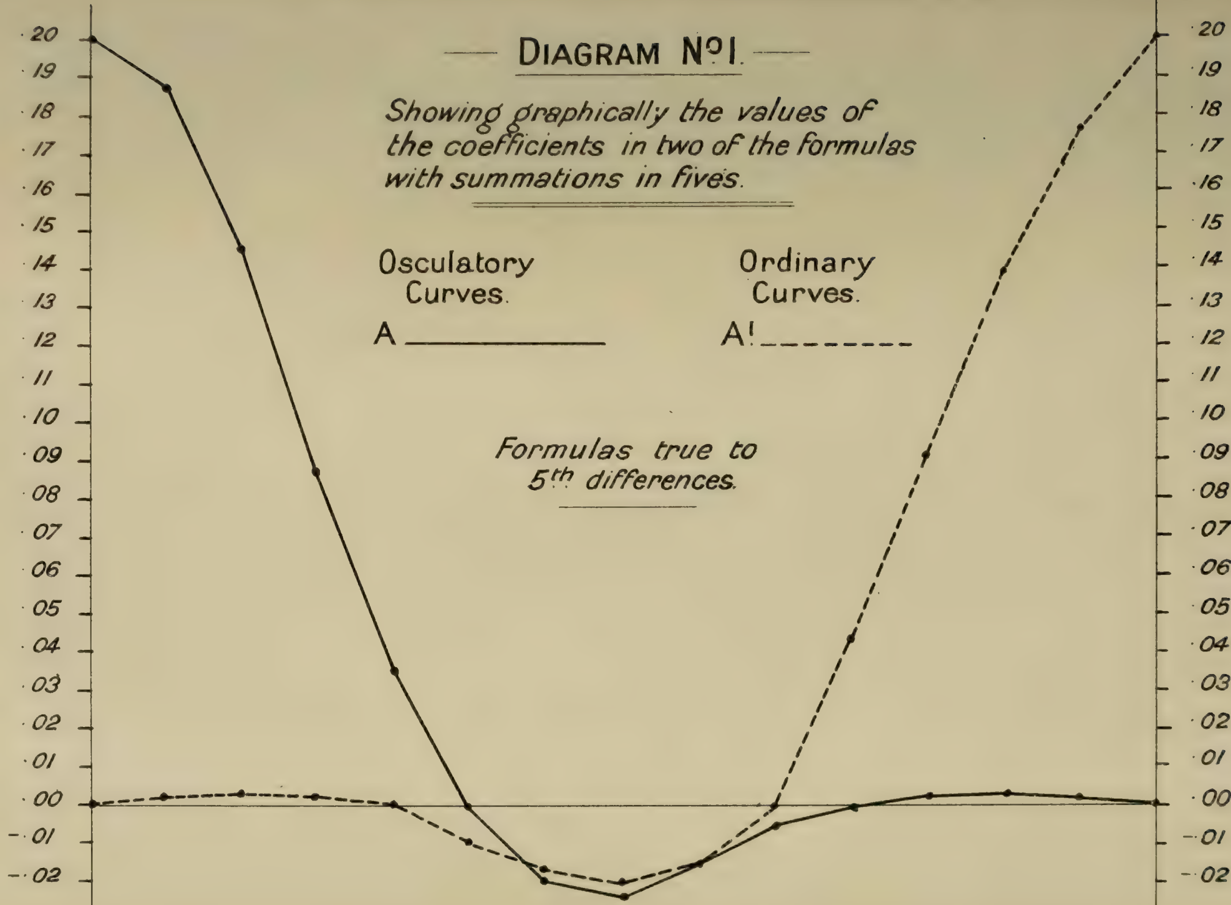

DIAGRAM No. 1.
Showing graphically the values of the coefficients in two of the formulas with summations in fives.
Osculatory Curves.
A
Ordinary Curves.
A'
Formulas true to 5th differences.
·20
·19
·18
·17
·16
·15
·14
·13
·12
·11
·10
·09
·08
·07
·06
·05
·04
·03
·02
·01
·00
-·01
-·02

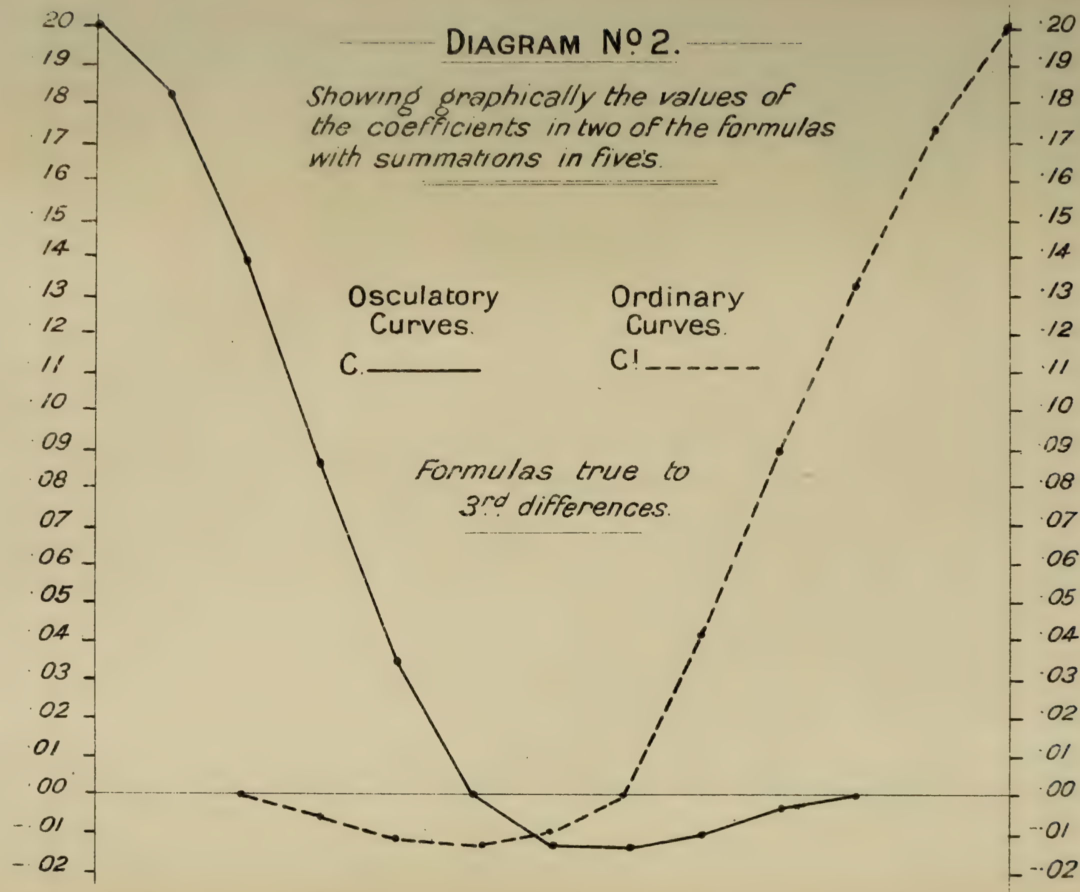
DIAGRAM No 2.
Showing graphically the values of the coefficients in two of the formulas with summations in fives.
Osculatory Curves.
C.
Ordinary Curves.
C!
Formulas true to 3rd differences.
20
19
18
17
16
15
14
13
12
11
10
09
08
07
06
05
04
03
02
01
00
-01
-02
·20
·19
·18
·17
·16
·15
·14
·13
·12
·11
·10
·09
·08
·07
·06
·05
·04
·03
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·00
-·01
-·02

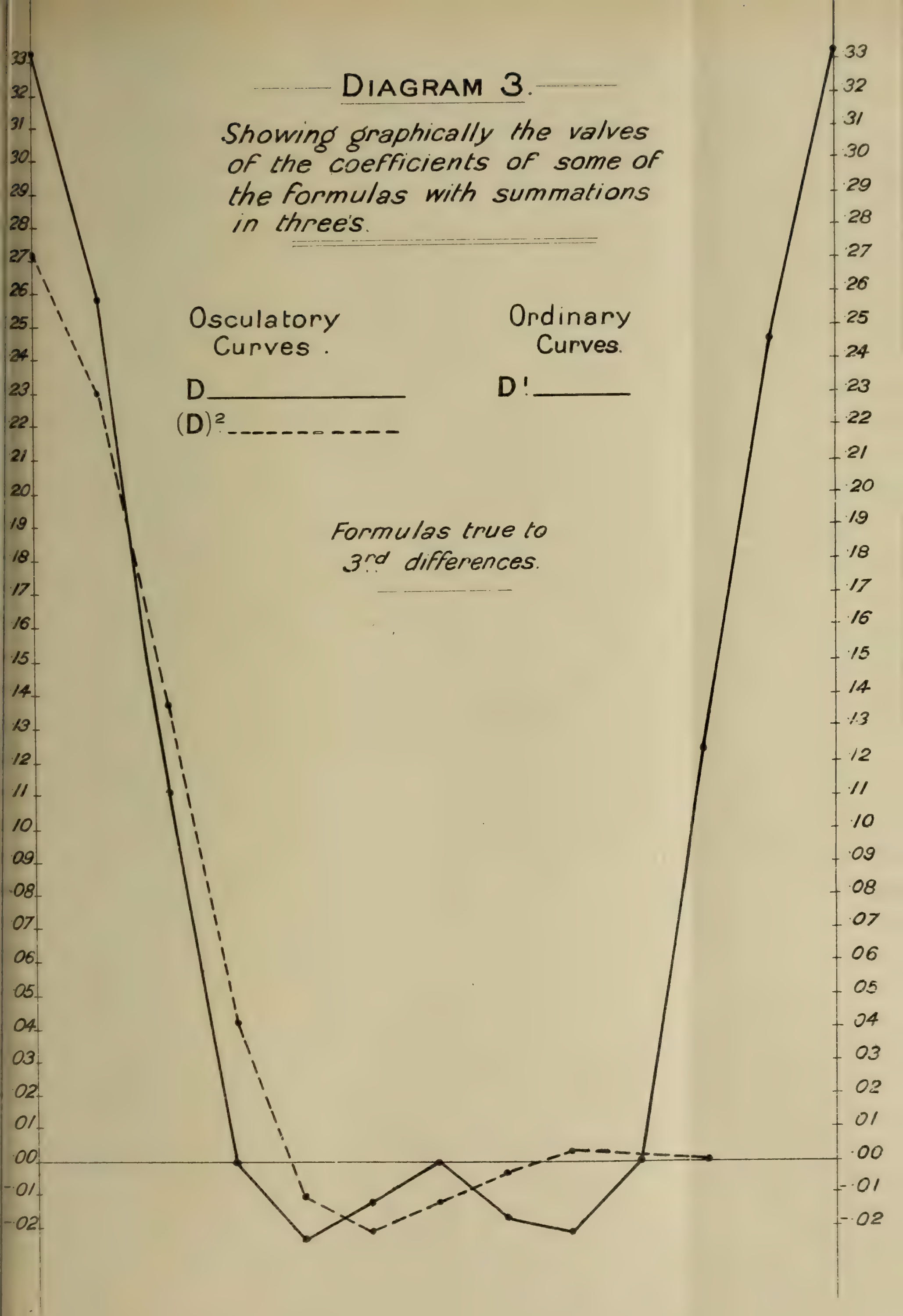
DIAGRAM 3.
Showing graphically the valves of the coefficients of some of the formulas with summations in three's.
Osculatory Curves.
D
(D)²
Ordinary Curves.
D¹
Formulas true to 3rd differences.
·33 ·32 ·31 ·30 ·29 ·28 ·27 ·26 ·25 ·24 ·23 ·22 ·21 ·20 ·19 ·18 ·17 ·16 ·15 ·14 ·13 ·12 ·11 ·10 ·09 ·08 ·07 ·06 ·05 ·04 ·03 ·02 ·01 ·00 -·01 -·02

we form a fourth difference as above, and complete the table. By this process repeated, the rates in Table 1 were obtained.

The graduation in Table 1 is free from fourth difference error, and it will be seen that in smoothness it is about the same as that by Mr. Spencer's 21-term formula. The expected deaths and the deviations show that the original facts have been adhered to closely, and that in this respect the method of adding the ends to the table is satisfactory.

Table 2 gives the graduation of Table 1, with a graduation by formula D superimposed. I venture to submit that the results are very good indeed, and that the graduation is smoother than any that has yet been produced. The use of formula D introduces a fourth difference error, but only to the extent of once the fourth differential coefficient, which is quite inappreciable. The deviation of the expected deaths from the actual is very small, the accumulated deviation amounting to only 2·8 deaths out of a total of 12,731; and at no point is the accumulated deviation of any magnitude.

Table 3 gives the graduation by formula D, and Table 4 that by $(D)^2$, or, what is the same thing, that produced by a double application of formula D.

The third differences of q_x' in Table 3 are not small, but when set out in a curve they run more or less regularly, and not with very marked angles, except in one or two places. The graduation therefore is fairly smooth, but the formula is too short to remove the longer waves which are presented by the unadjusted data. For many ordinary purposes a single graduation by formula D might be sufficient, and only a minimum amount of labour is involved.

In Table 4 we have a better graduation, in fact one as good as need be desired for ordinary purposes. Although the formula is applied twice, there is but little work. All the summations are in threes, and can be performed without mental effort, and with small risk of mistake; and 3 is the only coefficient of the subsidiary summations.

POSTSCRIPT.

Since the foregoing paper was set up in type, the following abbreviations of the three principal formulas have been arrived at, which render these formulas easy of application. It has been pointed out to me that as regards formulas A and C, Dr. Karup gives these abbreviations in much the same form but in less familiar notation.

Formula A. $5^5(u) = [5]^5\{2u_0 + 3[3]u_0 - 6\gamma_2 + \gamma_4\}$

Formula C. $5^4(u) = [5]^3\{3[3]u_0 - 2\gamma_3\}$

Formula D. $3^4(u) = [3]^3\{3u_0 + \gamma_1 - \gamma_2\}$

TABLE 1.

Government Female Annuitants (1883) *Ultimate Table.*
Graduated by Formula A.

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
20	·01440	+ 19	·2	+ ·2	+ ·2	20
1	·01585	+ 19	·3	+ ·3	+ ·5	1
2	·01649	+ 24	·4	+ ·4	+ ·9	2
3	·01651	+ 20	·5	+ ·5	+ 1·4	3
4	·01610	+ 26	·6	+ ·6	+ 2·0	4
25	·01550	− 48	·8	− 1·2	+ ·8	25
6	·01491	− 45	1·0	− 1·0	− ·2	6
7	·01459	+ 4	1·2	+ ·2	...	7
8	·01406	+ 76	1·4	+ ·4	+ ·4	8
9	·01287	+ 84	1·7	− 1·3	− ·9	9
30	·01106	+ 18	1·8	− ·2	− 1·1	30
1	·00939	− 59	1·8	+ 1·8	+ ·7	1
2	·00870	− 75	1·9	− ·1	+ ·6	2
3	·00917	− 25	2·3	− ·7	− ·1	3
4	·01021	+ 14	2·9	− ·1	− ·2	4
35	·01107	+ 26	3·5	− 1·5	− 1·7	35
6	·01150	+ 12	4·1	− ·9	− 2·6	6
7	·01164	− 4	4·8	+ 1·8	− ·8	7
8	·01175	− 6	5·6	− ·4	− 1·2	8
9	·01195	− 10	6·8	+ 4·8	+ 3·6	9
40	·01220	− 9	8·0	− 5·0	− 1·4	40
1	·01244	− 8	9·1	+ ·1	− 1·3	1
2	·01257	+ 12	10·5	− 1·5	− 2·8	2
3	·01250	+ 17	11·8	+ 2·8	...	3
4	·01215	+ 26	13·3	+ 1·3	+ 1·3	4
45	·01164	+ 8	14·6	− 2·4	− 1·1	45
6	·01114	+ 10	15·8	− 5·2	− 6·3	6
7	·01091	− 1	17·2	+ 4·2	− 2·1	7
8	·01103	− 20	19·4	+ 5·4	+ 3·3	8
9	·01160	− 24	22·7	− 3·3	...	9
50	·01261	− 6	27·2	+ 5·2	+ 5·2	50
1	·01386	− 9	33·1	− 3·9	+ 1·3	1
2	·01511	− 7	40·3	− 2·7	− 1·4	2
3	·01630	+ 6	47·4	− 3·6	− 5·0	3
4	·01734	+ 16	57·7	− 1·3	− 6·3	4
55	·01816	+ 2	66·9	+ 9·9	+ 3·6	55
6	·01882	+ 4	77·2	− 5·8	− 2·2	6
7	·01948	+ 17	87·1	+ 1·1	− 1·1	7
8	·02016	+ 11	98·5	+ 8·5	+ 7·4	8
9	·02090	+ 1	110·4	−12·6	− 5·2	9
60	·02187	+ 4	123·4	−14·6	−19·8	60
1	·02318	− 2	140·0	+24·0	+ 4·2	1
2	·02484	− 17	159·5	+ 2·5	+ 6·7	2
3	·02689	− 17	181·8	− ·2	+ 6·5	3
4	·02931	− 3	212·4	+ 3·4	+ 9·9	4
65	·03193	+ 12	242·6	−15·4	− 5·5	65
6	·03458	+ 22	271·9	+17·9	+12·4	6
7	·03723	+ 25	300·1	−16·9	− 4·5	7
8	·04000	+ 25	327·9	−25·1	−29·6	8
9	·04311	+ 7	358·1	+15·1	−14·5	9
70	·04681	− 14	391·9	− 4·1	−18·6	70
1	·05135	− 21	425·8	+24·8	+ 6·2	1
2	·05680	− 23	463·3	+31·3	+37·5	2
3	·06302	− 7	498·2	−19·8	+17·7	3
4	·06980	− 2	529·6	− 2·4	+15·3	4

TABLE 1—*continued.*

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
75	·07691	+ 14	554·8	−37·2	−21·9	75
6	·08428	+ 14	565·8	− 1·2	−23·1	6
7	·09189	+ 38	570·2	+22·2	− ·9	7
8	·09988	+ 14	569·7	+ 4·7	+ 3·8	8
9	·10839	+ 26	563·2	+ 1·2	+ 5·0	9
80	·11780	− 5	549·4	+ ·4	+ 5·4	80
1	·12825	− 43	531·2	−26·8	−21·4	1
2	·14000	− 38	500·6	+21·6	+ ·2	2
3	·15300	+ 33	469·7	−11·3	−11·1	3
4	·16682	+ 89	429·6	+30·6	+19·5	4
85	·18108	+ 37	388·6	−15·4	+ 4·1	85
6	·19611	−103	334·6	− 7·4	− 3·3	6
7	·21280	−210	283·4	−15·6	−18·9	7
8	·23152	−173	235·7	+25·7	+ 6·8	8
9	·25124	− 34	198·0	−10·0	− 3·2	9
90	·26986	+167	153·6	− 6·4	− 9·6	90
1	·28565	+417	111·7	− 3·3	−12·9	1
2	·29827	+676	79·0	+ 7·0	− 5·9	2
3	·30939	+805	58·5	− 4·5	−10·4	3
4	·32318	+613	39·1	+ ·1	−10·3	4
95	·34640	+420	27·0	− 4·0	−14·3	95
6	·38710	+229	18·2	+ 6·2	− 8·1	6
7	·45141	+ 36	15·3	− 2·7	−10·8	7
8	·54353	−157	8·7	− ·3	−11·1	8
9	·66575	...	4·7	+ ·7	−10·4	9
100	·81843	...	2·4	+ 2·4	− 8·0	100
1	1·00000	...	3·0	+ 1·0	− 7·0	1

TABLE 2.

Government Female Annuitants (1883) *Ultimate Table.*
Graduated by Formula A, *and then by* D.

Age	q_x	$\Delta^3 q^x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
20	·01440	+ 19	·2	+ ·2	+ ·2	20
1	·01585	+ 19	·3	+ ·3	+ ·5	1
2	·01649	+ 24	·4	+ ·4	+ ·9	2
3	·01651	+ 20	·5	+ ·5	+ 1·4	3
4	·01610	+ 26	·6	+ ·6	+ 2·0	4
25	·01550	− 48	·8	− 1·2	+ ·8	25
6	·01491	− 45	1·0	− 1·0	− ·2	6
7	·01459	+ 5	1·2	+ ·2	...	7
8	·01406	+ 74	1·4	+ ·4	+ ·4	8
9	·01287	+ 83	1·7	− 1·3	− ·9	9
30	·01107	+ 16	1·8	− ·2	− 1·1	30
1	·00940	− 57	1·8	+ 1·8	+ ·7	1
2	·00869	− 68	1·9	− ·1	+ ·6	2
3	·00910	− 23	2·3	− ·7	− ·1	3
4	·01006	+ 11	2·8	− ·2	− ·3	4
35	·01089	+ 20	3·5	− 1·5	− 1·8	35
6	·01136	+ 9	4·1	− ·9	− 2·7	6
7	·01158	0	4·8	+ 1.8	− ·9	7
8	·01175	− 8	5·6	− ·4	− 1·3	8
9	·01196	− 7	6·8	+ 4·8	+ 3·5	9
40	·01221	− 10	8·0	− 5·0	− 1·5	40
1	·01242	− 1	9·1	+ ·1	− 1·4	1
2	·01252	+ 9	10·4	− 1·6	− 3·0	2
3	·01241	+ 16	11·7	+ 2·7	− ·3	3
4	·01208	+ 18	13·3	+ 1·3	+ 1·0	4

TABLE 2—*continued.*

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
45	·01162	+ 18	14·5	− 2·5	− 1·5	45
6	·01119	+ 5	15·8	− 5·2	− 6·7	6
7	·01097	− 4	17·3	+ 4·3	− 2·4	7
8	·01112	− 11	19·6	+ 5·6	+ 3·2	8
9	·01169	− 16	22·9	− 3·1	+ ·1	9
50	01264	− 14	27·3	+ 5·3	+ 5·4	50
1	·01382	− 9	33·0	− 4·0	+ 1·4	1
2	·01507	− 1	40·2	− 2·8	− 1·4	2
3	·01625	+ 3	47·3	− 3·7	− 5·1	3
4	·01727	+ 7	57·5	− 1·5	− 6·6	4
55	·01812	+ 10	66·7	+ 9·7	+ 3·1	55
6	·01883	+ 8	77·3	− 5·7	− 2·6	6
7	·01947	+ 10	87·1	+ 1·1	− 1·5	7
8	·02014	+ 11	98·4	+ 8·4	+ 6·9	8
9	·02092	+ 5	110 5	−12·5	− 5·6	9
60	·02191	0	123·7	−14·3	−19·9	60
1	·02322	− 7	140·2	+24·2	+ 4·3	1
2	·02490	− 10	159·9	+ 2·9	+ 7·2	2
3	·02695	− 13	182·2	+ ·2	+ 7·4	3
4	·02930	− 2	212·3	+ 3·3	+10·7	4
65	·03185	+ 11	242·0	−16·0	− 5·3	65
6	·03447	+ 29	271·0	+17·0	+11·7	6
7	·03714	+ 25	299·4	−17·6	− 5·9	7
8	·03997	+ 18	327·6	−25·4	−31·3	8
9	·04315	+ 5	358·4	+15·4	−15·9	9
70	·04693	− 8	392·9	− 3·1	−19·0	70
1	·05149	− 18	427·0	+26·0	+ 7·0	1
2	·05688	− 19	463·9	+31·9	+38·9	2
3	·06302	− 11	498·2	−19·8	+19·1	3
4	·06973	+ 1	529·1	− 2·9	+16·2	4
75	·07682	+ 10	554·2	−37·8	−21·6	75
6	·08418	+ 20	565·1	− 1·9	−23·5	6
7	·09182	+ 19	569·7	+21·7	− 1·8	7
8	·09984	+ 18	560·5	+ 4·5	+ 2·7	8
9	·10844	+ 20	563·5	+ 1·5	+ 4·2	9
80	·11781	+ 26	549·5	+ ·5	+ 4·7	80
1	·12813	+ 9	530·7	−27·3	−22·6	1
2	·13960	− 24	499·2	+20·2	− 2·4	2
3	·15248	− 55	468·1	−12·9	−15·3	3
4	16[illegible]86	− 11	429·7	+30·7	+15·4	4
85	·18250	+ 18	391·6	−12·4	+ 3·0	85
6	·19885	− 12	339·2	− 2·8	+ ·2	6
7	·21580	−111	287·4	−11·6	−11·4	7
8	·23353	−170	237·7	+27·7	+16·3	8
9	·25192	−102	198·5	− 9·5	+ 6·8	9
90	·26986	+167	153·5	− 6·5	+ ·3	90
1	·28565	+417	111·6	− 3·4	− 3·1	1
2	·29827	+676	79·0	+ 7·0	+ 3·9	2
3	·30939	+805	58·6	− 4·4	− ·5	3
4	·32318	+613	39·1	+ ·1	− ·4	4
95	·34640	+420	26·9	− 4·1	− 4·5	95
6	·38710	+229	18·2	+ 6·2	+ 1·7	6
7	·45141	+ 36	15·3	− 2·7	− 1·0	7
8	·54353	−157	8·7	− ·3	− 1·3	8
9	·66575	...	4·7	+ ·7	− ·6	9
100	·81843	...	2·4	+ 2·4	+ 1·8	100
1	1·00000	...	3.0	+ 1·0	+ 2·8	1

TABLE 3.

Government Female Annuitants (1883) *Ultimate Table.*
Graduated by Formula D.

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
20	·01314	+ 46	·2	+ ·2	+ ·2	20
1	·01587	+ 40	·3	+ ·3	+ ·5	1
2	·01701	+ 43	·4	+ ·4	+ ·9	2
3	·01702	+ 49	·5	+ ·5	+ 1·4	3
4	·01630	+ 47	·6	+ ·6	+ 2·0	4
25	·01528	− 20	·8	− 1·2	+ ·8	25
6	·01445	− 154	1·0	− 1·0	− ·2	6
7	·01428	− 76	1·2	+ ·2	...	7
8	·01457	+ 169	1·4	+ ·4	+ ·4	8
9	·01378	+ 241	1·8	− 1·2	− ·8	9
30	·01115	+ 26	1·8	− ·2	− 1·0	30
1	·00837	− 200	1·6	+ 1·6	+ ·6	1
2	·00785	− 215	1·8	− ·2	+ ·4	2
3	·00985	− 22	2·4	− ·6	− ·2	3
4	·01237	+ 104	3·5	+ ·5	+ ·3	4
35	·01326	+ 192	4·2	− ·8	− ·5	35
6	·01230	+ 57	4·4	− ·6	− 1·1	6
7	·01053	− 83	4·4	+ 1·4	+ ·3	7
8	·00987	− 191	4·7	− 1·3	− 1·0	8
9	·01089	− 30	6·2	+ 4·2	+ 3·2	9
40	·01276	+ 98	8·3	− 4·7	− 1·5	40
1	·01357	+ 122	9·9	+ ·9	− ·6	1
2	·01302	− 57	10·8	− 1·2	− 1·8	2
3	·01209	− 88	11·4	+ 2·4	+ ·6	3
4	·01200	− 7	13·2	+ 1·2	+ 1·8	4
45	·01218	+ 116	15·2	− 1·8	...	45
6	·01175	+ 79	16·6	− 4·4	− 4·4	6
7	·01064	− 19	16·8	+ 3·8	− ·6	7
8	·01001	− 78	17·6	+ 3·6	+ 3·0	8
9	·01065	− 71	20·9	− 5·1	− 2·1	9
50	·01237	− 27	26·7	+ 4·7	+ 2·6	50
1	·01439	+ 16	34·3	− 2·7	− ·1	1
2	·01600	+ 47	42·7	− ·3	− ·4	2
3	·01693	+ 31	49·2	− 1·8	− 2·2	3
4	·01734	+ 13	57·7	− 1·3	− 3·5	4
55	·01770	− 19	65·2	+ 8·2	+ 4·7	55
6	·01832	− 42	75·2	− 7·8	− 3·1	6
7	·01933	− 19	86·5	+ ·5	− 2·6	7
8	·02054	+ 42	100·3	+ 10·3	+ 7·7	8
9	·02153	+ 76	113·7	− 9·3	− 1·6	9
60	·02211	+ 20	124·8	− 13·2	− 14·8	60
1	·02270	− 45	137·1	+ 21·1	+ 6·3	1
2	·02406	− 46	154·5	− 2·5	+ 3·8	2
3	·02639	+ 3	178·4	− 3·6	+ ·2	3
4	·02924	− 5	211·9	+ 2·9	+ 3·1	4
65	·03215	− 33	244·3	− 13·7	− 10·6	65
6	·03515	− 14	276·4	+ 22·4	+ 11·8	6
7	·03819	+ 57	307·8	− 9·2	+ 2·6	7
8	·04094	+ 123	335·6	− 17·4	− 14·8	8
9	·04326	+ 75	359·4	+ 16·4	+ 1·6	9
70	·04572	− 28	382·8	− 13·2	− 11·6	70
1	·04955	− 147	410·9	+ 9·9	− 1·7	1
2	·05550	− 136	452·7	+ 20·7	+ 19·0	2
3	·06329	− 15	500·3	− 17·7	+ 1·3	3
4	·07145	+ 136	542·2	+ 10·2	+ 11·5	4

TABLE 3—*continued.*

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
75	·07862	+ 127	567·2	−24·8	−13·3	75
6	·08465	+ 49	568·3	+ 1·3	−12·0	6
7	·09090	− 121	564·0	+16·0	+ 4·0	7
8	·09864	− 88	562·6	− 2·4	+ 1·6	8
9	·10836	− 26	563·0	+ 1·0	+ 2·6	9
80	·11885	+ 215	554·3	+ 5·3	+ 7·9	80
1	·12923	+ 104	535·3	−22·7	−14·8	1
2	·13924	− 59	497·9	+18·9	+ 4·1	2
3	·15103	− 313	463·7	−17·3	−13·2	3
4	·16564	+ 93	426·5	+27·5	+14·3	4
85	·18248	+ 194	391·6	−12·4	+ 1·9	85
6	·19842	+ 98	338·5	− 3·5	− 1·6	6
7	·21439	− 567	285·6	−13·4	−15·0	7
8	·23233	− 7	236·5	+26·5	+11·5	8
9	·25322	+ 23	199·5	− 8·5	+ 3·0	9
90	·27139	+ 302	154·4	− 5·6	− 2·6	90
1	·28677	− 230	112·1	− 2·9	− 5·5	1
2	·29959	+1111	79·4	+ 7·4	+ 1·9	2
3	·31287	+ 959	59·1	− 3·9	− 2·0	3
4	·32431	+ 672	39·2	+ ·2	− 1·8	4
95	·34502	+ 386	26·9	− 4·1	− 5·9	95
6	·38457	+ 101	18·1	+ 6·1	+ ·2	6
7	·44968	− 185	15·3	− 2·7	− 2·5	7
8	·54421	− 483	8·7	− ·3	− 2·8	8
9	·66917	...	4·7	+ ·7	− 2·1	9
100	·82271	...	2·5	+ 2·5	+ ·4	100
1	1·00000	...	3·0	+ 1·0	+ 1·4	1

TABLE 4.

Government Female Annuitants (1883) *Ultimate Table.*
Graduated twice by Formula D, *i.e., by Formula* $(D)^2$.

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
20	·01444	+ 17	·2	+ ·2	+ ·2	20
1	·01583	+ 17	·3	+ ·3	+ ·5	1
2	·01646	+ 19	·4	+ ·4	+ ·9	2
3	·01650	+ 25	·5	+ ·5	+ 1·4	3
4	·01612	+ 23	·6	+ ·6	+ 2·0	4
25	·01551	− 44	·8	− 1·2	+ ·8	25
6	·01492	− 53	1·0	− 1·0	− ·2	6
7	·01458	+ 9	1·2	+ ·2	...	7
8	·01405	+ 92	1·4	+ ·4	+ ·4	8
9	·01280	+ 95	1·7	− 1·3	− ·9	9
30	·01092	+ 16	1·8	− ·2	− 1·1	30
1	·00933	− 94	1·8	+ 1·8	+ ·7	1
2	·00898	−116	1·9	− ·1	+ ·6	2
3	·01003	− 43	2·5	− ·5	+ ·1	3
4	·01154	+ 61	3·2	+ ·2	+ ·3	4
35	·01235	+ 97	4·0	− 1·0	− ·7	35
6	·01203	+ 43	4·3	− ·7	− 1·4	6
7	·01119	− 42	4·7	+ 1·7	+ ·3	7
8	·01080	− 75	5·1	− ·9	− ·6	8
9	·01129	− 32	6·4	+ 4·4	+ 3·8	9
40	·01224	+ 21	8·0	− 5·0	− 1·2	40
1	·01290	+ 36	9·5	+ ·5	− ·7	1
2	·01295	+ 5	10·8	− 1·2	− 1·9	2
3	·01260	− 15	11·9	+ 2·9	+ 1·0	3
4	·01221	+ 6	13·4	+ 1·4	+ 2·4	4

TABLE 4—*continued.*

Age	q_x	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation	Accumulated Deviation	Age
45	·01183	+ 35	14·8	− 2·2	+ ·2	45
6	·01131	+ 73	16·1	− 4·9	− 4·7	6
7	·01071	− 31	16·9	+ 3·9	− ·8	7
8	·01038	− 35	18·2	+ 4·2	+ 3·4	8
9	·01105	− 54	21·6	− 4·4	− 1·0	9
50	·01241	− 26	26·8	+ 4·8	+ 3·8	50
1	·01411	+ 6	33·7	− 3·3	+ ·5	1
2	·01561	+ 29	41·7	− 1·3	− ·8	2
3	·01665	+ 26	48·5	− 2·5	− 3·3	3
4	·01729	+ 7	57·6	− 1·4	− 4·7	4
55	·01782	− 14	65·6	+ 8·6	+ 3·9	55
6	·01850	− 20	75·9	− 7·1	− 3·2	6
7	·01940	+ 2	86·9	+ ·9	− 2·3	7
8	·02038	+ 27	99·8	+ 9·8	+ 7·5	8
9	·02124	+ 34	112·3	−10·7	− 3·2	9
60	·02200	+ 16	124·2	−13·8	−17·0	60
1	·02293	− 11	138·6	+22·6	+ 5·6	1
2	·02437	− 26	156·5	− ·5	+ 5·1	2
3	·02648	− 25	179·1	− 2·9	+ 2·2	3
4	·02915	− 23	211·3	+ 2·3	+ 4·5	4
65	·03212	− 9	244·2	−13·8	− 9·3	65
6	·03514	+ 19	276·3	+22·3	+13·0	6
7	·03798	+ 57	306·2	−10·8	+ 2·2	7
8	·04055	+ 74	332·7	−20·3	−18·1	8
9	·04304	+ 41	357·6	+14·6	− 3·5	9
70	·04602	− 36	385·3	−10·7	−14·2	70
1	·05023	− 88	416·5	+15·5	+ 1·3	1
2	·05608	− 79	457·4	+25·4	+26·7	2
3	·06321	− 8	499·7	−18·3	+ 8·4	3
4	·07074	+ 69	537·0	+ 5·0	+13·4	4
75	·07788	+ 75	561·9	−30·1	−16·7	75
6	·08455	+ 24	567·7	+ ·7	−16·0	6
7	·09144	− 41	567·7	+19·7	+ 3·7	7
8	·09930	− 34	566·6	+ 1·6	+ 5·3	8
9	·10837	+ 22	563·2	+ 1·2	+ 6·5	9
80	·11824	+ 69	551·5	+ 2·5	+ 9·0	80
1	·12857	+ 35	532·6	−25·4	−16·4	1
2	·13958	− 33	499·1	+20·1	+ 3·7	2
3	·15196	− 45	466·5	−14·5	−10·8	3
4	·16606	− 3	427·6	+28·6	+17·8	4
85	·18155	+ 9	389·6	−14·4	+ 3·4	85
6	·19798	− 59	337·8	− 4·2	− ·8	6
7	·21532	−143	286·8	−12·2	−13·0	7
8	·23366	−134	237·9	+27·9	+14·9	8
9	·25241	− 44	198·9	− 9·1	+ 5·8	9
90	·27014	+145	153·7	− 6·3	− ·5	90
1	·28551	+399	111·6	− 3·4	− 3·9	1
2	·29808	+675	79·0	+ 7·0	+ 3·1	2
3	·30930	+808	58·5	− 4·5	− 1·4	3
4	·32316	+614	39·1	+ ·1	− 1·3	4
95	·34641	+420	27·0	− 4·0	− 5·3	95
6	·38713	+226	18·2	+ 6·2	+ ·9	6
7	·45146	+ 32	15·4	− 2·6	− 1·7	7
8	·54360	−140	8·7	− ·3	− 2·0	8
9	·66581	...	4·7	+ ·7	− 1·3	9
100	·81841	...	2·5	+ 2·5	+ 1·2	100
1	1·00000	...	3·0	+ 1·0	+ 2·2	1

ACTUARIAL NOTE.

On the application of the Law of Uniform Seniority to any Select Table graduated by Mr. G. F. Hardy's adaptation of Makeham's formula, so that the value of an annuity on two or more joint lives, commencing at any time during the period of selection, may be replaced by the value of an annuity on a single life, with payments varying so long as the effect of selection is operative, and at a different rate of interest. By Gilbert Goodman, A.I.A.

THE following formula appears on p. 158 of the "Account of the Principles and Methods adopted in the construction and graduation of the British Offices Life Tables":

$$\log_{10} l_{[x]+t} = \log_{10} l_{x+t} - f_{(t)} - \beta c^x \psi_{(t)}$$

for all values of t from 0 to 9 (where $l_{x+t} = \kappa s^{x+t} g^{c^{x+t}}$).

Hence

$$\log_{10} l_{[x]+t+n} = \log_{10} l_{x+t+n} - f_{(t+n)} - \beta c^x \psi_{(t+n)}$$

$$\therefore \quad \log_{10} {}_n p_{[x]+t} = \log_{10} {}_n p_{x+t} + (f_{(t)} - f_{(t+n)}) + \beta c^x (\psi_{(t)} - \psi_{(t+n)})$$

and $${}_n p_{[x]+t} = {}_n p_{x+t} \times 10^{\{(f_{(t)} - f_{(t+n)}) + \beta c^x (\psi_{(t)} - \psi_{(t+n)})\}}$$

whence $$a_{[x]+t} = \sum_{n=1}^{n=9-t} \{v^n {}_n p_{x+t} \times 10^{\{(f_{(t)} - f_{(t+n)}) + \beta c^x (\psi_{(t)} - \psi_{(t+n)})\}}\}$$
$$+ \sum_{n=10-t}^{n=\infty} \{v^n {}_n p_{x+t} \times 10^{\{f_{(t)} + \beta c^x \psi_{(t)}\}}\}$$

or, expressed in terms of Makeham's constants,

$$= \sum_{n=1}^{n=9-t} \{v^n s^n g^{c^{x+t}(c^n-1)} \times 10^{\{(f_{(t)} - f_{(t+n)}) + \beta c^x (\psi_{(t)} - \psi_{(t+n)})\}}\}$$
$$+ \sum_{n=10-t}^{n=\infty} \{v^n s^n g^{c^{x+t}(c^n-1)} \times 10^{\{f_{(t)} + \beta c^x \psi_{(t)}\}}\}$$

Taking now r lives, aged $[x]+t$, $[y]+t$, $[z]+t$, &c., we have

$$\log_{10} {}_n p_{[x]+t} + \log_{10} {}_n p_{[y]+t} + \log_{10} {}_n p_{[z]+t} + \ldots \text{ (to } r \text{ terms)}$$
$$= \log_{10} {}_n p_{x+t} + \log_{10} {}_n p_{y+t} + \log_{10} {}_n p_{z+t} + \ldots \text{ (to } r \text{ terms)}$$
$$+ r(f_{(t)} - f_{(t+n)}) + \beta(c^x + c^y + c^z + \ldots \text{ to } r \text{ terms})(\psi_{(t)} - \psi_{(t+n)})$$

and, therefore,

$$ {}_{n}p_{[x]+t:[y]+t:[z]+t\ldots(r)} = {}_{n}p_{x+t:y+t:z+t\ldots(r)} \times 10^{\{r(f_{(t)}-f_{(t+n)})+\beta(c^x+c^y+c^z+\ldots\text{ to } r \text{ terms})(\psi_{(t)}-\psi_{(t+n)})\}} $$

and

$$ a_{[x]+t:[y]+t:[z]+t\ldots(r)} = \sum_{n=1}^{n=9-t} \{v^n s^{rn} g^{(c^x+c^y+c^z+\ldots\text{ to } r \text{ terms})c^t(c^n-1)} \times 10^{\{r(f_{(t)}-f_{(t+n)})+\beta(c^x+c^y+c^z+\ldots\text{ to } r \text{ terms})(\psi_{(t)}-\psi_{(t+n)})\}}\} $$
$$ + \sum_{n=10-t}^{n=\infty} \{v^n s^{rn} g^{(c^x+c^y+c^z+\ldots\text{ to } r \text{ terms})c^t(c^n-1)} \times 10^{\{r.f_{(t)}+\beta(c^x+c^y+c^z+\ldots\text{ to } r \text{ terms})\psi_{(t)}\}}\} $$

Let now $(c^x+c^y+c^z+\ldots \text{ to } r \text{ terms}) = c^w$, then

$$ a_{[x]+t:[y]+t:[z]+t\ldots(r)} = \sum_{n=1}^{n=9-t} \{v^n s^{rn} g^{c^{w+t}(c^n-1)} \times 10^{\{r(f_{(t)}-f_{(t+n)})+\beta c^w(\psi_{(t)}-\psi_{(t+n)})\}}\} $$
$$ + \sum_{n=10-t}^{n=\infty} \{v^n s^{rn} g^{c^{w+t}(c^n-1)} \times 10^{\{rf_{(t)}+\beta c^w\psi_{(t)}\}}\} $$
$$ = \sum_{n=1}^{n=9-t} \{(vs^{r-1})^n s^n g^{c^{w+t}(c^n-1)} \times 10^{\{(f_{(t)}-f_{(t+n)})+\beta c^w(\psi_{(t)}-\psi_{(t+n)})\}} \times 10^{(r-1)(f_{(t)}-f_{(t+n)})}\} $$
$$ + \sum_{n=10-t}^{n=\infty} \{(vs^{r-1})^n s^n g^{c^{w+t}(c^n-1)} \times 10^{\{f_{(t)}+\beta c^w\psi_{(t)}\}} \times 10^{(r-1)f_{(t)}}\} $$

But
$$ a_{[w]+t} = \sum_{n=1}^{n=9-t} \{v^n s^n g^{c^{w+t}(c^n-1)} \times 10^{\{(f_{(t)}-f_{(t+n)})+\beta c^w(\psi_{(t)}-\psi_{(t+n)})\}}\} + \sum_{n=10-t}^{n=\infty} \{v^n s^n g^{c^{w+t}(c^n-1)} \times 10^{(f_{(t)}+\beta c^w\psi_{(t)})}\} $$

therefore

$$ a_{[x]+t:[y]+t:[z]+t\ldots(r)} = (\mathbf{V}a')_{[w]+t:\overline{9-t}|} + 10^{(r-1)f_{(t)}}\,{}_{9-t|}a'_{[w]+} $$
$$ = (\mathbf{V}_{\overline{10-t|}}\,a')_{[w]+t} $$

computed at rate i', where

$$ (1+i')^{-1} = vs^{r-1}, \text{ or } i' = (1+i)s^{1-r}-1; $$

and the successive terms of the annuity are $10^{(r-1)(f_{(t)}-f_{(t+1)})}$, $10^{(r-1)(f_{(t)}-f_{(t+2)})}$, &c., &c., up to $10^{(r-1)(f_{(t)}-f_{(9)})}$, and subsequently $10^{(r-1)f_{(t)}}$ for the remainder of life.

The above demonstration is independent of the form of the functions $f_{(t)}$ and $\psi_{(t)}$; the values adopted by Mr. G. F. Hardy in the graduation of the $O^{[M]}$ Table were, however,

$$ f_{(t)} = m(10-t)^2 + m'(c')^t $$

and
$$ \psi_{(t)} = n(10-t)^2, $$

where c', m, m', and n were taken equal to ·24, ·000040955, ·00112, and ·02386 respectively.

The following tables show the successive payments of single-life varying annuities, equivalent to uniform annuities on two, three, and four joint lives respectively, according to the $O^{[M]}$ Table; also the special rates of interest corresponding to certain specified normal rates, in respect of two, three, and four joint lives. These special rates of interest apply to any Select Table graduated by a method similar to that adopted with the $O^{[M]}$ Table, *provided that the same value of the Makeham constant s be employed in the graduation of such Table.*

Two Joint Lives

Period elapsed from date of selection $(t+n)$	Amount of successive Annuity Payments, in $(\mathrm{v}_{\overline{10-t}\rvert}a')_{[w]+t}$ ($O^{[M]}$ Table)									
	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$	$t=9$
1	1·00376	...	...	...	...	...	...	...	...	...
2	1·00584	1·00208	...	...		...	...	...	...	...
3	1·00738	1·00361	1·00153	...		...	...	...	...	...
4	1·00864	1·00487	1·00278	1·00125	...	...	...	...	...	...
5	1·00970	1·00592	1·00383	1·00230	1·00105	...	...	...	...	...
6	1·01056	1·00677	1·00469	1·00315	1·00190	1·00085	...	...	...	...
7	1·01122	1·00744	1·00535	1·00382	1·00256	1·00151	1·00066	...	...	...
8	1·01170	1·00791	1·00582	1·00429	1·00303	1·00198	1·00113	1·00047	...	...
9	1·01199	1·00820	1·00611	1·00457	1·00331	1·00227	1·00142	1·00076	1·00028	...
10 and upwards	1·01208	1·00829	1·00620	1·00467	1·00341	1·00236	1·00151	1·00085	1·00038	1·00009

Three Joint Lives

Period elapsed from date of selection $(t+n)$	Amount of successive Annuity Payments, in $(\mathrm{v}_{\overline{10-t}\rvert}a')_{[w]+t}$ ($O^{[M]}$ Table)									
	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$	$t=9$
1	1·00753	...	...	...	...	...	...	...	...	...
2	1·01172	1·00416	...	...	...	...	...	...	...	...
3	1·01481	1·00723	1·00306	...	...	...	...	...	...	...
4	1·01736	1·00976	1·00558	1·00251	...	...	...	...	...	...
5	1·01949	1·01187	1·00768	1·00461	1·00209	...	...	...	...	...
6	1·02122	1·01359	1·00939	1·00631	1·00380	1·00170	...	...	...	...
7	1·02257	1·01493	1·01073	1·00765	1·00512	1·00303	1·00132	...	...	...
8	1·02354	1·01589	1·01168	1·00860	1·00607	1·00397	1·00227	1·00094	...	...
9	1·02412	1·01646	1·01225	1·00917	1·00664	1·00454	1·00283	1·00151	1·00057	...
10 and upwards	1·02431	1·01665	1·01245	1·00936	1·00683	1·00473	1·00302	1·00170	1·00076	1·00019

Four Joint Lives										
Period elapsed from date of selection $(t+n)$	Amount of successive Annuity Payments, in $(\mathbf{v}_{\overline{10-t}\mid} a')_{[w]+t}$ (O[M] Table)									
	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$	$t=9$
1	1·01132	...	...	...	...	...	...	...	...	...
2	1·01763	1·00624	...	...	...	...	...	...	...	...
3	1·02230	1·01086	1·00459	...	...	...	...	...	...	...
4	1·02615	1·01467	1·00838	1·00377	...	...	...	...	...	...
5	1·02937	1·01785	1·01154	1·00691	1·00314	...	...	...	...	...
6	1·03200	1·02045	1·01412	1·00949	1·00570	1·00255	...	...	...	...
7	1·03405	1·02248	1·01613	1·01149	1·00769	1·00454	1·00198	...	...	...
8	1·03551	1·02392	1·01757	1·01292	1·00912	1·00597	1·00340	1·00142	...	...
9	1·03639	1·02479	1·01844	1·01378	1·00998	1·00682	1·00425	1·00227	1·00085	...
10 and upwards	1·03668	1·02508	1·01873	1·01407	1·01026	1·00710	1·00454	1·00255	1·00113	1·00028

Normal Rates of Interest	Special Rates of Interest ($\log_{10} s = -\cdot0026111$)		
	Two lives	Three lives	Four lives
Per-cent	Per-cent	Per-cent	Per-cent
$2\frac{1}{2}$	3·118	3·740	4·366
3	3·621	4·246	4·875
$3\frac{1}{2}$	4·124	4·752	5·384
4	4·627	5·258	5·893
$4\frac{1}{2}$	5·130	5·764	6·402
5	5·633	6·270	6·911

[The above demonstration is published in the *Journal* as of theoretical interest, and as showing the conditions under which the Law of Uniform Seniority can be applied in Select Tables, when graduated by Mr. G. F. Hardy's adaptation of Makeham's Law. The practical applications of the relations shown will, however, probably be somewhat limited. Under the O[M] Table, for instance, the volume recently published by the Institute of Actuaries and the Faculty of Actuaries includes a Table of Uniform Seniority, from which the value of a select annuity on any two lives whatever can be deduced, as an equivalent select annuity on two lives of equal age; and also Tables of $a_{[x][x]}$, interpolated for tenths of a year of age, at six

different rates of interest. Where, however, it is desired to ascertain the value of an annuity on two lives, $a_{[x]+t:[y]+t}$, at any period t after the date of selection, and also where annuities on three or more lives are involved, the relations shown by Mr. Goodman might be usefully applied, and the labour involved in computing the single-life annuity, at a special rate of interest, with variable payments during the period of selection, does not seem to be very considerable.—Ed. *J.I.A.*].

LEGAL NOTES.

By Arthur Rhys Barrand, F.I.A., *Barrister-at-Law.*

Deeds of assignment for the benefit of creditors are documents which frequently come under the notice of those who have to deal with the legal incidents connected with policies of life assurance, and in an earlier part of this volume attention has been called to two important recent cases bearing on the subject, those of Davis *v.* Petrie (*J.I.A.*, xli, 165), and Ponsford Baker & Co. *v.* The Union of London & Smith's Bank Limited (*J.I.A.*, xli, 167). To these may now be added a yet more recent case dealing with the same subject, that of *In re Saumarez, ex parte Salaman* [1907] 2 K.B. 170. The following are the particulars of the case. On 1 October 1905 a debtor being in difficulties, called a meeting of certain of his creditors, eleven in number, who were pressing for payment, and by a memorandum of that date, signed by the debtor and these creditors, the latter agreed not to take any further proceedings in respect of their claims, provided that 4*s.* in the pound was paid to them on or before 12 October 1905, and that the debtor transferred certain property belonging to him to trustees for the benefit of these creditors. On 15 January 1906 such a deed was executed, the composition of 4*s.* in the pound having already been duly paid. The deed in question provided for the property transferred being realized or otherwise dealt with, and for any of the property remaining undealt with to be reassigned to the debtor as soon as the creditors concerned should have received 20*s.* in the pound. Neither the memorandum nor the assignment was registered under the Deeds of Arrangement Act, 1887, and on the subsequent bankruptcy of the debtor in September 1906, the trustee in his bankruptcy

applied for a declaration that the memorandum and assignment were void as against the trustee, on the ground that they had not been so registered. Bigham, J., in deciding in favour of the validity of the deed, said: "It would be a perversion of the "meaning of the Act to say that a deed, which on its face is for "the benefit of certain named persons and not for any class "at all, could be described as for the benefit of the creditors "generally. The Act contemplates a deed which includes all "creditors who choose to assent to it." The trustee in bankruptcy appealed against this decision, but it was upheld by the Court of Appeal. Collins, M.R., in affirming the validity of the deed, said: "I do not see my way to differ from the decision "of Bigham, J., in this case. I say so with some regret, for I "think the decision tends to show a possible omission in the Act "of 1887, but it is not for us to deal with that—our duty is "limited to construing the Act . . . I cannot see my way to hold "that a deed for the benefit of certain specified creditors, with no "option to the other creditors to come in, is a deed for the "benefit of creditors generally."

In view of the extent to which life assurance companies invest in reversionary securities, attention may be called to a recent case which indicates some of the difficulties and dangers attending such investments. The case in question is that of *Bagot* v. *Chapman* [1907] 2 Ch. 222. Here a Mrs. Chapman and her husband executed a mortgage of a reversionary interest belonging to the former, in favour of the plaintiffs, in 1895, to secure £12,000. The deed contained covenants by the husband and wife for the payment of principal and interest. The husband induced his wife to execute the mortgage deed by representing that it was a power of attorney which would enable him to raise money at some future time if he should require it; but he stated that he was not going to use it. She understood that if her husband did raise money, it would be out of her reversionary interest, but she had no idea that any present charge was created, or that she was making herself personally liable to pay anything. She had no business experience and no independent advice. She was twenty-five years of age when the mortgage was executed, and her husband, who was a solicitor, and had been her father's solicitor, and was a trustee of his will, was about sixteen years older than herself. On an action being brought by the mortgagees for

foreclosure, and for judgment against the husband and wife personally on their covenants, the wife, amongst other defences, put in a plea of *non est factum,* that is, a denial that the deed in question was really her deed. In delivering judgment in favour of the wife, Swinfen Eady, J., said: "I am of opinion that the "deed is not binding upon the defendant Mrs. Chapman as, upon "the issue of *non est factum,* it must be determined that the "deed is not her deed. It is well settled that where a person is "induced to execute a deed by a false representation as to the "nature and character of the document he is signing—where "the document is of a totally different character from what he "was told it was—such a deed does not bind him." This case has been entered for appeal, but such appeal had not been heard when these notes were written. The difficulty disclosed by this case may of course arise with any sort of document, and in respect of any description of property, but it is one which is perhaps more likely to arise, as in this instance, with regard to reversionary transactions, and is more difficult to guard against in such circumstances, than where property of other descriptions is concerned.

A case in which life interests and reversions were involved, and which may usefully be compared with the preceding case, is that of *Walker* v. *Linom* [1907] 2 Ch. 105. Here a settlor, in contemplation of marriage, settled property on himself for life, such interest to be determinable on alienation, and, subject to such life interest, and to a discretionary trust in the event of such determination, to the plaintiff (the intended wife) for life, with remainder to the settlor on the failure of issue. The settled property had been purchased by the settlor and conveyed to him by a deed dated 8 May 1896. The settlement was effected by two documents, both executed on the same day, by one of which the property was conveyed to the trustees, and by the other the trusts were declared. The same solicitors acted for all the parties, and by an oversight the original purchase deed of 8 May 1896, conveying the property to the settlor, was not obtained from him. By means of the possession of this deed the settlor was enabled to mortgage, and did mortgage, the property, which, by means of this mortgage, followed by a transfer of mortgage and conveyance in exercise of the mortgagee's power of sale, became vested in the defendant, who held the deed of 8 May 1896. Neither the mortgagee nor any person claiming under him had any notice

of the settlements. When the defendant purchased the property from the transferees of the mortgages created by the settlor, he entered and remained in possession or in receipt of the rents and profits. The wife now brought an action against the purchaser, the trustees and her husband, for a declaration that the settlor's life interest had determined, and that the purchaser's interest in the property was subject to her interest under the settlement. Parker, J., in giving judgment in favour of the purchaser, said: "I cannot, under the circumstances of this case, find that anyone "concerned in the 1896 settlement, with the exception of George "Church Walker (the settlor), acted otherwise than honestly . . . "I do not think . . . there is anything in the authorities to "preclude me from holding, and I accordingly hold, that the "trustees, although they have the legal estate, are postponed to "the defendant . . . I hold . . . that the plaintiff is in no better "position than the trustees, and is postponed to the defendant."

Another recent case of importance in connection with life interests is that of *In re Bigge, Granville* v. *Moore* [1907] 1 Ch. 714. Here a testator, by her will, after giving certain legacies, left her residuary estate to trustees, and went on to direct that "my trustees shall out of the income thereof" pay certain annuities to persons named, and, subject thereto, pay the income to her sister for life, with remainder in trust for certain persons. The testatrix died in 1895, and after her death it was found that the income of her estate was insufficient to pay the annuitants in full, and a summons was now taken out by the trustees for the determination of the question as to whether the arrears of one of the annuities were a charge on the corpus of the residuary estate, or a continuing charge upon the income thereof, or were not so charged. Neville, J., in deciding that the annuity was only payable out of current income, said: "The "question whether an annuity is payable exclusively out of "income, or out of current income, or charged upon the corpus "of the estate, or whether it is payable out of accumulated "income—in other words, whether the arrears of the annuity in "any one year are payable out of the income of succeeding years "—in my opinion, after careful consideration of the cases that "have been cited to me, depends entirely upon the construction "to be placed upon the words in the particular will, construing "them, so far as possible, in their ordinary grammatical sense. ". . . . Turning to the words of the present will, and

"construing them according to their natural meaning, I find "that the annuity is to be paid out of the income of the estate, "and after payment of the annuity, the balance of the income is "to be paid to Julia Townsend during her life, and that payment "to her is to be 'subject thereto.' It seems to me that the "words 'subject thereto' mean subject to the payment of the "annuities out of the income. . . . I find nothing whatever to "indicate any intention to charge the annuity upon the corpus "of the estate. . . . It seems to me, therefore, that upon the "true construction of this will, the annuities were intended to be "paid out of the current income. . . . Consequently, so far as "the current income was insufficient to meet the annuities, the "annuities fail."

The question of the responsibility of an insurance company for the acts and statements of its agents is one of great interest and importance to all except those companies which are fortunate enough to be able to conduct their business successfully without the intervention of agents; and, in view of this, the recent case of *Kettlewell* v. *Refuge Assurance Company* [1907] 2 K.B. 242 is worthy of note. Here the plaintiff effected a whole-life industrial policy on the life of one James Kettlewell, with the defendants. The plaintiff paid the premiums for a year, but in April 1902, finding that she could not afford to continue paying them, she proposed to let the policy lapse. The defendants' district superintendent, however, informed her that if she continued paying premiums for five years from the date of issue, she would then be entitled to a free policy, and a similar representation was made to her by another of the defendants' agents. The representations so made were untrue, to the knowledge of the persons making them, and were made without the authority of the defendants. Relying on these representations, the plaintiff continued to pay premiums down to February, 1906, when she claimed a free policy, which the defendants declined to give her. She then brought an action to recover back the premiums paid by her. Evidence was given on behalf of the defendants to the effect that their agents had no authority to make any contract for the company. The County Court Judge, before whom the case came in the first instance, directed the jury that if the defendants' agents had been guilty of fraud, and the defendants had benefited by that fraud, and retained the premiums, they thereby ratified their agents' act, even though it was outside

the scope of their authority. On this direction the jury returned a verdict for the plaintiff, for the amount of premiums paid since the date of the false representations, and on appeal, this was upheld by the Divisional Court. In dismissing the appeal, Phillimore, J., said: "In this case two points have been argued "on behalf of the defendants. The first was that it is not now "open to the plaintiff to recover back the premiums which she "paid, because she has had, at any rate a part of the benefit that "she contracted for, and that, as the consideration has not "wholly failed, she cannot sue for the premiums as money had "and received to her use. The answer to that contention, I "think, is that in the true view of the facts, it is open to her to "say the policy never attached . . . Then his (the defendants' "counsel's) second point was that the agent had no authority to "make the representation. No doubt he had no authority to "make a fraudulent misrepresentation, but that is not enough "... If a principal appoints a person as his agent to obtain "bargains, any representation which the agent makes with the "object of obtaining a bargain, and which results in a bargain "being obtained, is one which the principal must make good, in "the sense that he cannot retain any benefit which the agent "may have procured for him by means of that representation, if "it turns out to have been untrue." Bray, J., in delivering judgment to the same effect, said: "We do not decide that the "defendants are bound by the agents' representations, so as to "make them liable for damages, for it is not necessary to decide "that here. What we decide is that the defendants cannot keep "the premiums which the plaintiff was induced to pay by the "fraud of the agent." In view of the importance of the point at issue, the company applied to the Court of Appeal for leave to appeal against the decision of the Divisional Court, and expressed their willingness, if such leave were granted, to submit to any terms as to costs that the Court might think fit to impose. Vaughan Williams, L.J., in delivering judgment granting leave to appeal, said the case was an extremely important one for the company, and raised an extremely arguable point, and that the Court would reserve to themselves, on the hearing of the appeal, the right to deal with the question of costs as they thought fit. At the time of writing the appeal had not been heard.

Another recent case, also dealing with the responsibility of an assurance company for the acts of its agents, and which is of

considerable interest and importance, is that of *Holdsworth* v. *Lancashire and Yorkshire Insurance Company* (1907) 23 T.L.R. 521. In this case the plaintiff effected an employers' liability policy with the Manchester Assurance Company, the business and liabilities of which were subsequently taken over by the defendants. The plaintiff was a builder and joiner, and this fact was known to the company's agent. The agent filled up the proposal form, which it was agreed should be the basis of the contract, and in which the plaintiff was described as a joiner only. A policy was issued on this basis, but the plaintiff declined to accept it with this mis-description of his occupation, and the agent then, with the authority of the chief clerk of the local office of the assurance company, added the words "and builder" to the plaintiff's description and initialled the alteration. He then handed the policy to the plaintiff, who thereupon paid the first premium. On one of his men being killed by an accident whilst engaged as a builder, the plaintiff paid compensation and claimed to be reimbursed by the defendants. On the facts as stated, the arbitrator found for the plaintiff, and on the case coming before Bray, J., in the form of a special case stated for the opinion of the Court, this award was upheld. In delivering judgment, Bray, J., pointed out that the plaintiff relied on three points, viz.: (1) Assuming that the policy was to be read as containing only the word "joiner", without the words "and builder", the company was liable in accordance with the decision in Bawden's case (*J.I.A.*, xli, 123); (2) It must be held as a fact that by the conduct of the company and the course of business in their office, the agent had authority to do what he did; (3) The company was estopped from denying their agent's authority, because they had received the premiums with constructive knowledge of the facts, and the knowledge of their agent was the knowledge of the company. Dealing with this last point first, Bray, J., said that, "In support of the third proposition, the case of Wing *v.* Harvey "(5 De G. M. & G. 265. Bunyon, 4th edit. p. 299) was cited, "and it seemed to him that that was an authority he was quite "unable to get over. . . . It was quite clear that if the directors "had received the premiums with knowledge of what passed "between Holdsworth (the plaintiff) and Ellisdon (the agent) they "would have been bound. . . That case (Wing's) was practically "on all fours with the present case, and he could not conceive "that it made any difference whether what took place happened "after the contract had already been made, or while it was in

"process of being made. The principle of Wing *v.* Harvey "showed that the knowledge of Ellisdon was the knowledge of "the company, and was decisive . . . As to the second point, he "felt a good deal of difficulty, and he preferred not to express "an opinion upon it. Then as to the first point, he was unable "to distinguish the case from Bawden *v.* London, Edinburgh "and Glasgow Assurance Company . . . The contract must be "treated as negotiated with a joiner and builder, and not with a "joiner only, and in that sense the knowledge of the agent "was the knowledge of the company. The judgment in "Bawden's case applied, and he must hold that even if "there had been no alteration at all in the policy, Holdsworth "would have been entitled to recover, and the fact that "an alteration had been made could not in any way prejudice "Holdsworth's claim . . . The award must therefore stand "in favour of Holdsworth." No fault can be found with the actual decision in view of the facts of the case, and apparently no appeal against it has been made, but the grounds upon which Mr. Justice Bray based his decision under the first head dealt with by him, and in which Wing's case is made an authority for deciding a case such as this in favour of the proposer, deserves the very careful consideration of all life assurance companies who employ agents. It is, however, worthy of note, perhaps, that the case has not yet been reported in the Law Reports, and does not now seem likely to appear there.

Attention has frequently been called to the somewhat remarkable decisions on the subject of the stamping of documents, which are given from time to time by the Inland Revenue authorities at Somerset House. One such decision which has come before me on more than one occasion, and to which, so far as I know, reference has not previously been made, may be referred to here, particularly as a good illustration of it came under my notice quite recently. It not infrequently happens that the deed of assignment, in the case of a mortgage transaction, takes the form of an absolute assignment, the evidence that the transaction is really one of mortgage being contained in other more or less formal documents. In such a case the Inland Revenue authorities appear to regard the deed as one relating to two distinct matters, and, therefore, as being liable to two distinct stamp duties; and to deal with it as being, in the first place, a conveyance other than a conveyance on sale,

and as such liable to a stamp duty of 10*s.*, and in the second place, as being a mortgage for the amount secured, and as such, liable to the ordinary mortgage duty. Thus, in the case referred to above, a deed in the form of an absolute assignment of a life policy, for a consideration of £20, was stamped with 2*s.* as an ordinary conveyance on sale. As a matter of fact it turned out to be only a mortgage to secure the amount of the consideration. On the deed being submitted to the Inland Revenue authorities for adjudication, an additional stamping of 8*s.* 8*d.* was impressed, making 10*s.* 8*d.* in all, 10*s.* being apparently in respect of the absolute assignment, and 8*d.* in respect of the mortgage.

As the Session of Parliament for 1907 is still continuing while these notes are being written, it is not possible here to refer to the legislation of the session more or less directly affecting assurance companies. Attention may, however, be called to one matter which has often given rise to adverse criticism in the past, and which has now been brought before Parliament, As is well known, the Courts have held that in estimating damages under Lord Campbell's Act, any amount to be received in respect of accident policies has to be taken into account, except in the case of the policies of one particular company which, by means of a private Act of Parliament, obtained relief in respect of its policies from the effect of this decision. The matter is referred to in an earlier part of the present volume of the *Journal* (*J.I.A.*, xli, 202), and the anomalous position created by such private legislation is discussed. During the session of Parliament now nearing its close, two other companies obtained private Acts conferring the same privilege on them as that enjoyed by the other company referred to above. This state of affairs only increased the existing anomaly, and it became evident that such a restriction as that imposed by the rule of law in question should apply to all companies or to none. Accordingly, a Bill was introduced under the title of the Fatal Accidents (Damages) Act, by which it was proposed to abrogate the old rule of the Courts as to the amount of assurance being taken into account. This Bill passed through all its stages in the House of Lords, but unfortunately failed to become law, and the anomaly therefore still exists.

REVIEW.

British Offices Life Tables, 1893.
Select Tables, $O^{[M]}$ *and* $O^{[NM]}$. *Valuation Tables*, O^{M}.*

When it is considered how greatly the work of the actuary depends for its value upon the reliability of the mortality tables he has occasion to employ, and how materially his labours are facilitated when he has access to an ample supply of monetary values based on those tables, it is not difficult to appreciate the indebtedness of the actuarial profession to those responsible for the production of the volumes of tables based on the British Offices Experience, 1893. The obligation is, indeed, a specially heavy one, as may readily be seen by comparing the output with that in connection with the 20 Offices' Experience. Not only has the more comprehensive analysis to which the crude data of the later experience were subjected provided a greater variety of mortality tables of the first importance, but these in their turn have been utilized as the foundation of a more generous tabulation of monetary values than was attempted when the earlier investigation was completed.

A considerable proportion of the first of the two volumes which form the subject of this notice consists of an amalgamation of material previously published. The decision on the part of the Institute and Faculty to incorporate in a single volume the monetary tables based on the $O^{[M]}$ experience, which had in the first instance been given to the profession in two distinct publications, was a praiseworthy, though, as some may think, an inevitable step. The omission from the $O^{[M]}$ volume of 1903 of such indispensable functions as log D, log N, C, M and R, was difficult to understand, and although, in a well-designed supplementary volume, Messrs. Baker and Raisin had supplied the missing tables, experience had shown how inconvenient it was to have to refer to two separate works in using the $O^{[M]}$ Table and the new volume thus remedies a troublesome state of affairs.

It is to be noticed that in reproducing, in respect of the $O^{[M]}$ Table, the values of functions tabulated for each of the first ten years of assurance and the ultimate table, the form of tabulation originally adopted, which terminates with the ultimate value at age 85, has been adhered to. One could wish that it had been found possible to give on the same folio the remainder of the ultimate values, viz., those for ages 86 to 105. As it is, these are to be found elsewhere and a good deal of time is lost, in evaluating a complicated benefit, in the search for the concluding values. Failing the complete tabulations suggested, it would have been useful to give at the foot of each table a reference to the page where the final values appear.

* British Offices Life Tables, 1893. Select Tables deduced from the Graduated Experience of Whole-Life Assurances on Male Lives. Published jointly by the Institute of Actuaries and the Faculty of Actuaries. London, C. & E. Layton, 1907.

British Offices Life Tables, 1893. Valuation Tables deduced from the Graduated Experience of Whole-Life Participating Assurances on Male Lives (O^{M} Table) at $2\frac{1}{2}$, $2\frac{3}{4}$, 3, $3\frac{1}{2}$, 4 and $4\frac{1}{2}$ per-cent. Published by the Institute of Actuaries, 1907.

The most important section of the new tables comprised in the volume consists of values of ${}_nV_{[x]}$, based on the $O^{[M]}$ Table, at six rates of interest. The publication of these values is a noteworthy event, since this is believed to be the first time that policy-values based on select tables have been systematically tabulated, and their appearance affords further evidence, if such were needed, of the extent to which present-day practice is influenced by the theory of select tables.

The following comparison of the new values with those based on Dr. Sprague's Select ($H^{[M]}$) Tables incorporates part of a similar tabulation given by Mr. Ackland in his comprehensive survey of the British Offices Experience in 1902 (*J.I.A.*, xxxvii, p. 176). It will be seen that with the exception of a few values, relating to advanced ages attained, the $O^{[M]}$ reserves are throughout greater than the corresponding $H^{[M]}$ values.

Comparison of Whole-Life Policy-Values.

3 PER-CENT. $100{}_nV_{[x]}$. 3 PER-CENT.

Age at Entry	Table	Duration						
		5	10	15	20	30	40	50
[20]	$O^{[M]}$	5·12	10·16	15·61	21·72	35·72	51·24	66·44
	$H^{[M]}$	3·91	8·12	13·61	19·64	34·10	50·11	65·90
[25]	$O^{[M]}$	6·00	11·94	18·31	25·34	40·92	57·24	72·07
	$H^{[M]}$	5·26	10·92	17·14	24·31	40·13	56·83	72·19
[30]	$O^{[M]}$	7·05	14·03	21·42	29·40	46·45	63·15	77·11
	$H^{[M]}$	6·62	13·14	20·67	28·77	46·08	63·15	77·15
[35]	$O^{[M]}$	8·31	16·48	24·96	33·91	52·16	68·75	81·41
	$H^{[M]}$	7·89	15·86	24·46	33·45	52·01	69·08	80·94
[40]	$O^{[M]}$	9·82	19·31	28·93	38·79	57·88	73·83	...
	$H^{[M]}$	9·56	18·80	28·46	38·53	57·98	73·94	...
[45]	$O^{[M]}$	11·60	22·54	33·29	43·93	63·38	78·22	...
	$H^{[M]}$	11·39	21·94	32·93	43·71	63·73	77·65	...
[50]	$O^{[M]}$	13·70	26·18	37·96	49·20	68·44	...	...
	$H^{[M]}$	13·52	25·69	37·64	49·21	68·60	...	...
[55]	$O^{[M]}$	16·13	30·17	42·82	54·38	72·87	...	...
	$H^{[M]}$	15·89	29·41	42·50	54·52	71·97	...	...
[60]	$O^{[M]}$	18·91	34·45	47·70	59·28	...	...	...
	$H^{[M]}$	18·26	33·42	47·34	58·71	...	...	...

On a question of terminology it may be suggested that the description of the new reserves as "Select Values" is not free from objection. The values of ${}_nV_{[x]}$ given are obviously average values which apply to "mixed" lives, and the term "Select Values" may perhaps be regarded as more strictly applicable to reserves based on the assumption that the lives are select at the date of valuation.

A similar criticism applies to the inclusion of these policy values in the synopsis of tables among the functions "tabulated as at date of assurance only."

It may be remarked that the policy values are shown to two decimal places only, as against three in Mr. R. P. Hardy's H^M Tables. This welcome innovation will not only save labour but admits of the values being set out with greater clearness. No provision has been made in the volume for dealing with paid-up policies, and it would have been of advantage if extended values of $A_{[x]+t}$, or even the ultimate values only, could have been published. These can, of course, be computed from the corresponding annuity values, but the operation absorbs time, and considering the frequency with which paid-up policies are met with in practice it might not unreasonably have been anticipated that these assurance values would be given. An incidental result of their exclusion is that there is not in the whole volume a single value of A_x for any age above 75, where the select values cease. At the important rate of $4\frac{1}{2}$ per-cent the usual annuity and assurance values and commutation columns have not yet been published—an unaccountable omission.

As regards the $O^{[NM]}$ Table, the functions now given are practically the same as in Messrs. Baker and Raisin's work, the only additions being temporary annuities at $2\frac{1}{2}$ per-cent and 3 per-cent.

The vexed question of contingent assurance premiums is again brought to the fore in the volume under notice. Originally, it will be remembered, values of A^1_{xy} and P^1_{xy} were tabulated on the assumption that the life assured would experience the mortality shown by the $O^{[M]}$ Table and the counter life that of the $O^{[af]}$ Table. Subsequently Messrs. Baker and Raisin substituted the $O^{[NM]}$ for the $O^{[M]}$ Table in this combination, and their values are now reproduced, but are accompanied by a note in the introduction, in which it is stated that for the "principal assuring ages (30 to 50)" the actual deaths in the contingent survivorship assurance class were 132 in number as compared with 116 expected according to the $O^{[NM]}$ Table, and that, at these entry ages taken alone, an average addition of 3 years to the age of the assured life will approximately represent the actual mortality. That is no doubt the case; but, remembering that the unadjusted data in the special class referred to are comparatively meagre, and are limited to the experience of the first 10 years of assurance; and that the observations for other age-groups, including that relating to ages at entry 23-27, which, it may be suggested, are entitled to be included among the principal assuring ages, do not confirm the indication expressed above, the view may be put forward that, whatever the future may have in store, there is at the present time but slender foundation for the suggestion that the mortality of the assured life is likely to prove heavier than that indicated by the $O^{[NM]}$ Table. As regards the counter life there would appear to be no theoretical justification for adopting the $O^{[af]}$ basis as a standard, in the very common case where the counter life is a male.

In addition to the values of A^1_{xy} and P^1_{xy} opportunity has been

taken to publish the corresponding joint annuity values, one life being assumed to be subject to $O^{[NM]}$ mortality and the other to $O^{[af]}$ mortality. Since the life assured, as well as the counter life, may conceivably be of either sex, this table is likely to prove a pitfall for the unwary and must be handled with considerable care.

The second of the two volumes under notice is published by the Institute alone, and consists entirely of values based on the O^{M} Table, including whole-life limited payment and endowment assurance premiums and policy values, in addition to ordinary whole-life reserves, at various rates of interest. Many would probably have preferred the adoption of the $O^{M(5)}$ Table instead of the O^{M} as the basis of some of the values here tabulated, but the latter table has the merit of affording values which are strictly comparable with those founded on the H^{M} Table, while giving effect to the improvement in mortality which is a predominant feature of the later experience.

Values of ${}_tP_x$ are given not only for quinquennial values of t up to 30, but also for $x+t=45, 50 \ldots 70$, and thus form a distinctly serviceable table. As regards endowment assurances it is satisfactory to find both $A_{x\overline{t|}}$ and $P_{x\overline{t|}}$ extensively tabulated. It is not however clear why no premiums are given for ages 10 to 14, especially as, later on, values of policies effected at these entry ages have been wisely included.

The ordinary whole-life policy values are open to no criticism as regards extent as they are given for every age at entry, from 10 onwards, and all durations, the tabulation having been carried out to two decimal places as in the $O^{[M]}$ volume. It is pointed out in the Introduction that in certain cases where the attained age exceeds 89, ${}_nV_x > {}_nV_{x+1}$, this anomaly being due to the fact that the annuities upon which the policy values are founded were based on integral values of l_x. Since ${}_nV_x > {}_nV_{x+1}$ when ${}_1V_x > {}_1V_{x+n}$, these disturbances may readily be traced by examining the values of ${}_1V_x$. To take an example, it will be found that, at 3 per-cent interest, ${}_1V_{96}$ is less than the corresponding reserves for all values of x from 74 to 95 inclusive, and it follows therefore at each of these entry ages the policy-value appertaining to attained age 97 is less than the value relating to the next lower age at entry and the same duration. The defect pointed out is fortunately of no practical importance, but it throws light upon the method adopted in the case of the O^{M} and $O^{M(5)}$ Tables in terminating the l_x column. It is difficult to see what advantage is gained by cutting down values of l_x to the nearest integer, especially as a different result would be reached by varying the radix, which, of course, is arbitrarily chosen, and the plan unfortunately introduces irregularities at the concluding ages, and gives rise to inconsistencies such as those referred to above. Even if l_x itself were the function graduated the adjusted results would hardly emerge in the form of whole numbers, and the preferable course would appear to be to regard the graduated curve, whether consisting of values of l_x, q_x, or colog p, as when Makeham's law applies, as inviolate, afterwards making the dependent values agree by computing them to the necessary number of decimal

places. The $O^{[M]}$ Table affords an instance in which the method here suggested as the correct one has been carried out. The question raised is one which might be of importance in theoretical investigations, and it may therefore be opportune to point out that the final values in the $O^{M(5)}$ and O^{M} Tables, as published in the 1902 volume, cannot, strictly speaking, be regarded as based on the $O^{M(5)}$ constants, which appear on p. 105 of that work, but deviate appreciably from the true values. A comparison of the values of colog p on page 153 of "Account of Principles and Methods" with those which appear on p. 5 and p. 107 of the 1902 volume of O^{M} and $O^{M(5)}$ Tables, for ages above 85, will bear out what has been here said.

Many will be grateful to the Institute for tabulating values of limited payment policies. It will be remembered that some years ago Messrs. Hume and Stott gave values of these policies, based on the H^{M} Table, at 3 per-cent interest (*J.I.A.*, xxxiv, p. 397), but the present tabulation is a much more extensive one and is amply justified by the greater favour with which this class of assurance is being viewed by the insuring public. There is an excellent set of endowment assurance policy-values given, these relating to entry ages from 10 to 69, and quinquennial ages at maturity from 40 to 70. It is satisfactory to find the values for age at maturity 40 published, as, although assurances payable at this age cannot be said to be at all fashionable at the present time, it is becoming increasingly common to issue endowment assurance policies for periods of 15 and 10 years, and the values referred to will be very useful if it should be thought necessary to resort to interpolation. The use of Carment's handy little volume of tables is often attended with a slight drawback owing to the absence of values relating to age at maturity 45, which is often met with in practice, as, although by making use of the corresponding value for maturity age 50 (*i.e.* taking x five years older, the term of the assurance remaining constant) very little harm is done, the trifling error is not always on the safe side. The following little table illustrates this remark, and gives an idea of the progression of these policy values when the term and duration of the assurance remain constant, the age at entry and, in consequence, the maturity age varying.

Endowment Assurance Policy-Values.

O^{M} $100_{n}V_{x\overline{t}|}$. 3 PER-CENT.

$t=15$			$t=25$			$t=35$		
Age at Entry x	$n=5$	$n=10$	Age at Entry x	$n=10$	$n=20$	Age at Entry x	$n=10$	$n=25$
25	27·91	60·78	15	30·76	72·84	10	18·59	59·26
30	27·76	60·57	20	30·72	72·69	15	18·77	59·19
35	27·59	60·33	25	30·61	72·47	20	18·92	59·07
40	27·41	60·03	30	30·46	72·21	25	19·04	58·99
45	27·21	59·63	35	30·40	71·91	30	19·30	59·02
50	26·95	59·05	40	30·49	71·54	35	19·88	59·28
55	26·59	58·17	45	30·77	71·04	...	...	...

As in the case of the select tables the usefulness of the $4\frac{1}{2}$ per-cent policy-values is considerably limited by the non-publication of the corresponding annuity-values and commutation columns.

The inevitable comparison of the latest volume with Mr. R. P. Hardy's "Valuation Tables" suggests that the former contains not only much more but also much less than its well-worn predecessor. Many will no doubt regard it as a distinct advantage to have all the classes of policy-values in a single volume, though, on the other hand, the necessity for turning to another volume for the values of A_x and a_x and the temporary annuity-values which are a feature of Hardy's Tables and are so constantly required in dealing with conversions of policies, is somewhat of a drawback. Further, one will sorely miss the extremely useful little tables of logarithms and reciprocals which appear in the earlier volume, and one could wish that it had been found possible to include these, not only in the present work, but also at the end of every volume of monetary tables issued by the Joint Mortality Committee.

With the O^M volume is issued a separate print giving the values of temporary annuities at $2\frac{3}{4}$ per-cent, according to both the O^M and $O^{M(5)}$ Tables.

It would be improper to conclude this review without expressing appreciation of the valuable work which has been performed by the compilers of the volumes dealt with above. A careful study of the Tables reveals the magnitude of the task which has been accomplished, and the thanks of the profession will be given without stint to all who have taken part in the preparation of the volumes, and especially to the members of the Committee of the Institute, who have, in conference with a standing Committee of the Faculty as regards the earlier volume, borne the brunt of the work.

J. S.

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS OF THE INSTITUTE, April 1907.

Examination for Admission to the Class of Associate (Part I).

Examiner—Prof. S. L. Loney, M.A.

Supervisors—Messrs. A. G. Hemming and W. P. Phelps, M.A.

First Paper.

1. Without using logarithms multiply 2·347639 by 4·6237, and divide unity by 2·7182818, in each case correct to three places of decimals. [Contracted methods are preferred.]

2. A plot of land, of frontage 47 feet and depth 180 feet, is let at a ground rent of £18; what rent to the nearest shilling is this per acre?

3. Show how to obtain the square root of an expression of the form $a + \sqrt{b}$, where $\sqrt{b}$ is a surd.

Express

$$2\sqrt{2}\sqrt{4+\sqrt{15}} - \frac{\sqrt{2}}{\sqrt{4-\sqrt{15}}}$$

as the sum of two simple surds.

4. Write down the roots of the quadratic equation

$$ax^2 + bx + c = 0.$$

If a, b, c be all real, prove that the roots of the equation

$$\frac{1}{x+a} + \frac{1}{x+b} + \frac{1}{x+c} - \frac{3}{x} = 0$$

are real.

5. Prove the formula for the sum of a number of quantities which are in geometrical progression, and find under what conditions the sum of an infinite geometric progression can be obtained.

Show that the sum of an infinite geometrical progression, whose common ratio is positive and less than unity, can never be less than four times its second term.

6. Find the number of permutations of the letters in the word *proportional* taken all at a time.

Find also the number of different choices of five letters that can be made from the same word.

7. Prove the Binomial Theorem for a positive integral exponent.

Find which term in the expansion of $(7+4x)^{19}$ has the greatest coefficient.

8. Define a logarithm, and state the fundamental properties on which their use is based.

Find the value of x to three places of decimals, given that

$$8^x = 4^x + 6 \times 2^x.$$

9. Show that the number of solutions in positive integers, including zero, of the equation $x + 2y = 2n$, where n is an integer, is $n+1$.

At an entertainment the prices of admission were 1*s.*, 2*s.*, and 10*s.*, and the total receipts were £50; in how many ways can the audience have been made up?

10. If the probabilities of the happening of two independent events are each known, find the probability that one at least will happen.

A pack of well-shuffled playing cards is dealt out, as in whist, to four players A, B, C and D successively. Find the chance that A and C may have all their cards of the same colour, given that $\log(\underline{|26}) = 26{\cdot}60562$ and $\log(\underline{|52}) = 67{\cdot}90665$.

11. Two persons, A and B, play for a stake, each throwing alternately two dice, A commencing. A wins if he throws 6, B if he throws 7, the game ceasing as soon as either event happens.

What ratio will A's chance of winning bear to B's?

12. Show that $\Delta \dfrac{u_x}{v_x} = \dfrac{v_x \Delta u_x - u_x \Delta v_x}{v_x v_{x+1}}$.

If $x^{(-m)}$ denote the quantity $\dfrac{1}{x(x+1)\ldots(x+m-1)}$, show that $\Delta^n x^{(-m)} = (-1)^n m(m+1)\ldots(m+n-1)x^{(-m-n)}$.

Second Paper.

13. A man sells out £20,000 $2\frac{1}{2}$ per-cent Consols at $86\frac{1}{8}$, and invests the resulting money in Canada 3 per-cent Stock at $97\frac{3}{4}$; taking brokerage and other expenses of sale in each case to be £$\frac{5}{8}$ per-cent of the face-value of the stock, find the change in his income to the nearest shilling.

14. A man has a certain quantity of money in his pocket; he pays away half what he has and half-a-sovereign more; he then pays away half of what he then has and half-a-sovereign more; he repeats the process again, and finally has £2. 3*s.* left; how much had he to start with?

15. If P be a rational integral algebraic expression containing x, which vanishes when $x=a$, show that $x-a$ is a factor of P.

Find the factors of the expression

$$a^3(b-c)+b^3(c-a)+c^3(a-b).$$

16. Show that the product of any four consecutive numbers increased by unity is a perfect square, and that the product of any four consecutive odd numbers increased by sixteen is also a perfect square.

17. Find the sum of n terms of the series

(1) $1^2+2^2+3^2+\ldots;$

(2) $\dfrac{1}{1.2.4}+\dfrac{1}{2.3.5}+\dfrac{1}{3.4.6}+\ldots$

18. Prove that ${}_nC_r + {}_nC_{r-1} = {}_{n+1}C_r$.

Show also that the number of ways in which $a+b$ different things may be arranged in a row in such a manner that no two of the b things may be adjacent is $\dfrac{\lfloor a \lfloor a+1}{\lfloor a-b+1}$.

19. If n be a positive integer, find the sum of the coefficients in the expansion of $(1+x)^n$ in powers of x.

Prove that the number of combinations of $2n$ things taken n at a time, when n of the things are alike and the rest all different, is 2^n.

20. Making use of the Tables, find

(1) the seventh root of the fourth power of 3·247;

(2) the number of digits in 3^{47}.

21. Assuming the truth of the Exponential Theorem, obtain the expansion of $\log_e(1+x)$, and hence find the value of $\log_e 2$ correct to four places of decimals.

Find the sum to infinity of the series

$$\frac{1}{2\,.\,3}-\frac{2}{3\,.\,4}+\frac{3}{4\,.\,5}-\frac{4}{5\,.\,6}+\ldots$$

correct to four places of decimals.

22. Explain the meaning of the words Probability or Chance, and show how they are mathematically measured.

The sum of two positive integers (excluding zero) is 100; find the chance that their product exceeds 1,200.

23. Prove Lagrange's Interpolation Formula

$$u_x=u_a\frac{(x-b)(x-c)\ldots(x-k)}{(a-b)(a-c)\ldots(a-k)}+u_b\frac{(x-a)(x-c)\ldots(x-k)}{(b-a)(b-c)\ldots(b-k)}$$
$$+\ldots+u_k\frac{(x-a)(x-b)\ldots}{(k-a)(k-b)\ldots},$$

explaining carefully its use.

24. Define the symbols Δ, E, and Σ, and prove that Σ and Δ^{-1} are equivalent.

Show that $\mathrm{E}^p a^x\phi(x)=a^x.(a\mathrm{E})^p\phi(x)$, and that

$$\Sigma a^x\phi(x)=\frac{a^x}{a-1}\left[\phi(x)-\frac{a}{a-1}\Delta\phi(x)+\frac{a^2}{(a-1)^2}\Delta^2\phi(x)-\ldots\right],$$

where $\phi(x)$ is any rational integral algebraic function of x.

Examination for Admission to the Class of Associate (Part II).

Examiners—Messrs. H. Bearman, W. H. Hodgson, A. C. Thorne, and H. M. Trouncer, M.A.

First Paper.

1. Prove from first principles that if there be two annuities-certain, one payable at the end of every rth interval of a year and the other at the end of every tth interval of a year, the amounts payable annually, the effective rate of interest and the time for which they are to run being the same, the ratio of the value of the first annuity to that of the second is independent of the time for which they are to run.

2. From the following data, find the value of ${}_{2|}q_{\overline{30:31}}$:

Age (x)	Alive at beginning of year of age (x)	Entered during year of age (x)	Left during year of age (x)	Died during year of age (x)
30	12335	420	232	91
31	12432	397	245	94
32	12490	404	256	98
33	12541	381	239	101

3. Obtain a formula for the net annual premium, payable for n years, for a Whole-Life Assurance on (x), it being a condition that, should death occur within n years, in addition to payment of the sum assured, the excess of the net premiums actually paid over those which would have been paid under an ordinary Whole-Life Policy would be returned.

4. A certain sum of cash is to be employed in either of the following ways:

(*a*) To convert an ordinary Whole-Life Policy into an Endowment Assurance.

(*b*) To limit the number of future premiums payable under an ordinary Whole-Life Policy.

Deduce formulæ from which it will be possible to obtain, in the case of (*a*), the date at which the Endowment Assurance would mature, and, in the case of (*b*), the number of future premiums payable.

5. (*a*) Write down the formulæ for the Single Premium, and the Annual Premium, payable during t years only but ceasing on the first death, for an annuity deferred n years on the joint lives and the life of the survivor of (x) and (y), it being assumed that the mortality experienced during the period of n years will accord with a certain table (A) and after n years with another table (B), n being greater than t.

(*b*) If the only data available under table (A) consist of a table of Term annuities on Single lives, how would you compute the Single Premium?

6. Explain why the ratio of the average number of deaths between ages (x) and $(x+n)$ to l_x is not an adequate measure of the force of mortality at age (x) and deduce the perfect measure of the latter: if the average number of deaths between ages $(x-n)$ and $(x+n)$ were taken, would the ratio be more satisfactory? Give reasons for your answer.

7. Give a short account of how you would proceed to construct a table of the values of μ_x and e_x.

8. It is desired to raise £100,000 by an issue of Debentures. £5,000 is to be set aside each year to pay interest and provide for the redemption of the Debentures, the sum to be apportioned as follows:

(1) Interest at $4\frac{1}{2}$ per-cent is to be paid at the end of each year on the Debentures then outstanding.

(2) The balance of the £5,000 is to be invested, to yield $2\frac{1}{2}$ per-cent, to provide for triennial drawings of the Debentures at a premium of 5 per-cent, the first drawing to be at the end of the third year from the date of the issue.

Obtain an expression for the number of years, at the expiration of which the whole loan would be paid off.

Second Paper.

9. (*a*) Having calculated tables of the functions

$$(1+i)^n,\ v^n,\ s_{\overline{n}|} \text{ and } a_{\overline{n}|}$$

what checks could be used to test the accuracy of the figures?

(*b*) Interest tables frequently contain values of these functions at 1 per-cent, $1\frac{1}{8}$ per-cent, &c. For what purpose, and under what conditions, can these columns be used?

10. Assuming that deaths are uniformly distributed in each year of age, find the probability that of three lives (x), (y) and (z)

(*a*) At least two will die within t years.

(*b*) That (x) will die in the tth year, (y) and (z) surviving him.

(*c*) That (x) will die within t years and that (y) will die before (z) or within t years after the death of (z)

11. Derive and give a verbal interpretation of the following formulæ:

(*a*) $${}_nV_x = \left(P_x + \frac{P_x}{i} + 1\right)A_{x+n} - \left(P_x + \frac{P_x}{i}\right)$$

(*b*) $$P_x = \frac{|_nA_{x+m} + v^n{}_np_{x+m}\ {}_{n+m}V_x - {}_mV_x}{1 + |_{n-1}a_{x+m}}$$

12. Brewery 4 per-cent Debentures, redeemable in the year 1917 at 105, are at present quoted in the market at 95. A proposal is being brought forward to convert the Debentures into 3 per-cent Stock, redeemable at par in 1937, £133. 6*s*. 8*d*. of the new Stock being allotted for each £100 of the present holding. To what extent should the Market quotation of the Stock be affected by the proposed conversion on the assumption that the approximate yield at Market price should remain the same?

13. An Endowment Assurance to mature at age $(x+m)$ was issued n years ago, on a life then aged (x), with the condition that, if death occurred within the m years, the premiums paid were to be returned together with half of the sum assured, but that on maturity the sum assured only would be payable. Find by the retrospective method the value of the policy at the present time and prove the identity of this expression with that obtained by valuing by a prospective formula. Assume net premiums throughout.

14. Explain clearly the principles and uses of an ordinary Conversion Table. How would you proceed to form such a table and what steps would you take to prevent an error accumulating?

Show, by the use of first and second differences only, that the value of $A_{x:\overline{n-\frac{1}{m}}|}$ may be obtained by entering the table with $a_{x:\overline{n-\frac{m+1}{m}}|}$.

15. What is meant by Select Tables? State and explain the symbols usually adopted.

Given complete tables of the D and N columns, explain in detail how you would construct tables of select temporary annuities by the aid of logarithms.

16. Given the formula

$$\int_0^\omega u_x dx = \frac{1}{m}(u_0 + u_{\frac{1}{m}} + u_{\frac{2}{m}} + \ldots + u_\omega) - \frac{1}{2m}(u_0 + u_\omega) + \frac{1}{12m^2}\left(\frac{du_0}{dx} - \frac{du_\omega}{dx}\right), \text{ \&c.}$$

and assuming that the terminal values of the function and its differential coefficients vanish, deduce an approximation to the value of $\int_0^\omega u_x dx$, involving n, u_{-n}, u_n, u_{3n}, u_{5n}, &c.

Show how to apply the formula so deduced to calculate the value of $\bar{a}_{25}$.

Third Paper.

17. A Bank employs 300 clerks: they enter at age 18 at a salary of £30 per annum, which is increased £10 a year up to a maximum of £300: at age 60 they retire on a pension. Assuming that 10 per-cent leave the Bank after exactly five years' service and that after twenty years' service 5 per-cent are promoted from the clerical staff to higher positions, how would you proceed to find the ultimate state of the clerical staff in the following particulars?

(*a*) Total annual sum payable in salaries.

(*b*) Number superannuated each year.

(*c*) Number of pensioners on the books.

18. The leaseholder of property, which has 30 years to run, of annual value £100, wishes to sell the remainder of his lease after reserving to himself the rental for the next 10 years. A purchaser wishes to realize 5 per-cent on his outlay for the entire term, it being assumed that he will not be able to re-invest at more than $3\frac{1}{2}$ per-cent.. What must he give for the property?

19. On the assumption that $a_x^{(m)} = a_x + \frac{m-1}{2m}$, prove algebraically and by general reasoning that the addition to be made to P_x in order to obtain $P_x^{(m)}$ is $(P_x + d)\left(\frac{m-1}{2m} P_x^{(m)}\right)$.

20. Given the equation

$$\log {}_tp_x = t \log s + c^x(c^t - 1)\log g$$

deduce the values of l_x and μ_x.

If Gompertz's formula for the law of mortality holds, prove that for n joint lives of different ages we may substitute one or any number, m, joint lives of equal ages; and if Makeham's formula holds, prove that we can substitute the same number, n, joint lives of equal ages.

21. What do you understand by a Commutation Table? Explain the relations of the columns to each other.

Obtain an expression, in commutation symbols, which will give a close approximation to the annual premium for a deferred annuity, payable during the remainder of a life, now aged x, after the expiration of n years, the annuity to be payable by half-yearly instalments and apportionable to the day of death.

22. Prove the formulæ

(*a*) $\bar{A}_x = 1 - \delta\bar{a}_x$

(*b*) $\bar{A}_x = \mu_x\bar{a}_x - \dfrac{d\bar{a}_x}{dx}$.

23. Having given a series of equidistant terms, show how to sub-divide each interval into n parts.

Calculate the values of p_{31}, p_{32}, p_{33} and p_{34}, having given that

$$p_{30} = .99405 \qquad p_{40} = .99085$$
$$p_{35} = .99262 \qquad p_{45} = .98847$$

24. Explain the principles that should guide you, in the collection and disposal of your data, if you were called upon to construct a Mortality Table from Census Returns and Death Registers.

If the population by the Census, taken on 1 April 1891, be P and that by the Census, taken 1 April 1901, be rP, show that the true mean population of the 10 years, 1 January 1891 to 31 December 1900, will be represented by the expression

$$\frac{rP - P}{r^{\frac{1}{40}} \log_e r}.$$

Examination for Admission to the Class of Fellow (Part III).

Examiners—Messrs. A. Levine, M.A., E. A. Rusher, J. Spencer, and R. R. Tilt.

First Paper.

1. Give a short account of the method of construction of the London Life Table, 1891-1900.

2. It is desired to investigate the Sickness Experience of a Friendly Society. Draft a form of card for use in collecting the necessary data, and explain how you would proceed to deduce tables of ungraduated sickness rates for various periods of attack.

Having given the following ungraduated data, apply Mr. G. F. Hardy's Friendly Society formula to obtain the graduated values of u_{30}, u_{31}, and u_{32}.

Age (x)	u_x	Age (x)	u_x	Age (x)	u_x
21	·0409	28	·0526	35	·0760
22	·0429	29	·0563	36	·0778
23	·0422	30	·0587	37	·0828
24	·0530	31	·0595	38	·0846
25	·0505	32	·0647	39	·0836
26	·0459	33	·0669	40	·0916
27	·0499	34	·0746	41	·0956

3. Describe the method suggested by Mr. G. F. Hardy of deducing formulas for graduation by summation based on the assumption of constant third differences.

Show how, without actual trial, the suitability of any given formula for employment in practice may be determined.

4. Explain shortly the method adopted in the graduation of the $O^{[NM]}$ Table, dealing separately with

(*a*) the ultimate table, and

(*b*) the first five years of assurance.

5. State in what form the results of the investigation made by the Actuarial Society of America into 98 special classes of risks are published, and mention generally the distinctive features disclosed by those results.

How far do you regard the experience as serviceable in dealing with questions of extra risk in other countries?

6. For what purposes and by what methods have actuarial tables been constructed from statistics relating to the families of the Peerage?

7. In the calculation of office rates of premium, do you consider it desirable to employ the same mortality basis for participating as for non-participating assurances? Consider specially

(*a*) Whole-Life Assurances;

(*b*) Endowment Assurances.

What practical difficulties have to be borne in mind?

8. What bases as to mortality, interest, and loading would you employ in calculating office premiums for Double Endowment Assurances? Give reasons for your answer.

In what circumstances and to what extent do you regard this plan as suitable in dealing with proposals on under-average lives?

Second Paper.

9. What alternative bases as regards mortality have been suggested for the computation of premiums for Contingent Survivorship Assurances? State which, in your opinion, is most suitable and give your reasons.

How far would you regard the mortality basis adopted in the calculation of the premiums as appropriate for the calculation of valuation reserves for Contingent Survivorship Assurances?

10. Investigate the origin of the theoretical error involved in the valuation of Endowment Assurances in groups by Mr. Lidstone's method, assuming that the Mortality Table follows Makeham's Law.

In a group of 400 Endowment Assurances maturing in 1929 and assuring together £250,000, all the policies mature at ages varying from 45 to 65 except one for £1,500 on a life now aged 53. How would you deal with this policy in the valuation? Give reasons for your answer.

11. What are the different methods adopted in practice for the valuation of Whole-Life Limited-Payment Policies?

An office whose limited-payment premiums are obtained by commuting its ordinary whole-life premiums by the $H^{[M]}$ Table at $3\frac{3}{4}$ per-cent interest makes a net premium valuation of its ordinary business by the $O^{M(5)}$ Table at 3 per cent. What would you consider a suitable basis for the valuation of its limited-payment policies, assuming that the office transacts a large business under this plan?

12. A progressive mutual life office, whose new business has largely increased in recent years, has hitherto valued by the combined H^{M} and $H^{M(5)}$ Tables at 3 per-cent, and has declared at four successive quinquennial distributions a cash bonus equal to 30 per-cent of the ordinary premiums paid. It has been decided to adopt the new British Offices Experience for future valuations. State what basis of valuation you would recommend and discuss the effect of the change on the reserves and on the relative amounts of profit from the various sources.

13. What are the different methods that have been proposed for valuing joint-life policies in offices transacting industrial business and consequently having a large number of these risks?

Which do you think the most suitable for practical purposes?

Give reasons for your answer.

14. In a proprietary company transacting a large new business the expenses of the life department are fixed at 10 per-cent of the premium income, and the shareholders and participating policy-holders are entitled to share in the divisible surplus in the proportions 1 : 8 respectively. What steps would you take to satisfy yourself as to the probability of being able to maintain, in future, a given rate of compound reversionary bonus?

15. What principles would govern you in valuing the liabilities and assets of a life assurance company for purposes of transfer, and to what points would you give special consideration before deciding upon the terms of such transfer?

16. A company guarantees, in the case of participating policies, a minimum surrender-value, after payment of two years' premiums, of a certain fixed proportion of the total premiums paid. It is desired to adopt a basis for surrender-values which, while conforming to this guarantee, shall be similar for all ordinary classes of policies and for all durations.

Discuss the basis and formula you would suggest, dealing specially with any difficulties that may arise in the early policy years, and also in the case of lives subject to extra premium on account of occupation.

Third Paper.

17. An office, whose participating premiums are the $H^{[M]}$ 3 per-cent rates loaded with a percentage and a constant, makes annual valuations by the $O^{M(5)}$ Table at $2\frac{3}{4}$ per-cent. Explain how you would estimate and divide the profit from

(*a*) Interest.

(*b*) Mortality.

18. A is absolutely entitled to one-fifth of the under-mentioned fund on the death of B. Draft letters (*a*) to a proposed vendor informing him of the conditions on which your company is prepared to purchase, and (*b*) to the company's solicitors instructing them to proceed with the purchase.

£10,000 Consols.

Rs. 25,000 Indian Rupee Paper $3\frac{1}{2}$ per-cent.

£12,000 advanced at $4\frac{1}{2}$ per-cent, on Mortgage of a Freehold Farm.

£4,000 advanced in equal shares to four beneficiaries and £2,000 advanced to the vendor.

19. What preliminary enquiries would you make before entertaining a proposal for a loan on a Life Interest? Some offices make a practice of refusing to purchase such interests though willing to lend upon them. What are the difficulties which give rise to this distinction, and why do they not apply in the case of loans? Show how the distinction is intensified in the case of Reversionary Life Interests.

20. A (a naval officer), aged 25, is entitled on the death of B, a male, aged 65, to a life interest in an estate producing from well secured ground rents, £3,000 a year. He desires to raise £1,000, giving a charge on his life interest to be paid by six half-yearly instalments accruing from the death of B. Give a formula for the amount of the charge, shewing clearly the nature of the assurance and your method of calculating the premium. What mortality tables and rates of interest would you use?

21. An advance of £5,000 is to be made on an absolute reversion of ample value in consideration of a reversionary charge to be paid on the death of the tenant for life, a male, aged 60. The reversioner is to have the option to redeem the charge during the first five years on payment of £5,000 with compound interest at 5 per-cent per annum, and if he make a cash payment at the end of that period the option is to be extended for a further five years.

Investigate formulas for the amount of the reversionary charge and the amount of the cash payment to be made at the end of five years.

*22. A, aged 30, is entitled at the death of his father, aged 65, should he then be living, to a large settlement fund. Find the amount of the reversionary charge which he should give for an immediate advance of £3,000 and an annuity of £250 per annum to be paid to him quarterly whilst he and his father are both living.

*23. A sum of £8,000 $2\frac{1}{2}$ per-cent Consols is charged with the payment of the following annuities:

£50 to a female, aged 72.

£40 ,, ,, ,, 75.

£60 ,, male, aged 74, and an annuity of £30 to any widow he may leave (he has a wife living, aged 70).

£10 to the trustees so long as the trust lasts.

The surplus income is to be distributed by the trustees periodically at their discretion amongst five persons in equal shares, and the capital is to be distributed in similar shares as the annuitants die. Find the value of a fifth share.

* In answering these questions the Candidate is to set out his work as in an actual numerical valuation, but to exhibit the final results in terms of the actuarial functions involved, without inserting the numerical values of the functions, stating, however, the mortality tables and rates of interest which he would use.

*24. Estimate the market value of a whole-life policy for £4,000 on a male, born 4 July 1845. The annual premium is £80, due 1 June, and the bonus additions to 31 December 1903, amount to £1,600. The basis of distribution is a cash bonus of a uniform percentage of the premiums paid during the quinquennium with an option to take an equivalent reversionary bonus. At the last three divisions the cash bonus has been 25 per-cent of the premiums paid, with an interim bonus at the same rate.

Examination for Admission to the Class of Fellow (Part IV).

Examiners—Messrs. H. W. Andras, J. R. Hart, E. C. Thomas, and J. D. Watson.

First Paper.

1. Distinguish between real and personal property.

In what circumstances can the same class of property be in some cases "real" and in other cases "personal" property? How would such a distinction affect

(*a*) Estate or other duties; and

(*b*) The doctrine of notice?

2. A has a general power of appointment over settled funds at his death, and appoints a sum to B, who asks you to lend on his reversionary interest. What legal points would require your consideration?

3. Define "Insurable Interest," "Warranty," and "Fraudulent Misrepresentation."

Is an "Indisputable Policy" legal in England?

4. The Rules of a Bank Provident Fund securing annuities to the widows of members dying after 15 years' active service provide that in the event of death within the first 15 years of service the contributions are to be returned with compound interest at 4 per-cent per annum. If death occur after 15 years, the accumulated contributions of *unmarried members only* are to be returned. The contribution is a fixed percentage of the annual salary, and is payable monthly. How would you calculate the value of the "Return of Contribution" benefit in a valuation based on 4 per-cent interest?

* In answering this question the Candidate is to set out his work as in an actual numerical valuation, but to exhibit the final results in terms of the actuarial functions involved, without inserting the numerical values of the functions, stating, however, the mortality tables and rates of interest which he would use.

5. Describe the difference between a cash account and a revenue account, and show how the latter acts as a connecting link between two consecutive balance-sheets.

How would you treat the following items in the published annual accounts of a Life office?

(*a*) Unpaid half-yearly instalments of annual premiums.
(*b*) Interest accrued but not yet payable.
(*c*) One-third credit premiums.
(*d*) Reductions of premium by application of bonus.

6. State the advantages and disadvantages, with the respective modes of transfer, of "Inscribed", "Registered", and "Bearer" Stock Exchange securities.

Explain the difference between Debentures and Debenture Stock and the meaning of a floating charge.

7. State the leading points for consideration in connection with an application to a life assurance company for a loan to a municipal authority upon security of the rates.

8. The minimum rate of discount of the Bank of France recently continued at 3 per-cent for several years, and that of the Bank of England showed considerable fluctuations during the same period. How do you account for this?

What were the chief reasons for raising the Bank of England minimum rate of discount to 6 per-cent in the autumn of 1906?

Second Paper.

9. What is the effect of an assignment of a lease upon the rights and liabilities of lessor, lessee, and assignee?

10. In the absence of special stipulation, what evidence of title is required to be shown by (*a*) a vendor on a sale, and (*b*) a borrower on a mortgage, of freehold and leasehold land respectively?

11. Describe briefly the nature of the proceedings following the making of a Receiving Order against a debtor in England.

Discuss the legal position in England, Scotland, and Ireland, respectively, with regard to searches in bankruptcy.

12. What are the arguments for and against the employment of select tables of mortality, withdrawal, and superannuation in (*a*) the calculation of rates of contribution for a Staff Pension Fund and (*b*) the valuation of its liabilities?

13. What conditions are imposed on Friendly Societies registered in the United Kingdom with regard to the following points?

(*a*) The grant of annuities.
(*b*) The assurance of adult lives.
(*c*) The assurance of children's lives.
(*d*) Income tax.

14. The following notice appeared in a money article last year:

Money and Exchange.	New York. To-day.	New York. Yesterday.
Exchange on London-sight ...	4.85.70	4.85.70
,, Cable transfers ...	4.86.70	4.86.65
,, 60 days' sight ...	4.80.60	$4.80\frac{1}{2}$

Explain, in detail, what these figures mean.

15. A life office has power to invest in British or American railway

(*a*) Debenture Stocks and Bonds.

(*b*) Preference Stocks.

(*c*) Ordinary and common Stock.

In which country would you advise an investment at the present time and in which class of security. Give reasons.

16. In connection with a proposed mortgage of a freehold, a statement of a year's gross rental is furnished showing the income derived from the following items:

Mansion House and Park, Farms, Cottages, Shooting, Timber, Quarries, and Mines.

Discuss each of these items from a mortgagee's point of view, stating what further enquiries you would make.

Third Paper.

17. A Life office is asked to lend upon one of its policies. How would you deal with the application, if received from

(*a*) The assured, after notice of a mortgage has been received;

(*b*) The trustees under a settlement of the policy;

(*c*) A mortgagee of the policy?

18. What are the various forms of constitution of Life Assurance Companies in the United Kingdom? State briefly the advantages or disadvantages of each.

What should the Memorandum and Articles of Association of a Life Assurance Company, registered under the Companies Acts, respectively contain?

19. In connection with the valuation of a Widows' Fund upon the basis of its own experience, you are supplied with tables showing the relative ages of husbands and wives

(*a*) At date of marriage.

(*b*) At death of the husband.

(*c*) At date of valuation.

How would you expect the three tables to compare? Which would you consider the most satisfactory basis for a valuation by the collective method, and why? Assume the material to be sufficiently extensive to give reliable results.

20. When the country is prosperous and trade is good, would you advise a life office to invest its money in (*a*) Mortgages, (*b*) Stock Exchange securities, (*c*) Reversions? Give reasons.

21. It is desired to arrange a loan on Stock Exchange Securities. How would you advise a life office as a lender to carry out the transaction?

22. Upon the quinquennial valuation of assets of a Life Office there are disclosed deficiencies from book values in respect of some of the (*a*) mortgages and (*b*) convertible securities.

How would you deal with these deficiencies in the revenue account and balance sheet? Give reasons.

23. It has been stated that the gold reserves of the United Kingdom are inadequate. What basis is there for this statement? and what remedies would you suggest?

24. Draft a brief report to the Directors of a Life Office, explaining clearly in respect of a financial year

(*a*) The causes of a change in the average rate of interest earned on the funds.

(*b*) Particulars of the profit or loss from mortality.

(*c*) The strain of the Office expenditure.

PROCEEDINGS OF THE INSTITUTE.—Session 1906–1907.

First Ordinary Meeting, 26 *November* 1906.

The first ordinary meeting of the Session 1906-1907 was held at the Hall of the Institute, on the 26th day of November 1906.

The President (Mr. Frank B. Wyatt) in the Chair.

The President delivered an Inaugural Address.

Second Ordinary Meeting, 17 *December* 1906.

The President (Mr. Frank B. Wyatt) in the Chair.

A paper entitled "On the Error introduced into Mortality Tables by Summation Formulas of Graduation", by Mr. George King, was read by the Author.

The following gentlemen took part in the discussion:—Messrs. A. W. Watson, T. G. Ackland, G. J. Lidstone, H. P. Calderon. F. B. Galer, R. Todhunter, and the President, who also read a communication on the subject from Dr. T. B. Sprague.

Third Ordinary Meeting, 28 *January* 1907.

The President (Mr. Frank B. Wyatt) in the Chair.

A paper entitled "Further Notes on some Legal Aspects of Life Assurance Practice", by Mr. A. R. Barrand, was read in abstract by the Author.

The following gentlemen took part in the discussion:—Messrs. J. R. Hart, E. J. MacGillivray (a visitor), J. H. Barnes, W. C. Sharman, C. R. V. Coutts, W. T. May, H. E. W. Lutt, Dr. A. W. Findlay, E. C. Doust-Smith, and J. E. Faulks.

Fourth Ordinary Meeting, 25 *February* 1907.

The President (Mr. Frank B. Wyatt) in the Chair.

A paper entitled "Comparative Bonuses under Whole-Life and Endowment Assurances", by Mr. H. J. Rietschel, was read by the Author.

The following gentlemen took part in the discussion:—Messrs. H. T. Adlard, S. J. H. W. Allin, H. H. Austin, C. R. V. Coutts, R. P. Hardy, W. P. Elderton, H. W. Manly, W. P. Pulley, H. J. Baker, and the President.

Fifth Ordinary Meeting, 25 *March* 1907.

The President (Mr. Frank B. Wyatt) in the Chair.

Messrs. Hugh Francis Cowan, F.F.A., and Charles Carlyon Nicholl, B.A., F.F.A., were duly elected Associates of the Institute.

The President announced that, for the Messenger Prizes offered by the Council in 1905 for essays on "The Methods of Ascertaining the Rates of Mortality amongst the general population of a Country, District or Town, or amongst different classes of such population, by means of Returns of Population, Births, Deaths and Migrations", one essay was received, for which a Prize of Twenty-five Guineas had been awarded to the Author, Mr. Charles H. Wickens, A.I.A., of Melbourne, Australia.

A paper entitled "On the Relation between the Theories of Compound Interest and Life Contingencies", by Mr. J. Mayhew Allen, was read by the Author.

The following gentlemen took part in the discussion:—Messrs. T. P. Thompson, C. W. Kenchington, D. C. Fraser, and S. G. Warner.

Sixth Ordinary Meeting, 29 *April* 1907.

The President (Mr. Frank B. Wyatt) in the Chair.

Mr. Gilbert Edward Shearer, F.F.A., was duly elected an Associate of the Institute.

A paper entitled "On Extra Premuims", by Mr. H. E. W. Lutt, was read by the Author.

The following gentlemen took part in the discussion:—Messrs. J. R. Hart, C. R. V. Coutts, W. H. Hodgson, J. Burn, J. Bacon, E. A. Rusher, W. Penman, H. W. Manly, and S. G. Warner.

Mr. H. N. Sheppard, A.I.A., of New York, exhibited specimens of Life Policies standardized in the State of New York, and made a few explanatory remarks respecting them.

The Sixtieth Annual General Meeting, 3 *June* 1907.

The President (Mr. Frank B. Wyatt) in the Chair.

The Proceedings of the Annual General Meeting will be found on page 609.

REPORT, 1906–1907.

The Council have the pleasure to report to the members upon the progress of the Institute during the session of 1906–1907, the fifty-ninth year of its existence.

There has been an *increase* of 34 in the number of members, as compared with the previous year. At the end of the official year in which the Institute was incorporated by the Royal Charter the number of members

was 434, while ten years later, at 31 March 1895, it was 775. Since that time the numbers have been as follows:

On 31 March 1896, 788,
,, 1897, 826,
,, 1898, 860,
,, 1899, 834,
,, 1900, 822,
,, 1901, 818,
,, 1902, 842,
,, 1903, 828,
,, 1904, 856,
,, 1905, 881,
,, 1906, 922,
,, 1907, 956.

The following schedule shows the additions, changes, and losses in the membership, which have occurred during the year ending 31 March last:

Schedule of Membership, 31 *March* 1907.

	Honorary Members	Fellows	Associates	Students	Corresponding Members	Total
i. Number of Members in each class on 31 March 1906 .	1	232	301	367	21	922
ii. Withdrawals by						
(1) Death . . .	...	...	1	1	...	48
(2) Resignation or otherwise . .	...	2	10	34	...	
	1	230	290	332	21	874
iii. Additions to Membership						
(1) By Election . .	...	...	4	...	...	
(2) By Order of Council	...	...	...	65	...	82
(3) By Re-instatement	...	2	5	6	...	
	1	232	299	403	21	956
iv. Transfers						
(1) By Examination: *from* Associates	...	...	13	...	...	...
to Fellows .	...	13	...	...	...	...
	1	245	286	403	21	956
(2) By Examination: *from* Students	...	...	...	3	...	...
to Fellows .	...	3	...	...	...	...
	1	248	286	400	21	956
(3) By Examination: *from* Students	...	...	...	17	...	...
to Associates .	...	...	17	...	...	...
v. Number of Members in each class on 31 March 1907 .	1	248	303	383	21	956

[*Continued on page* 604.]

Dr. *Revenue Account for the*

	£ s. d.	£ s. d.
Amount of Funds at the beginning of the year made up as under—		
General Fund	9,018 0 3	
Messenger Legacy Fund	391 14 1	
Brown Prize Fund	269 6 7	
	9,679 0 11	
British Offices Valuation Tables Expenses Fund	221 6 5	
		9,900 7 4
Subscriptions—		
Fellows	712 19 0	
Associates	600 12 0	
Students	405 6 0	
Probationers	79 16 0	
	1,798 13 0	
Fines for Re-instatement	0 12 6	
		1,799 5 6
Application Fees—		
Associates	8 8 0	
Students	42 0 0	
Probationers	37 16 0	
		88 4 0
Examination Fees for year 1906		297 3 0
Lecture and Class Fees		215 17 0
Sales of Publications—		
Journal	179 7 8	
Text-Book, Part I	41 15 6	
Text-Book, Part II	153 15 4	
Government Annuity Tables	2 6 4	
Select Life Tables	4 4 8	
Frequency-Curves and Correlation	39 16 2	
Short Collection of Actuarial Tables	5 14 9	
Hardy's Friendly Societies	1 6 11	
Legal, Financial, and Statistical Lectures	5 8 10	
Transactions of Second International Congress	3 15 0	
Syllabus and Examination Questions	6 14 7	
British Offices Life Tables	30 11 2	
		474 16 11
Dividends and Interest—		
General Fund	269 0 3	
Messenger Legacy Fund	11 15 0	
Brown Prize Fund	8 1 7	
British Offices Valuation Tables Expenses Fund	1 8 5	
		290 5 3
Refunded by the Faculty of Actuaries, being one-half share of expenses on account of the Combined Volume of British Offices Select Tables		136 7 6
		£13,202 6 6

Balance Sheet,

LIABILITIES.

	£ s. d.	£ s. d.	£ s. d.
General Fund		7,971 1 10	
Messenger Legacy Fund	233 9 2		
Accumulated Dividends	143 14 11		
		377 4 1	
Brown Prize Fund	200 0 0		
Accumulated Dividends	77 8 2		
		277 8 2	
			8,625 14 1
Examination Fees for year 1907			155 8 0
Sundry unpaid accounts			20 19 2
			£8,802 1 3

year ending 31 *March* 1907. **Cr.**

	£	s.	d.	£	s.	d.
Journal—						
Printing of Nos. 224, 225, 226, and 227	404	12	0			
Clerical assistance	60	0	0			
Expenditure on account of Index to Forty Volumes	50	15	3			
				515	7	3
Library—						
Binding, Purchases, and Index Cards	67	18	9			
Expenditure on Account of a New Edition of the Catalogue	29	12	6			
Additional Bookcase	42	0	0			
				139	11	3
"Frequency-Curves and Correlation" (W. P. Elderton). Printing and Editorial Expenses				276	8	6
Publications' Account—Binding				30	2	4
Meetings				69	2	3
Examination charges				81	4	7
Lecturer, and Tutors for classes in Parts I and II				464	2	0
Messenger Prize Award				26	5	0
Office Expenditure—						
Rent	600	0	0			
Salaries and Pension	360	8	8			
House expenses	70	3	9			
Corporation Duty	13	4	2			
Fire Insurance	26	16	0			
Stationery and Printing	130	2	2			
Postage and Telegrams	45	16	11			
Furniture and Fittings	4	10	10			
Sundries	5	16	4			
				1,256	18	10
Expenditure on account of the Volume of British Offices Valuation Tables				276	7	9
Expenditure on account of the Combined Volume of British Offices Select Tables				272	15	0
Amount written off in respect of decrease in value of Stock Exchange Securities				1,168	7	8
Amount of Funds at the end of the year, as per Balance Sheet				8,625	14	1
				£13,202	6	6

Examined and found correct, 29 April 1907.

GEO. A. BROWN,
HUGH LUGTON,
STANLEY HAZELL, } *Auditors.*

31 *March* 1907.

ASSETS.

	£	s.	d.
£3,000 Natal 3 per-cent Inscribed Stock at 82	2,460	0	0
£1,200 Metropolitan Railway 3½ per-cent Debenture Stock at 94	1,128	0	0
£1,750 Great Eastern Railway 4 per-cent Debenture Stock at 114	1,995	0	0
£1,000 Great Northern Railway Preferred Ordinary Stock at 96½	965	0	0
£1,350 Great Western Railway 4¼ per-cent Debenture Stock at 123	1,660	10	0
Cash on Current Account	593	11	3
	£8,802	1	3

[The Institute also possesses certain copyrights and stocks of publications (see p. 604).]

Examined and found correct, 29 April 1907.

GEO. A. BROWN,
HUGH LUGTON,
STANLEY HAZELL, } *Auditors.*

136 candidates have been admitted as Probationers, and 61 as Students conditionally on their passing Part I of the Examination. These are not included in the above Schedule of Membership.

The Council have, with great regret, to report the loss by death, since the last Annual Meeting, of one Fellow, Mr. A. C. R. Cockman; one Associate, Mr. H. E. Stamp; and two Students, Mr. C. H. Coventry and Mr. C. D. Hill.

The Annual Subscriptions, together with admission and other fees, amounted to £2,235. 12*s*. 6*d*., as compared with £2,169. 7*s*. 6*d*. received in the previous year. The total Income for the year was £2,845. 6*s*. 8*d*., and the total Expenditure £3,101. 15*s*. 3*d*. The Revenue Account and Balance Sheet are given herewith (pp. 602, 603).

In view of the low prices of Stock Exchange securities, it has been thought advisable to write down to the value of the day those held by the Institute. This has accordingly been done, and the necessary entry of £1,168. 7*s*. 8*d*. appears in the Revenue Account.

The stock in hand of the Institute publications on 31 March was as follows:

No. of Copies	Description of Work
15,899	Parts of *Journal*.
501	Index to Vols. 1 to 10.
969	,, to Vols. 21 to 30.
737	*Text-Book*, Part I (New Edition).
1,086	,, Part II (Second Edition).
675	Government Joint-Life Annuity Tables.
759	Select Life Tables.
279	A Short Collection of Actuarial Tables.
1,819	Frequency-Curves and Correlation (W. P. Elderton).
189	Messenger Prize Essay (Friendly Societies).
6 *in cloth* 2,887 *in paper*	Lectures on Finance and Law (Clare and Wood Hill).
1,621	Lectures on the Companies Acts (A. C. Clauson).
1,549	Lectures on the Law of Mortgage (W. G. Hayter).
804	Lectures on the Measurement of Groups and Series (A. L. Bowley).
692	Transactions of the Second International Congress of Actuaries.
2,246	Syllabus and Examination Questions.

The following papers were submitted at the sessional meetings of the Institute, namely:

26 *November* 1906.—An Inaugural Address by the President.—Mr. Frank B. Wyatt.

17 *December* 1906.—"On the Error introduced into Mortality Tables by Summation Formulas of Graduation."—Mr. George King.

28 *January* 1907.—"Further Notes on some Legal Aspects of Life Assurance Practice."—Mr. A. R. Barrand.

25 *February* 1907.—"Comparative Bonuses under Whole-Life and Endowment Assurances."—Mr. H. J. Rietschel.

25 *March* 1907.—"On the Relation between the Theories of Compound Interest and Life Contingencies."—Mr. J. Mayhew Allen.

29 *April* 1907.—"On Extra Premiums."—Mr. H. E. W. Lutt.

For the Examinations held in the United Kingdom and the Colonies on 19, 20, 22, and 23 April last, 302 entries were received, namely

121	for	Part	I.
116	„	„	II.
54	„	„	III.
11	„	„	IV.

The results of the Examinations will be duly announced.* The Council warmly acknowledge the valuable services of the Honorary Examiners and Supervisors.

In their last Report, the Council announced that further Monetary Tables, based on the British Offices Experience, were in preparation. They have now the pleasure to report the recent publication of an inclusive volume of SELECT TABLES, based upon the experience of the $O^{[M]}$ and $O^{[NM]}$ Tables. These Tables have been published jointly by the Institute and the Faculty of Actuaries, and incorporate, in one volume, practically the whole of the Select Tables published by the Joint Mortality Committee in June 1903, and also (by arrangement with the Authors and Publishers of the volume) the Select Tables issued by Messrs. H. J. BAKER and A. H. RAISIN in October 1904. The opportunity has also been taken to include, in the volume now published, important additional Tables based upon the $O^{[M]}$ and $O^{[NM]}$ Experience, including Whole-Life $O^{[M]}$ Policy-values for all entry ages and durations, at six different rates of interest; further complete Tables of Term Annuities according to the $O^{[NM]}$ Table; and certain Annuity Tables on two Joint Lives. It is hoped that it will be found greatly for the convenience of members of the profession to have the Select Tables already published, together with these useful additional functions, included in a single comprehensive volume.

The Council have also the pleasure to announce the early publication by the Institute of a volume of VALUATION TABLES, based upon the O^M experience, which, it is hoped, will usefully supplement the Tables already published on that basis by the Joint Mortality Committee. The new volume, to the cost of which 41 English offices have generously contributed, will include, at different rates of interest, complete Tables of Policy-values for Whole-Life Assurances with uniform premiums, and with a limited number of premiums, and for Endowment Assurances; also Annual and Single Premiums for various classes of Assurance, and other valuation factors; and these Tables will, it is hoped, be a further practical contribution to the working tools of the Actuary. Complete Tables of Term Annuity-values, according to the O^M and $O^{M(5)}$ Tables at $2\frac{3}{4}$ per-cent (a rate at which these functions were not included in the volume published by the Joint Mortality Committee) will also be issued, in a separate form, with the volume of O^M Tables.

The Council desire to express their appreciation of the services of the committee, consisting of Messrs. R. P. Hardy, T. G. Ackland, and G. J. Lidstone, who, in conference with a committee appointed by the Faculty of Actuaries, and with Messrs. Baker and Raisin, have made the necessary arrangements for the publication of the volume of Select Tables recently issued; and who have also undertaken the preparation and publication, on behalf of the Institute, of the volume of O^M Tables about to be issued.

* These results, for the United Kingdom and Colonies combined, are given on pp. 607–8.

The revised Catalogue of the Library of the Institute, and the complete Index to the first 40 volumes of the *Journal*, are approaching completion, and will, it is hoped, be issued at a very early date.

Mr. W. Palin Elderton has, at the suggestion of the Council, been good though to prepare a treatise on "Frequency-Curves and Correlation", which has been published by the Institute during the Session. It is felt that this work, dealing as it does with an important and difficult subject, will be of permanent value, not only to the actuarial profession, but to many others who are interested in the study of these special statistical methods.

The Fifth International Congress of Actuaries was held in Berlin in September last, under the auspices of the German Federation for Insurance Science (founded in 1899); and greater State recognition, attention and encouragement were obtained than have been accorded to any previous Congress. His Excellency, the Graf von Posadowsky, Minister of the Interior, accepted the position of Honorary President, and the proceedings passed off most successfully under the presidency of Dr. Ferdinand Hahn, of Magdeburg. The President, Mr. Frank B. Wyatt, two Vice-Presidents, Messrs. T. G. Ackland and E. Woods, and four Fellows, Messrs. H. W. Andras, W. Hughes, H. W. Manly, and W. P. Phelps, attended the Congress as official delegates, and several other Members of the Institute were also present. One hundred and ten papers were presented, twelve of them being by Members of the Institute. The two volumes of "Reports, Memoirs, and Proceedings", which were published with commendable promptitude before the meeting, and a further volume containing a full report of the proceedings and discussions, form a most valuable addition to the literature of the profession.

It was with great regret that the Council had to record the death of the distinguished President, Dr. Hahn, on the 12th of December last.

In 1905, the Council, from the Messenger Legacy Fund, offered two prizes for the two best Essays on "The Method of ascertaining the Rates "of Mortality amongst the general population of a Country, District, or "Town, or amongst different classes of such population, by means of Returns "of Population, Births, Deaths, and Migration." The offer resulted in the submission of one essay only,—by Mr. C. H. Wickens, A.I.A., of the Commonwealth Bureau of Census and Statistics, Melbourne, Australia, and to this Essay the Council had the pleasure of awarding a prize of Twenty-five Guineas.

During the past Session it came under the notice of the Council, that the Fellows and Associates of a Society known as the "Institute of Accountants" were using, to denote their status in that Society, the letters "F.I.A." and "A.I.A.", printed after their names. It was felt that this should not, in the interests of the Institute of Actuaries, be permitted; and action was taken which finally resulted, on 1st February last, in an order by the Hon. Mr. Justice Joyce, restraining the Society in question from the use of the letters aforesaid.

Believing that the Members of the Institute would be glad to see in its *Journal* periodical reports of Law cases involving questions of interest to the profession, the Council have arranged that these shall appear, and have been fortunate in securing the services, to this end, of Mr. A. R. Barrand, who has joined the staff of the *Journal* for that purpose.

A gratifying incident of the Session has been the co-optation by the Royal Patriotic Fund Corporation of the President of the Institute of Actuaries for the time being to be a Member of the Corporation and of its Executive Committee for a period of three years, ending 31st December, 1909.

EXAMINATIONS, 1907.

Examinations were held on the 19th, 20th, 22nd, and 23rd of April, 1907, in the United Kingdom and the Colonies, at London, Liverpool, Edinburgh, Adelaide, Melbourne, Sydney, Montreal, Toronto, Ottawa, with the following results, the names in each class being arranged in alphabetical order:—

Part I.

One hundred and twenty-one candidates sent in their names, of whom one hundred and twelve presented themselves, and seventy-five passed, namely:—

Class I:

Baker, S. H.
Bullwinkle, L. A.
Harvey, P. N.
Henry, A.
Howell, P.
Lambert, A.
Mol, W. J. B.
Monilaws, S. H.
Morton, F. W.
Newland, E. A.

Class II:

Allen, S.
Beeston, H. L.
Carey, N. L.
Casebow, P. C.
Cashman, T.
Cox, H.
Crang, J. S.
Curtis, A. T. G.
Duffell, J. H.
Green, J. S.
Green, W. J.
Hanson, E. G.
Harding, D. A.
Holgate, B.
Hutchings, L. H.
James, R. W.
Keable, H. B.
Keachie, M. M.
Ledger, R. J.
Miller, A. A.
Nash, K. O.
Pattison, G. B.
Prout, H. J.
Shine, J. N.
Sturgeon, R. W.
Tayler, H. H.
Taylor, F. G.
Taylor, F. R. S.
Taylor, H. G. B.
Thornton, K. L.
Trembath, A. E.
Turner, J. G.
Wilson, W. C.

Class III:

Bailey, F. A.
Chandler, F. P.
Charles, A. H.
Coard, G. A.
Currie, J. T.
Emery, C. G.
Fidler, W. E.
Grant, F. J.
Guthrie, I. H.
Hudson, C. H.
Latham, F. N. W.
Mace, D.
Macleod, J.
Mills, C. K.
Pocock, H. G. G.
Rushton, T. A.
Sanders, B. G. T.
Shinmi, S. M.
Simmons, F. V.
Singer, C. P.
Smith, F. J.
Smith, R. T.
Smith, S. A. G.
Spiegel, E. W. R.
Stutfield, M.
Tomlinson, B.
Vineberg, H. E.
Welch, L. G.
Wenyon, H. J.
Wilkinson, C. S.
Williams, T. W.
Wright, A. W.

Part II.

One hundred and sixteen candidates sent in their names, of whom one hundred and four presented themselves, and twenty-six passed, namely:—

Class I:

None.

Class II:

Derrick, V. P. A.
Edwards, H. H.
Holness, A. S.
Hughes, T.
Phillips, T. A.
Warren, C. F.

Class III:

Allen, A. O.
Brown, A. E.
Burrows, V. A.
Deck, J. G.
Eldridge, E. E. B.
Emery, W. S.
Fulford, W. J.
Gunningham, S. J.
Harris, E. A.
Humphry, E. W.
Jones, E. S.
Laing, John M.
Langstaff, M. P.
Levey, R.
Ley, J.
Parker, J. G.
Reeve, G. M.
Reynolds, W. D.
Turner, S.
Underwood, R. E.

Part III.

Fifty-four candidates sent in their names, of whom forty-nine presented themselves, and twelve passed, namely:—

Class I:

None.

Class II:

Harriss, W. J.
Melville, H. E.

Class III:

Atkins, L. G.
†Blanchard, N.
Daman, G. W.
File, L. K.
Goodman, G.
Langstaff, J. M.
McKechnie, J. B.
Maltby, C. H.
Raynes, H. E.
Robertson, J. L.

Part IV.

Eleven candidates sent in their names, of whom nine presented themselves, and five passed, namely:—

Class I:

None.

Class II:

†Carter, N. J.
†Laing, James M.

Class III:

Falk, O. T.
Hudson, A. J.
†Mackenzie, M. A.

Those marked (†) have now completed the Examination for the Class of Fellow.

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Sixtieth Annual General Meeting of the Institute of Actuaries was held at Staple Inn Hall, Holborn, on Monday evening, 3 June 1907, Mr. F. B. Wyatt (the President) in the Chair.

The Report of the Council (given on p. 600) having been taken as read,

The PRESIDENT, in moving the adoption of the report, called attention to the further large increase in the number of members, now raised to 956. They were approaching a thousand members; if they looked back to the date of the Charter they would find that they then numbered only 775. He noticed, too, that the year showed a marked increase in the number of Fellows—from 232 to 248. Previous to last year, the number of Fellows did not increase very rapidly, and, incidentally, he thought they had here a matter for congratulation, since it meant that a considerable number of Associates had qualified for the higher degree. Death had been very kind to the members of the Institute during the year, for only four had passed away. He regretted to say, however, that they had lost one of the youngest and most distinguished of their Fellows, Mr. A. C. R. Cockman, at the very early age of 34.

With regard to the accounts, he observed that during the year there had been no increase, but a small decrease in the amount of the funds. In making that statement he was not taking into consideration the sum of £1,168. 7*s.* 8*d.* by which the funds had been written down. The real decrease in the General Fund, apart from that, was more than £100, and without troubling them with detailed figures he found their normal income was now about £300 more than their normal expenditure, and, curiously enough, that £300 was almost exactly the amount of the interest on their invested funds of £8,625. He submitted that this margin of £300 was not too large. As doubtless they were aware, it might be necessary to spend more in a very useful direction—extended tuition or lectures or something of that kind,—and he thought, therefore, that this was perhaps an answer to the criticism that had sometimes been made, that they were suffering from a plethora of wealth.

During the year they had had a number of very interesting and instructive papers read before the Institute, some highly technical and some thoroughly practical. He hoped the standard would be maintained during the coming session; and understood that already, by the energies of their honorary secretaries, arrangements had been made for two or three excellent papers.

He now came to what was, perhaps, the most important part of the report—the definite announcement of the issue of the volume of Select Tables, which had been carried out jointly by the Institute and the Faculty of Actuaries, the work being done almost entirely by a small committee in London, consisting of Mr. R. P. Hardy, Mr. Ackland and Mr. Lidstone. He thought the Institute could not be too profuse in their thanks to those three gentlemen. Referring to the approaching publication by the Institute of a volume of Valuation Tables based on the O^M Experience, under the supervision and direction of the same Committee, the President said he should

like to mention how much they were indebted to Mr. R. P. Hardy for his generosity in placing gratuitously at the service of the Institute a number of valuable tables which he had constructed. The Library Committee, presided over by Mr. Ackland, had done some very valuable work for the Institute during the year in the preparation (with the assistance of the Honorary Librarians) of the Catalogue of the Library, and in the large and responsible work of preparing the Index to the first forty volumes of the *Journal*. They were also indebted to Mr. Elderton for a very valuable statistical work on the lines advocated by Professor Karl Pearson.

With regard to the International Congress of Actuaries, which a number of their members attended last year, he had already given them a description of it (*J.I.A.*, xl, 6, *et seq.*), so that he would only repeat that it was in every way a thorough success and was well supported, both by the German Government and those who attended it. There were scarcely enough English actuaries there, and he did not think there were enough papers contributed by their English and Scottish friends. He hoped this would be remedied at the next Congress, to be held in 1909, at Vienna, and that contributions for that Congress would be forthwith prepared. The success of the last Congress was almost entirely due to the good administration, energy and courtesy of its organizing secretary, Professor Alfred Manes, who, they would be pleased to hear, was with them in the Hall at the present meeting.

A slight disappointment awaited the Council with regard to the Messenger Prize Essay, only one essay being sent in. Perhaps there was some excuse to be found in the fact that the Congress took some of their best papers.

A body calling itself the "Institute of Accountants" having adopted the letters "F.I.A." and "A.I.A." to denote membership of their Society, the Hon. Secretaries of the Institute of Actuaries, with the utmost promptitude, instructed their solicitors to apply for an injunction, which injunction was granted by Mr. Justice Joyce on the 1st of February last.

In these times, one of the most important things they had to watch was the effect of the ever-changing legislation, and it sometimes became a very difficult task. The members, therefore, would be pleased to hear that Mr. Barrand had undertaken, at the invitation of the Council and the Editor, to make concise reports in the *Journal* on all law cases which affected life assurance practice. At the end of the report they would find the statement that the Royal Patriotic Fund Corporation had made the President of the Institute of Actuaries for the time being a member of the Corporation and also a member of the Executive Committee. He (the speaker) had had the pleasure of attending two meetings, and only that afternoon he attended the annual meeting, which was presided over by the Duke of Connaught, who referred to the fact that the Corporation were now sure of their financial position. The assistance rendered by the Council of the Institute, in making valuations of the several funds under the administration of the Corporation, it was gratifying to know, had been fully appreciated.

The matters mentioned in the report did not give a full idea of the work which had been carried on by the Council. He had himself been a member of the Council for twenty years, and he never remembered a time when there was so much to do. There had been an unusually large number of committee meetings, on subjects which were still *sub judice*. The first of these was

the consideration of amendments to the Life Assurance Companies Acts of 1870 and 1872. A committee was appointed, and had held very many and very lengthy sittings, and a list of amendments had been drawn up and approved by the Council; so that when the time came they would be ready with suggestions. The same plan has been carried out by the Life Offices' Association, by the Faculty of Actuaries, and, he thought, by the Associated Scottish Life Offices; so that combined statements of views and suggestions would be submitted to the Board of Trade. He understood, however, that there was no probability of the matter coming on this session.

As to other matters, the Council considered that the time had arrived when the whole of the regulations as to the admission of members and the Syllabus of the examinations should be reconsidered. There was a committee sitting on this subject, and, as its deliberations had not yet come to an end, he was not in a position to make any statement. He should like, however, to say that the object they had before them was to raise the status of the profession of an Actuary, maintain the highest standard, and at the same time uphold the dignity of the Institute.

Mr. Ernest Woods remarked that, as senior Vice-President, it fell to his lot to second the motion for the adoption of the report. The President had dealt with it so fully that he had left him very little to say, but there were one or two subjects to which he should like to allude. First of all he thought the most important part of the report from the point of view of practical every day work was that which referred to the publication of the Valuation Tables based on the O^M Experience. These Tables would doubtless be frequently used by actuaries. The International Congress at Berlin was a very pleasant experience to all who attended it, and he hoped that on future occasions more of the leading actuaries would be present. It was not, perhaps, the occasion to criticise the Congress, but he could not help thinking that a little too much work was crammed into the proceedings. Those who were most constant in their attendance found it very laborious to be present the whole day, and he trusted that at the Vienna Congress there would not be quite so many sittings. The President had referred to the amount of the funds, and personally he did not think the £300 a year which had been mentioned was at all an excessive margin, because in his opinion the whole of that amount and even more might well be spent on the education of the younger members. It was most essential that they should have extended facilities for thoroughly mastering their work.

Mr. T. P. Wansbrough regretted that he felt it his duty to record his vote against the adoption of the report, not so much on account of what it contained as because of its notable omissions. He observed, first of all, that there was no reference whatever to a phrase used in the President's inaugural address, that the Institute numbered nearly sufficient members, although the President had alluded to it indirectly in his remarks. Secondly, there was no reference in the report to the fact the President also mentioned in his inaugural address, that the Council were attempting to disqualify Associates from acting as actuaries. Thirdly, there was only a very meagre reference indeed to the results of the examinations held this year; and yet he should have thought that the results of those examinations were sufficiently striking to have called for further comment. The speaker then proceeded to discuss, at considerable length, the Institute Examinations,

and the class of questions set for Candidates for the Fellowship, and generally expressed his views as to the due recognition of the status and rights of the class of Associate.

The President explained that any remarks made by him in his Presidential Address were on his sole responsibility, and were not necessarily endorsed by the whole Council. He desired also to point out that what he then said had not been properly represented by Mr. Wansbrough; also that in intimating that in his opinion the Institute already numbered nearly sufficient members, he had solely the intention, in which he trusted he had succeeded, of discouraging incompetent young men from attempting the course of study for an Actuary. The President further explained that the results of the examinations in the Colonies were not yet to hand, and that, at the date at which the Council Report was drawn up, the results for the United Kingdom were not even available. As regards the status and rights of the Associates, he was confident that the Council would not act in a narrow-minded or prejudiced way in matters which affected the interests of both senior and junior members.

The resolution for the adoption of the Report was then put and carried, with one dissentient.

Election of Officers.

Messrs. A. R. Barrand and O. F. Diver were appointed scrutineers of the ballot for the election of Officers for the ensuing year.

The ballot was then taken, and the President subsequently announced that the Council nominations had been adopted, as follows:—

President.

Frank Bertrand Wyatt.

Vice-Presidents.

Frederick Schooling.
Thomas Gans Ackland.
George Todd, M.A.
George Francis Hardy.

Council.

Thomas Gans Ackland.
Henry Walsingham Andras.
*Arthur Digby Besant, B.A.
Thomas G. C. Browne.
Henry Cockburn.
Francis Ernest Colenso, M.A.
Joseph Ernest Faulks, B.A.
*George Francis Hardy.
Ralph Price Hardy.
Arthur George Hemming.
Charles Daniel Higham.
Lewis Frederick Hovil.
William Hutton.
Abraham Levine, M.A.
George James Lidstone.
Henry William Manly.
Geoffrey Marks.
William Peyton Phelps, M.A.
Edward Arthur Rusher.
Gerald Hemmington Ryan.
Frederick Schooling.
Edward Robert Straker.
*Robert Ruthven Tilt.
George Todd, M.A.
*Ralph Todhunter, M.A.
Harold Moltke Trouncer, M.A.
Samuel George Warner.
Ernest Woods.
Frank Bertrand Wyatt.
Thomas Emley Young, B.A.

Treasurer.

Ernest Woods.

Honorary Secretaries.

Samuel George Warner.
Joseph Ernest Faulks, B.A.

* Not Members of the previous Council.

Mr. A. J. Hicks then moved the re-election of Messrs. H. Lugton and J. S. Hazell, and the election of Mr. A. G. Scott, as Auditors for the ensuing year.

Dr. A. W. Findlay seconded the motion, which was carried unanimously.

Mr. R. Todhunter, in moving a vote of thanks to the President, Vice-Presidents, Council, Officers, Examiners and Supervisors, for their services during the past year, said that no one who read the Report could doubt that the President and other members of the Executive had given a very great amount of trouble, time and thought to the administration of the affairs of the Institute. The most interesting paragraphs of the Report were those which referred to the literary output of the year. It was seldom the members had to congratulate themselves on such valuable additions to their actuarial literature as the Council had given them during the past year. Whether they could do anything more practical to ensure appreciation of the services of the Council than simply to accord them a vote of thanks he was not sure. Perhaps the best way in which they could do so would be by maintaining the attendances and the interest of the debates at the sessional meetings.

Mr. W. P. Elderton, in seconding the motion, said that the tremendous amount of work that had been done by the Council had been already mentioned by one or two previous speakers, but, during the last few months it had been brought before him somewhat forcibly in connection with the difficulty of obtaining the use of the halls or class-rooms for his students on extra nights, on account of frequent Committee meetings being held. There was one other class of person, Examiners and Supervisors, if he might take them together, which was included in the vote of thanks proposed. He was afraid it was an unpopular class. People who inflicted on others what those others considered to be a misfortune, were never likely to be very popular. For his own part he did not think they deserved the opprobrium they received. Strictly, he thought it should be directed rather against examinations in general, than against the examiners. Examiners undertook a very thankless task, involving a great amount of work which, so far as his small experience went, they performed wonderfully well.

The resolution was then put and carried.

The President, on behalf of his colleagues and himself, thanked the members heartily for their renewal of confidence. The Council was pleased to number among its body four new members, who would bring the light of their intelligence to bear upon their deliberations which, he assured the members, were carried out with only one object, the good of the Institute.

Mr. R. Cross, in proposing a vote of thanks to the auditors for their services during the past year, said that many years ago he served as an auditor, and, though the duties were not very onerous, they were certainly most important.

Mr. J. Chatham seconded the motion, which was carried unanimously.

The President then adjourned the meeting to Monday, 25 November 1907.

Additions to the Library.

The following works have been added to the Library since the publication of the *Journal* for October 1906:

By whom presented (when not purchased).

Accountants and Auditors, Society of
List of Members, &c., 1906–7. — *The Society.*

Accountants, Institute of Chartered, in England and Wales.
List of Members, 1907. — *The Institute.*

Actuarial Society of America.
Transactions, 1906–7. — *The Society.*
Containing *inter alia*—
"A Practical Interpolation formula, with a Theoretical Introduction", by R. Henderson.
"Formula for obtaining the cost of Insurance with its application in a Method of Computing the Values of Cancelled Policies", by J. D. Craig.
"A few Suggestions as to the Assessment of Expenses", by P. C. H. Papps.
"Mortality Rates experienced by the New York Life Insurance Company among its Deferred Dividend Policies, taken by amounts assured", by A. Hunter.
"Practical application of the Piece-Work System in Life Insurance Offices", by A. Hunter.
"Notes on the Select and Ultimate Method", by H. N. Sheppard.
"A Review of the Mortality in certain Occupations in England and America", by R. G. Hunter.
"A Practical Rule for calculating Annual Dividends", by R. W. Weeks.
"On Surplus Apportioned Annually", by H. W. Robertson.
"Is the New York Standard Life Policy Act Constitutional?" by W. S. Nichols.
"A Distribution Formula", by M. M. Dawson.

Actuarial Society of New South Wales.
Transactions, 1907. — *The Society.*

Actuaries, Faculty of
Transactions, 1906–7. — *The Faculty.*
Containing *inter alia*—
"An Analysis of the Profit from Endowment Assurances", by J. Chatham.
"Notes on Summation and Interpolation", by A. J. C. Fyfe.
"The Actuary in Scotland": A Presidential Address, by A. Hewat.
"Some Notes on Death Duties from an Actuarial point of view", by J. C. Denmead.
"Paid-up Policies and Surrender Values", by Dr. A. E. Sprague.
"Notes on the British Offices Life Annuity Tables, 1893", by T. G. Ackland.

Altenburger (J.).
Versuch Einer allgemeinen Theorie der mechanischen Ausgleichungs-Methoden. — *The Author.*

	By whom presented (when not purchased).
American Mathematical Society.	
Transactions, 1906–7.	*The Society.*
American Statistical Association.	
Transactions, 1906–7.	*The Association.*
Andrae (Dr. A.).	
Die Sterblichkeit in den land und forstwirtschaftlichen Berufen nach den Erfahrungen der Gothaer Lebensversicherungsbank. Berlin, 1906.	*The Gotha Life Office.*
Archer (J. A.).	
Compound Interest, Annuity, and Sinking Fund Tables. Large 8vo. 1907.	*The Publishers.*
Another Copy.	*Purchased.*
Australian Mutual Provident Society.	
Fifty-eighth Annual Report, 1907.	*The Society.*
Austria.	
Bericht der Arbeiter-Unfall-Versicherungs-Anstalt für das Königreich Böhmen, 1905.	*The Austrian Government.*
Versicherungswissenschaftliche Mitteilungen der Mathematisch-Statistische Vereinigung. 1904–7.	*The Society.*
Bachelier (L.).	
Théorie des Probabilités continues. 4to. Paris, 1906.	*The Author.*
Bankers.	
Journal of the Institute of Bankers. List of Members, 1907.	*The Institute.*
Belgium.	
Bulletin de l'Association des Actuaires Belges.	*The Association.*
Bulletin de la Prevoyance. March, 1907. (Rapport à propos d'une investigation sur les Risques Surélevés.	*L'Assoc. des Actuaires Belges.*
Bulletin du Comité permanent des Congrès internationaux d'Actuaires. 1906.	*The Permanent Committee.*
Compte Rendu des Opérations et de la Situation de la Caisse Générale d'Epargne et de Retraite. 1906.	*The Belgian Government.*
Bergholm (P.).	
An Enquiry into the Mortality among Scandinavians in the Congo State, during the period 1878–1904.	*H. Cockburn.*
"Biometrika."	*Purchased.*
Vol. V, containing, *inter alia*—	
"The non-inheritance of Sex in Man", by F. A. Woods.	
"On the Inheritance of the Sex Ratio", by D. A. Heron.	
"Skew Frequency Curves", by Prof. Karl Pearson.	
"On the Curves which are most suitable for describing the Frequency of random samples of a Population", by Prof. Karl Pearson.	
"Statistical Studies in Immunity", by Dr. J. Brownlee.	
"The Calculation of the Moments of a Frequency-Distribution", by W. F. Sheppard.	
Boddy (H. M.).	
Some defects in Life Assurance legislation. Cape Town, 1907.	*The Author.*

	By whom presented (when not purchased).
British Almanac and Companion, 1858 and 1867.	*W. Stott.*
British Offices Life Tables, 1893.	
Select Tables deduced from the graduated Experience of Whole-Life Participating and Non-Participating Assurances on Male Lives. 8vo. 1907. Valuation Tables deduced from the graduated Experience of Whole-Life Participating Assurances on Male Lives (O^M Table), at $2\frac{1}{2}$, $2\frac{3}{4}$, 3, $3\frac{1}{2}$, 4 and $4\frac{1}{2}$ per cent. 8vo. 1907. Values of Temporary Annuities at all Entry Ages from 10 to 99 inclusive, and in respect of all durations; at $2\frac{3}{4}$ per-cent interest according to the O^M and $O^{M(5)}$ Mortality Tables. 8vo. 1907.	*Official.*
British Trade Year Book.	
Covering the years 1880–1906, by J. Holt Schooling. 8vo. 1906.	*The Author.*
Canada.	
Canadian Annual Financial Review, compiled by W. R. Houston. Ob. 8vo. Toronto, 1906, 1907.	*Purchased.*
Canadian Bank Shares as Investments, 1897–1906. Toronto. 1907.	*John Mackay & Co.*
Crelle (A. L.).	
Tables de Calcul où se trouvent les Multiplications et Divisions toutes faites de tous les nombres au dessous de mille et qui facilitent et assurent le calcul. 3me edit. by C. Bremiker. Fol. Berlin, 1869.	*F. E. Colenso.*
De Savitch (S.).	
The Elementary Theory of Life Assurance. (In Russian). 8vo. St. Petersburg, 1900.	*H. W. Manly.*
Deutscher Verein fur Versicherungs-Wissenschaft.	
Katalog der Bücker-Kammer, 1902. 8vo. Berlin, 1902. List of Members, 1907. Veröffentlichungen, 1906. Zeitschrift für die gesamte Versicherungs-Wissenschaft, 1906–7.	*The Society.*
Dougharty (H.).	
Annuities and Sinking Funds, Simple and Compound Interest Tables, together with Notes. 8vo. 1906.	*The Author.*
Economic Relation of Life Insurance to Society and State.	
Report to American Academy of Political and Social Science. Pa., 1897.	*T. G. C. Browne.*
Eedy (A. M.).	
Office System and Methods in the business of Life Assurance. N.S.W. 1906.	*The Author.*
Elderton (W. Palin).	
Frequency-Curves and Correlation. 8vo. 1906.	*Official.*

By whom presented (when not purchased).

Equitable Life Assurance Society.

Reports to the Court of Directors. Cases submitted to Counsel, with Opinions. Nine addresses by W. Morgan and three by A. Morgan. Statements of the Results of Decennial Investigations in 1829, 1839, and 1849. In one Vol., 1854.
Deed of Settlement, with Bye-Laws and Orders, 1854. (Revised in 1881.) — *The Society.*

Fackler (E. B.).

Notes on Life Insurance. The Theory of Life Insurance practically explained. 8vo. New York, 1907. — *Purchased.*

Fox (M.).

Report on the Auckland (N.Z.) Diocesan Pension Fund, 1904. — *The Author.*

France.

Bulletin de l'Institut des Actuaires Français. — *The Institute.*

Recueil de documents relatifs aux Assurances sur la vie, rèunis par le Ministère du Commerce. No. 1. Formules et Barèmes des Primes ou Cotisations minima des opérations d'assurances sur la vie. 8vo. Paris, 1906. — *Purchased.*

Gamborg (V. E.).

4—Cifrede Logaritmer til de Trigonometriske Funktioner (med decimalinddeling af graden). 8vo. Copenhagen, 1906. — *The Author.*

Germany.

Denkschrift betreffend die von den Organisationen der Privatangestellten im October 1903, angestellten Erhebungen über ihre wirtschaftliche Lage der Kosten einer Pensions-und Hinterbliebenenfürsorge dieser Berufskreise. 4to. Berlin, 1907. — *E. Wolfer.*

Rechenschaftbericht der Verwaltung der Warschauer Gegenseitigen Unfall-Versicherungs-Gesellschaft. 4to. Warschau, 1906. — *The Company.*

Giornale di Mathematica Finanziaria, 1907. — *The Editor.*

Gotha.

Rechenschaftsbericht der Lebensversicherungsbank auf Gegenseitigkeit zu Gotha, 1906. Fol. — *The Gotha Life Office.*

Green (C. H.).

The Workmen's Compensation Act, 1897; the Employer and the Insurance Companies. 8vo. 1898. — *T. G. C. Browne.*

Hardy (R. P.).

Report and Valuation Balance Sheet, as at 31 December 1905, of the Great Western Railway Enginemen and Firemen's Mutual Assurance, Sick and Superannuation Society. Swindon, 1906. — *The Author.*

Hendrick (B. J.).

The Story of Life Insurance (being an attempt to describe the causes that led to the American Insurance Scandals of 1905–6). 8vo. 1907. — *Purchased.*

By whom presented (when not purchased).

Holland.

Archief voor de Verzekerings-Wetenschap. 1906–7.	*The Society.*
Jaarboekje van de Vereeniging voor Levensverzekering. 1907.	*The Algemeene Maatschappij.*
Mededeelingen der Vereeniging voor Levensverzekering. 1906–7.	*The Society.*
Nationale Levensverzekering-Bank. Verslag over het jaar 1906; en over het Dreijarig Tijdvak 1904–6. Sterfte en Sterfte-Oorzaken. Rotterdam, 1906.	*The Company.*

Institute of Commercial Research in the Tropics.

Journal, 1907.	*The Institute.*

Insurance Institute of Ireland.

Journal, 1906–7, containing *inter alia*— "The Law of Insurable Interest as affecting Life Assurance", by F. W. May. "The Investment side of Life Assurance", by H. J. Lapham.	*The Institute.*

Journal of the Federation of Insurance Institutes.

Vol. IX, containing *inter alia*— "Developments of Accident Insurance", by J. C. M'Bride. "Legal Aspects of Life Assurance", by J. L Mounsey. "Life Assurance. Some modern aspects of competition: Notes on Assurances without medical examination and other schemes presently in vogue", by W. R. M'Ilvenna. "The Insurance of Under-average Lives", by W. Hughes.	*Purchased.*

Lash (Z. A.).

The Banking System of Canada. Toronto, 1907.	*John Mackay & Co.*

Laurent (H.).

La Géométrie analytique Génerale. 8vo. Paris, 1906.	*The Author.*

Life Offices Association.

Memoranda as to the Meetings of the Standing Committee. 1906–7.	*The Association.*

London Mathematical Society.

Proceedings, 1906–7.	*The Society.*

Mathieson (F. C.) & Sons.

Stock Exchange ten-year record of Prices and Dividends. 1897–1906. Large 8vo. 1907.	*The Publishers.*

Messenger (H. J.).

Comments upon the Gain and Loss Exhibit. Hartford, Conn., 1906. The Rate of Sickness. Hartford, Conn., 1907.	*The Author.*

Moir (H.).

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Acts of Parliament.	
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Colonial Acts and Bills.	
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Benevolent and Provident Societies Act. Ontario, 1897.	*The Canadian Government.*
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Foreign Insurance Companies' Deposits Act Amendment Act, New Zealand, 1904. Government Accident Insurance Act. New Zealand, 1899. Government Life Insurance Acts. New Zealand, 1886, 1888, 1890, 1891, 1893, 1894, 1899.	*The Government of N.Z.*
Insurance Acts. Ontario, 1897, 1901, 1903, 1904. Insurance for the benefit of Wife and Children. Canada, N.D.	*The Canadian Government.*
Life and Accident Insurance. New Brunswick, 1905.	*The Government of N.B.*
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Life Assurance Act, 1891. Cape Town.	*The Cape Government.*
Life Assurance Companies Act. Queensland, 1901. Life Assurance Companies Amendment Acts. South Australia, 1882, 1887.	*Purchased.*
Life Assurance Companies Act Amendment Act. Western Australia, 1903.	*The Government of W.A.*
Life Assurance Policies' Acts. New Zealand, 1884, 1885, 1903.	*The Government of N.Z.*
Life Insurance Act. Manitoba, 1902.	*The Canadian Government.*
Life Insurance Companies Acts. Newfoundland, 1888, 1901, 1904.	*The Government of Newfoundland.*

	By whom presented (when not purchased)
Parliamentary Papers—*(continued)*.	
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Married Women's Property Act Amendment Act. Nova Scotia. 1904.	*The Government of N.S.*
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Report of the Registrar for the years 1903–1905. — *Purchased.*

Life Assurance Companies.

Returns, 1906. — *The Board of Trade.*

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Annuities. Account to 5 January 1907.

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Return showing Liabilities and Assets at the close of each financial year, from 1835–6 to 1906–7.

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By whom presented (when not purchased).

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Periodicals.	
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Poterin du Motel (H.).	
Théorie des Assurances sur la vie. 8vo. Paris, 1899.	*Purchased.*
Poussin (R.).	
Traité Élémentaire des Assurances sur la vie. Principes et Applications. 8vo. Paris, 1906.	*Purchased.*
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Issued by the following Companies prior to 1870: Clerical, Medical and General Life Assurance Society. Guardian Assurance Company. National Mutual Life Assurance Society. National Provident Institution. Pelican and British Empire Life Office.	*The respective Offices.*
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By whom presented (when not purchased).

Schaertlin (Dr. G.).
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Spain.
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"A System of Life Insurance Bookkeeping and Office Records", by T. W. Wallace.
"British Life Assurance Accounts", by C. C. Ferguson.
"Some points in Life Assurance law", by A. Bruce.
"Sickness Insurance", by E. Willans.
"Investments of Canadian Life Offices", by T. Bradshaw.
"The Past, Present and Future of Life Insurance", by L. K. File.
"A Description of Mortality Tables, with some Results of a recent Investigation", by W. A. P. Wood.
"Deferred Claims in Liability Insurance", by C. N. Neely.
"New Features in Personal Accident Contracts", by B. G. Walker.
"Re-insurances of Life Offices", by G. C. Moore.

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	By whom presented (when not purchased).
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Proceedings, 1905.	*The Institute.*
Welman (A.J.).	
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White (William).	
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Yorkshire, Insurance Institute of	
Proceedings, 1905–6, containing *inter alia*— "A Summary of the Law of Employer's Liability", by J. L. Booth. "Urinary abnormalities in persons applying for Life Assurance Policies", by Dr. A. Rabagliati.	*The Institute.*
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The Principles and Practice of Insurance and their modern developments. A Lecture given before the Society of Arts. 1907.	*H. Cockburn.*

The following additional copies of works already in the Library have also been added:—

Manchester, Insurance Institute of	
Reports, 1896–7, 1897–8, 1901–2 and 1902–3.	*The Institute.*
Manly (H.).	
The Principles of Book-keeping by Double Entry. 4th edit. by H. W. Manly. 8vo. 1875.	*F. B. Wyatt.*
Parliamentary Papers.	
Colonial Acts and Bills. Foreign Insurance Companies' Deposits Acts. New Zealand, 1894, 1895, 1900. Government Insurance and Annuities Act. New Zealand, 1874. Government Insurance Association Act. New Zealand, 1884. Life Assurance Companies Act. New Zealand, 1873.	*The Government of N.Z.*
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Proceedings, 1901–6.	*The Institute.*

COMBINED INDEX TO THE FIRST FORTY VOLUMES OF THE *JOURNAL*.

For the greater convenience of students and readers of the *Journal*, and in view also of the fact that the Combined Index to the First Twenty Volumes has for some time been out of print, the Council have authorized the Editor to prepare and issue, in place of an Index to Volumes XXXI to XL, a Combined Index to the First Forty Volumes. This Index has now been completed, and is issued to Members of the Institute with the present number of the *Journal*.

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(*Corrected to* 1 *January* 1907.)

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1902 †Anderson, Thomas Frederic, F.F.A., *Royal Exchange Assurance Corporation, Royal Exchange,* E.C.

1891 †Anderson, William Smith, *Gresham Life Assurance Society, St. Mildred's-house, Poultry,* E.C.

1885 †Andras, Henry Walsingham, F.S.S. (LIBRARIAN), *Alliance Assurance Co., Ltd. (Provident Life Fund),* 50 *Regent-street,* W.

1885 †Ansell, Hubert, *Anglo-American Debenture Corporation, Ltd.,* 20 *Birchin-ln.,* E.C.

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1896 †Archer, Joseph Alfred, *Ecclesiastical Commission, Millbank,* S.W.

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1903 †Bacon, James, *c/o T. G. Ackland, Esq.,* 5 *&* 6 *Clement's-inn, Strand,* W.C.

1850 †Bailey, Arthur Hutcheson, F.S.S. (PAST PRESIDENT, 1878–82), 26 *Mount Ephraim-rd, Streatham,* S.W.

1896 †Baker, Henry James, *Metropolitan Life Assur. Soc.,* 13 *Moorgate-street,* E.C.

1885 †Barnes, Joseph Howard, F.S.S., *Pelican and British Empire Life Office,* 70 *Lombard-street,* E.C.

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1889 †Bell, Frederick, *Alliance Assurance Co., Limited (Imperial Life Assurance Fund), 47 Chancery-lane,* W.C.

1886 †Berry, Berry Alfred, B.A., *London Life Association, Ltd., 81 King William-street,* E.C.

1895 †Besant, Arthur Digby, B.A., *Clerical, Medical & General Life Assur. Soc., 15 St. James's-square,* S.W.

1879 Besso, Marco, *Via Gregoriana 54 in the Villino Besso, Rome.*

1894 †Blackadar, Alfred Kimball, M.A., F.A.S., *Government Insur. Department, Ottawa, Canada.*

1883 †Blakey, James, *National Debt Office,* E.C.

1897 †Bradshaw, Thomas, F.A.S., *Imperial Life Assurance Co. of Canada, Toronto, Canada.*

1899 †Brown, Edward Harold, *Prudential Assurance Company, Holborn-bars,* E.C.

1904 †Brown, Henry, B.A., *Commercial Union Assur. Co., 24, 25 & 26 Cornhill,* E.C.

1901 †Brown, Hugh Wylie, F.F.A., *Scottish Union & National Insur. Company, 35 St. Andrew-square, Edinburgh.*

1875 Browne, Thomas G. C., *Guardian Assurance Company, 11 Lombard-street,* E.C.

1887 Browne, Willis, *India Office,* S.W.

1901 †Buchanan, James, D.Sc., M.A., F.F.A., *Scottish Widows' Fund and Life Assurance Society, 9 St. Andrew-square, Edinburgh.*

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1888 †Calderon, Henry Philip, *Law Accident Insurance Society, Limited, 215 Strand,* W.C.

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1906 †Catchlove, Chas. Hamilton Leyland, *Australian Mutual Provident Society, Sydney, Australia.*

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1889 †Chatham, James, F.F.A., F.S.S., *North British and Mercantile Insurance Co., 64 Princes-street, Edinburgh.*

1875 Cherriman, J. B., Prof., M.A., *c/o The Bank of Montreal, Abchurch-lane,* E.C.

1883 Chisholm, James, F.F.A., F.A.S., *Crossfield, Alberta, Canada.*

1895 †Clarke, Arthur Harold, *Clerical, Medical and General Life Assurance Society, 15 St. James's-square,* S.W.

1863 Clirehugh, William Palin, F.S.S., *London and Lancashire Life Assurance Company, 66 & 67 Cornhill,* E.C.

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1897	†Fellows, Rowland Hill, F.S.S., *Pelican and British Empire Life Office*, 70 *Lombard-street*, E.C.
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1905	†Gemmill, William, *Transvaal Chamber of Mines, Johannesburg, South Africa.*
1901	†Gibson, Rev. John Paul S. R., B.A. *Holmesdale, Brooklands-avenue, Cambridge* (*Reinstated,* 1905).
1902	†Gillies, George, *Union Assurance Society,* 81 *Cornhill,* E.C.
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1878	Gordon, Charles, F.F.A., *South African Mutual Life Assurance Society, Cape Town, South Africa.*
1901	†Gordon-Smith, Randolph, F.F.A., *Scottish Amicable Life Assur. Soc.,* 35 *St. Vincent-pl., Glasgow.*
1882	†Graham, James, F.F.A., *Australian Widows' Fund Life Assurance Society, Collins-street-west, Melbourne, Australia.*
1904	†Grant, Milton Daniel, B.A., *Government Insurance Department, Ottawa, Canada.*
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1888	†Hemming, Arthur George, F.S.S., *London Assurance Corporation,* 7 *Royal Exchange,* E.C.
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1888	†Hopkins, William Raynes, *London and Lancashire Life Assur. Co.,* 66 & 67 *Cornhill,* E.C.
1890	†Hovil, Lewis Frederick, *National Provident Institution,* 48 *Gracechurch-street,* E.C.
1871	†Hughes, William, F.A.S. (PAST-PRESIDENT, 1902-4), 62 *Palace-road, Tulse-hill,* S.W.
1906	†Humphreys, Henry Thompson, *Sun Life Assurance Society,* 63 *Threadneedle-street,* E.C.
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1893 †Hutton, William, F.F.A., *Scottish Amicable Life Assur. Society*, 35 *St. Vincent-place, Glasgow.*

1903 †Jarman, William Rees, B.A., *National Debt Office*, E.C.

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1902 †Kenchington, Charles William, *Prudential Assurance Company, Holborn-bars*, E.C.

1897 †Kentish, Owen, *Economic Life Assurance Soc.*, 6 *New Bridge-street*, E.C.

1874 †King, George, F.F.A., F.A.S. (Lecturer), 15 *Walbrook*, E.C.

1887 †Kyd, Thomas, F.F.A., *Northern Assurance Company*, 1 *Union-terrace, Aberdeen.*

1882 Lancaster, Sir William John, *South Lynn, Putney-hill*, S.W.

1894 †Laughton, Alexander Millar, F.F.A., *National Mutual Life Assoc. of Australasia, Limited, Corner of Collins and Queen-streets, Melbourne, Australia.*

1887 †Lemon, William Kent, Barrister-at-Law, 1 *Vanbrugh-terrace, Blackheath*, S.E.

1896 †Levine, Abraham, M.A., *Alliance Assurance Co., Ltd., Bartholomew-lane*, E.C.

1896 †Lewis, John Norman, F.F.A., *London Assurance Corporation*, 7 *Royal Exchange*, E.C.

1892 †Lidstone, George James, *Equitable Life Assurance Soc., Mansion-house-street*, E.C.

1901 †Little, James Fulton, *Mutual Life Association of Australasia, Perth, West Australia.*

1899 Low, George Macritchie, F.F.A., *Scottish Equitable Life Assur, Society*, 28 *St. Andrew-square, Edinburgh.*

1899 †Lutt, Harold Edward William, *Northern Assurance Company*, 60 & 61 *London-wall*, E.C.

1898 †Macaulay, Thomas Bassett, F.A.S., *Sun Life Assurance Co. of Canada, Montreal, Canada.*

1874 McClintock, Emory, F.A.S., *Mutual Life Insurance Company of New York, New York, U.S.A.*

1894 †McDonald, John, *Prudential Assurance Company, Holborn-bars*, E.C.

1883 †McGowan, James, B.A., *The Treasury, Cape Town, South Africa.*

1885 Mackenzie, Alexander George, F.F.A., 29 *Chester-terrace, Regent's-park*, N.W.

1900 †Macnaghten, Steuart Edye, A.C.A., *Equity & Law Life Assur. Soc.*, 18 *Lincoln's-inn-fields*, W.C.

1901 †Macphail, Donald, F.F.A., *Yorkshire Insurance Company, Cape Town, South Africa.*

1870 †Manly, Henry William, F.A.S. (Past President, 1898-1900), *Glenthorne*, 157, *Highbury New-park*, N.

1890 †Marks, Geoffrey (Librarian), *National Mutual Life Assur. Soc.*, 39 *King-street, Cheapside*, E.C.

1900 †Marr, Vyvyan, F.F.A., *Edinburgh Life Assurance Co.*, 22 *George-street, Edinburgh.*

1902 †May, Basil, *Royal Exchange Assur. Corp., Royal Exchange*, E.C.

1897 †May, George Ernest, *Prudential Assurance Company, Holborn-bars*, E.C.

1906 †May, Walter Thomas, *Liverpool and London and Globe Insurance Co.*, 1 *Cornhill*, E.C.

1897 †Miller, Neville, *London Assurance Corporation*, 7 *Royal Exchange*, E.C.

1905 †Milligan, Charles Livingstone, *Alliance Ass. Co., Ltd. (Provident Life Fund)*, 50 *Regent-street*, W.

1893 †Milner, John William, *North British & Mercantile Insur. Co.*, 61 *Threadneedle-street*, E.C.

1892 †Milton, Henry, M.A., *Law Debenture Corporation, Ltd.*, 41 *Threadneedle-street*, E.C

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.	
1899	†Moir, Henry, F.F.A., F.A.S., *Provident Savings Life Assur. Soc.*, 346 *Broadway, New York, U.S.A.*
1890	†Molyneux, Arthur Ernest, *Provident Clerks' and General Mutual Life Assurance Assoc.*, 27 *&* 29 *Moorgate-street*, E.C.
1901	†Moorhouse, Alfred, *Friends' Provident Institution, Bradford, Yorkshire.*
1896	†Moran, Joseph Flack, *Reversionary Interest Society*, 30 *Coleman-street*, E.C.
1900	†Morgan, Benjamin Charles, M.A., *Commercial Union Assur. Co.*, 24, 25 *&* 26 *Cornhill*, E.C.
1895	†Muter, Percy, *New Zealand Government Life Insurance Department, Wellington, New Zealand.*
1888	†Nash, Willie Oscar, *Law Reversionary Interest Soc., Limited, Thanet-house*, 231 & 232 *Strand* (*opposite the Law Courts*), W.C.
1906	†Neill, Samuel Bennett, *China Mutual Life Insurance Co., Shanghai, China.*
1883	Neison, Francis G. P., F.S.S., 19 *Abingdon-st., Westminster*, S.W.
1888	†Newman, Philip Lewin, B.A., *Yorkshire Insurance Co., York.*
1865	Newton, Algernon, M.A., *c/o London & Westminster Bank*, 94 *&* 96 *High-st., Kensington*, W.
1887	†Nightingale, Harry Ethelston, *Royal Exchange Assurance Corporation, Royal Exchange*, E.C.
1903	†Norris, Charles Arthur, *National Mutual Life Association of Australasia, Limited, Melbourne, Australia.*
1901	†Norton, William Ernest, *National Provident Institution*, 48 *Gracechurch-street*, E.C.
1905	†Oakley, Henry John Percy, *North British and Mercantile Insurance Company*, 61 *Threadneedle-street*, E.C.
1899	†Parker, Robert Peter, *Sun Life Assurance Society*, 63 *Threadneedle-street*, E.C.
1864	Pearson, Arthur, *Betchworth-house, The Bank, Highgate*, N.
1905	†Penman, William, Jr., *Atlas Assurance Company*, 92 *Cheapside*, E.C.
1891	†Phelps, William Peyton, M.A., *Equity and Law Life Assur. Soc.*, 18 *Lincoln's-inn-fields*, W.C.
Under the Charter.	Priestley, John George, 44 *St. German's-road, Forest-hill*, S.E.
1891	†Pulley, William Pritchard, *Norwich Union Life Insur. Soc.*, 71 *&* 72 *King William-st.*, E.C.
1903	†Rae, Joseph, *Finance Department, Town-hall, Upper-street*, N.
1899	†Raisin, Arthur Herbert, *Pelican and British Empire Life Office*, 70 *Lombard-street*, E.C.
1897	†Rees, Martin, *Law Reversionary Interest Soc., Limited, Thanet-house*, 231 *&* 232 *Strand* (*opposite the Law Courts*), W.C.
1901	†Reeve, Charles Ernest, *Royal Exchange Assurance Corporation, Royal Exchange*, E.C.
1902	†Richmond, George William, *Scottish Widows' Fund and Life Assur. Society*, 28 *Cornhill*, E.C.
1904	†Rietschel, Hermann Julius, *Sun Life Assurance Society*, 63 *Threadneedle-street*, E.C.
1898	†Robinson, George Frederick, *Legal and General Life Assur. Society*, 10 *Fleet-street*, E.C.
1905	†Robinson, Hugh Thomas Kay, *Clergy Mutual Assur. Soc.*, 2 *&* 3 *The Sanctuary*, S.W.
1888	†Rusher, Edward Arthur, F.S.S., *Prudential Assurance Company, Holborn-bars*, E.C.
1882	†Ryan, Gerald Hemmington, F.A.S., (TREASURER), *Pelican and British Empire Life Office*, 70 *Lombard-street*, E.C.
1898	†Salmon, Richard George, F.S.S., *Sun Life Assurance Society*, 63 *Threadneedle-street*, E.C.

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.

1883 Saunders, Harris Charter Lindon, F.R.A.S., *"Marquise," Twickenham.*

1886 †Schooling, Frederick, F.A.S. (Vice-President), *Prudential Assurance Company, Holborn-bars,* E.C.

1901 †Searle, George Morley, *Sun Life Assurance Society,* 63 *Threadneedle-street,* E.C.

1901 †Sharman, William Charles, *Prudential Assurance Company, Holborn-bars,* E.C.

1905 †Sherriff, Francis Henry, *Provident Clerks' and General Mutual Life Assurance Assoc.,* 27 *&* 29 *Moorgate-street,* E.C.

1896 †Sim, William Abernethy, F.F.A., *Scottish Union and National Insurance Co.,* 35 *St. Andrew-square, Edinburgh.*

1875 †Smither, Arthur, *Inces, Scaynes-hill, Hayward's-heath.*

1881 †Somerville, William Finlay, *Liverpool and London and Globe Insurance Co.,* 1 *Dale-street, Liverpool.*

1877 †Sorley, James, F.S.S., F.R.S.E., 82 *Onslow-gardens,* S.W.

1898 †Spencer, John (Sub-Editor of Journal), *English and Scottish Law Life Assurance Assoc.,* 12 *Waterloo-place,* S.W.

1894 †Sprague, Alfred Ernest, D.Sc., M.A., F.F.A., *Edinburgh Life Assurance Co.,* 22 *George-street, Edinburgh.*

1857 Sprague, Thomas Bond, M.A., LL.D., Hon. F.F.A., F.S.S., F.R.S.E. (Past President, 1882–86), 29 *Buckingham-ter., Edinburgh.*

1906 †Spurgeon, Ernest Frank, *Prudential Assurance Company, Holborn-bars,* E.C.

1896 †Stahlschmidt, Louis, *St. John's College, Agra, India.*

Under the Charter Stevens, Charles, *Aberdeen Ho., Preston, Brighton.*

Date of becoming a Fellow.

1888 Stewart, John, F.F.A., *City of Glasgow Life Assur. Co.,* 30 *Renfield-street, Glasgow.*

1906 †Stewart, Lionel William, *Alliance Assurance Co., Ltd., Bartholomew-lane,* E.C.

1898 Stirling, Robert, F.F.A., *Rock Life Assurance Company,* 15 *New Bridge-street,* E.C.

1892 †Straker, Edward Robert, *Pelican and British Empire Life Office,* 70 *Lombard-street,* E.C.

1878 †Straker, Frank Arthur, *Legal and General Life Assur. Society,* 10 *Fleet-street,* E.C.

1902 †Strong, William Richard, *London Guarantee & Accident Co.,* 61 *Moorgate-street,* E.C.

1884 †Stuart, John Moody, F.F.A., *Leeds Permanent Benefit Building Society, Victoria-buildings, Park-lane, Leeds.*

1900 †Sutherland, John, M.A., *Australasian Temperance and General Mutual Life Assurance Society, Swanston-street, Melbourne, Australia.*

1906 †Symmons, Frank Percy, *Prudential Assurance Company, Holborn-bars,* E.C.

1889 †Tarn, Arthur Wyndham, *Guardian Assurance Company,* 28 *King-street, Covent-garden,* W.C.

1887 Teece, Richard, F.F.A., F.A.S., F.S.S., *Australian Mutual Provident Society, Sydney, Australia.*

1872 Templeton, Col. John M., C.M.G., *National Mutual Life Association of Australasia, Melbourne, Australia.*

1864 †Terry, James, *Hernlee, Lyme Regis, Dorset.*

1889 †Thiselton, Herbert Cecil, F.F.A., F.A.S., *Commercial Union Assurance Co.,* 24, 25 *&* 26 *Cornhill,* E.C.

1901 †Thodey, Robert, *Australian Mutual Provident Society, Sydney, Australia.*

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.	
1893	†Thomas, Ernest Charles, *Gresham Life Assurance Society, St. Mildred's-house, Poultry,* E.C.
1899	†Thomas, Robert Arthur Caradoc, *Pelican and British Empire Life Office,* 12 *Dalhousie-sq., Calcutta.*
1905	†Thompson, Thomas Percy, B.A., *Pelican and British Empire Life Office,* 70 *Lombard-street,* E.C.
1895	†Thomson, Herbert Archer, B.A., *Umberslade, Boscobel-road, St. Leonard's-on-Sea.*
1893	†Thorne, Alfred Charles, *Equity & Law Life Assur. Soc.,* 18 *Lincoln's-inn-fields,* W.C.
1891	†Tilt, Robert Ruthven, *General Reversionary & Investment Co., Ltd.,* 26 *Pall-mall,* S.W.
1902	†Tinner, Thomas, *Comptroller's Depart., London County Council, Spring-gardens,* S.W.
1881	†Todd, George, M.A. (VICE-PRESIDENT), *Economic Life Assurance Society,* 6 *New Bridge-street,* E.C.
1894	†Todhunter, Ralph, M.A., *University Life Assur. Soc.,* 25 *Pall-mall,* S.W.
1899	†Trouncer, Harold Moltke, M.A. *London Life Association, Ltd.,* 81 *King William-street,* E.C.
1878	Turnbull, Andrew Hugh, F.F.A., F.R.S.E., 18 *Whitehouse-loan, Edinburgh.*
1889	Wallace, Thomas, F.F.A., *North British & Mercantile Insurance Co.,* 64, *Princes-street, Edinburgh.*
1905	†Wandless, John Robert, *Canada Life Assurance Co.,* 14 *King William-street,* E.C.
1906	†Wares, Harold Wallace, *Yorkshire Insurance Company, York.*
1888	†Warner, Samuel George (HON. SEC.), *Law Union & Crown Insur. Co.,* 126 *Chancery-lane,* W.C.
1893	†Watson, Alfred William, *Manchester Unity Friendly Soc., Nottingham.*
1895	†Watson, James Douglas, F.A.S., *English & Scottish Law Life Assr. Assoc.,* 12 *Waterloo-place,* S.W.
1904	†Weatherill, Henry, *National Debt Office,* E.C.
1880	†Whittall, Wm. Joseph Hutchings, F.A.S., 15 *St. James's-sq.,* S.W.
1905	†Wilson, John Sydney, *Australian Widows' Fund Life Assurance Society, Melbourne, Australia.*
1864	Wilson, Robert, 44 *Talfourd-rd., Camberwell,* S.E.
1888	†Wilson, Robert, Jr., *General Assurance Company,* 103 *Cannon-street,* E.C.
Under the Charter.	Winser, Thomas Boorman, F.R.G.S., F.R.N.S., 81 *Shooter's-hill-road, Blackheath,* S.E.
1899	†Winter, Arthur Thomas, *Pelican and British Empire Life Office,* 70 *Lombard-street,* E.C.
1897	†Wintle, Lancelot Andrewes, *Economic Life Assurance Soc.,* 6 *New Bridge-street,* E.C.
1904	†Wood, Arthur Barton, B.A., F.A.S., *Sun Life Assurance Co. of Canada, Montreal, Canada.*
1884	†Woods, Ernest, F.A.S. (VICE-PRESIDENT), *Guardian Assurance Company,* 11 *Lombard-street,* E.C.
1902	†Woolmer, Alfred Henry, *Star Life Assurance Society,* 32 *Moorgate-street,* E.C.
1902	†Workman, William Arthur, *Equitable Life Assur. Society, Mansion-house-street,* E.C.
1902	†Worthington, William, *Royal Insurance Co., Liverpool.*
1875	†Wyatt, Frank Bertrand, F.A.S. (PRESIDENT), *Clergy Mutual Assurance Soc.,* 2 & 3 *The Sanctuary,* S.W.
1906	†Young, Arthur Stanley, *Metropolitan Life Assurance Society,* 13 *Moorgate-street,* E.C.
1874	Young, Thomas Emley, B.A., F.R.A.S., F.A.S. (PAST-PRES., 1896-8), 108 *Evering-road, Stoke Newington,* N.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute. Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate

1900 2 Adams, Cecil Francis, *New Zealand Insurance Co., Accident Branch, Palmerston North, New Zealand.*

1905 (2) Adamson, William, F.F.A., *Scottish Accident Life & General Insurance Co., Ltd.,* 115 *George-street, Edinburgh.*

1869 2 Adey, Theodore Henry, *Scottish Provident Institution,* 17 *King William-street,* E.C.

1899 2 Ansell, George Frederic, *National Debt Office,* E.C.

1904 2 Ashley, Charles Henry, *British Widows' Assurance Co.,* 1 *Old-street,* E.C.

1883 2 Ashley, John Geo., M.A., *War Office,* S.W.

1901 2 Ashton, William Richard, *Commercial Union Assur. Co.,* 24, 25 *&* 26 *Cornhill,* E.C.

1904 2 Atkins, Leonard George, *Law Union & Crown Insurance Co.,* 126 *Chancery-lane,* W.C.

1881 2 Ayling, Charles Stephen, *Commercial Union Assur. Co.,* 26 *New Bridge-street,* E.C.

1905 2 Bain, William Algernon, *Manufacturers' Life Insurance Co., Toronto, Canada.*

1903 2 Ball, Sidney Robertson, *English and Scottish Law Life Assurance Association,* 12 *Waterloo-place,* S.W.

1905 2 Barford, Frederick William, M.A., *The High School, Perth, West Australia.*

1904 2 Barrett, William Goodsman, *United Kingdom Temperance and General Provident Institution,* 1 *Adelaide-place, London-bridge,* E.C.

1885 Barton, Arthur, *Royal Insurance Company, Maidstone.*

1894 3 Barton, Robert Whitchurch, *Clerical, Medical & General Life Assurance Society,* 26 *Mosley-street, Newcastle-on-Tyne.*

1903 2 Baxter, Edwin Herbert, *North British and Mercantile Insurance Co.,* 61 *Threadneedle-street,* E.C.

Date of becoming an Associate.

1901 2 Benjamin, Stanley O., *Australian Mutual Provident Society, Sydney, Australia.*

1881 Birks, Edmund Alfred, *Yorkshire Insurance Co., York.*

1906 2 Blake, Francis Seymour, *London County Council, Spring-gardens,* S.W.

1906 2 Blehl, Ernest M., A.M., A.A.S., *Philadelphia Life Insurance Co., North American Building, Philadelphia, Pa., U.S.A.*

1898 (2) Blount, Edward Thos. J., F.F.A., F.S.S., *Standard Life Assurance Co., Shanghai, China.*

1906 2 Boag, Harold, 14 *Avondale-terrace, Gateshead.*

1873 2 Boon, Gerald Inglis, *United Legal Indemnity Insur. Soc., Limited,* 222 *Strand,* W.C.

1906 2 Borrajo, Edward Joseph William, *Prudential Assurance Company, Holborn-bars,* E.C.

1889 (2) Bremner, Thomas William, F.F.A., *Mutual Life Insurance Co. of New York, Sydney, Australia.*

1905 (2) Brodie, Robert Raynal, F.F.A., *Scottish Provident Institution,* 6 *St. Andrew-sq., Edinburgh.*

1906 2 Bromby, Wilfrid, *Australasian Temperance and General Mutual Life Assurance Society, Melbourne, Australia.*

1896 (2) Brown, George Andrew (Auditor), *Clerical, Medical & General Life Assurance Society,* 1 *King William-street,* E.C.

1899 2 Brown, Harold, *Scottish Union and National Insurance Co.,* 3 *King William-street,* E.C.

1886 Buckley, Thomas John Wesley, 9 *St. Andrew-street, Holborn-circus,* E.C.

1882 Burke, David, F.S.S., *Royal Victoria Life Insur. Co., Montreal, Canada.*

1900 2 Burnley, Isaac, *Australian Mutual Prov. Society, Sydney, Australia.*

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1906 [2] Burrows, George Eastoe, *Alliance Assurance Co., Ltd., Bartholomew-lane,* E.C.

1895 [3] Butterfield, William Thos., A.C.A., 9 *Market-street, Bradford, Yorkshire.*

1905 (2) Cameron, Finlay James, F.F.A., *Friends' Provident Institution, Bradford, Yorkshire.*

1876 Carter, Eric Mackay, 33 *Waterloo-street, Birmingham.*

1906 [2] Carter, George Stanley, *Life Association of Scotland,* 18 *Bishopsgate-street-Within,* E.C.

1904 (2) Cathles, Lawrence Maclagan, F.F.A., *Franklin Life Insurance Co., Springfield, Ill., U.S.A.*

1905 [2] Chubb, William, *Sun Life Assurance Company of Canada, Montreal, Canada.*

1898 [2] Coates, Thomas Linnaeus, *Mutual Life Insur. Co. of New York,* 16, 17 *&* 18 *Cornhill,* E.C.

1904 [2] Collier, Charles Aubrey, 6 *Old Palace-yard,* S.W.

1871 Cook, Arthur James, M.J.I., *Victoria Mutual Assur. Society, Farringdon-street,* E.C.

1899 [3] Cook, William Playfair, *Guardian Assurance Company,* 11 *Lombard-street,* E.C.

1897 [2] Coop, Charles Rowland, *United Kingdom Temperance and General Provident Institution,* 28 *High-street, Birmingham.*

1905 [2] Cooper, John James, *Sun Life Assurance Co. of Canada, Montreal, Canada.*

1891 [2] Coote, Ernest Charles, *Alliance Assurance Co., Ltd., Bartholomew-lane,* E.C.

1900 [2] Corbett, Edwin Somerville, *Australasian Temperance and General Mutual Life Assurance Society, Sydney, Australia.*

1871 Coutts, Edwin Arthur, *North British and Mercantile Insurance Company,* 12 *Low-pavement, Nottingham.*

1900 [2] Covington, Oliver Henry, *Prudential Assurance Company, Holborn-bars,* E.C.

Date of becoming an Associate.

1884 Craig, Robert Alexander, *Abstainers' and General Assur. Co., City Buildings, Birmingham.*

1904 [2] Daman, Gerard William, B.A., 28 *Oakley Street, Chelsea,* S.W.

1906 [2] Davis, Mervyn, B.A., *Connecticut General Life Insur. Co., Hartford, Conn., U.S.A.*

1906 [2] Defries, Frederick, *Union Assurance Society,* 81 *Cornhill,* E.C.

1901 [2] Diamond, George Frederick, *City Mutual Life Assur. Society, Hunter-st., Sydney, Australia.*

1855 Dix, James, *Hurstdale, Wood-la., Highgate,* N.

1901 (2) Donald, Alexander Graham, M.A., F.F.A., *Scottish Provident Institution,* 6 *St. Andrew-square, Edinburgh.*

1881 Donaldson, John, *Australian Widows' Fund Life Assurance Society, Collins-street-west, Melbourne, Australia.*

1899 [2] Dougharty, Harold, F.S.S., F.C.I.S., *London and Lancashire Life Assurance Company,* 66 *&* 67 *Cornhill,* E.C.

1902 [2] Doust-Smith, Ernest Charles, *Prudential Assurance Company, Holborn-bars,* E.C.

1881 Dovey, William Roadly, F.F.A., F.A.S., 62 *Weston-park, Crouch End,* N.

1905 [2] Downes, Edward George, *Union Assur. Soc.,* 81 *Cornhill,* E.C.

1906 [2] Downes, Sidney Cecil, *Prudential Assurance Company, Holborn-bars,* W.C.

1870 Dowson, John, *Royal Insur. Company, Liverpool.*

1898 [2] Doyle, Arthur James, 54 *Bourke-st., Sydney, Australia.*

1901 [2] Earle, Arthur Percival, *Reliance Life Insurance Co., Farmers' Bank-buildings, Pittsburgh, Pa., U.S.A.*

1868 Eaton, Henry William, *Liverpool & London & Globe Insurance Company, William-street, New York, U.S.A.*

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1904 [2] Ecroyd, Cuthbert W., *Friends' Provident Institution, Ocean Chambers,* 44 *Waterloo-street, Birmingham.*

1905 [2] Elderton, Robert Lapidge, *National Provident Institution,* 48 *Gracechurch-street,* E.C.

1905 [2] Ellis, Reginald George Gregson, 12 *Manson-pl., Queen's-gate,* S.W.

1872 [2] Evans, William, F.F.A., F.R.S.E., 38 *Morningside-park, Edinburgh.*

1905 [2] Falk, Oswald Toynbee, B.A., F.S.S., *National Mutual Life Assur. Soc.,* 39 *King-street, Cheapside,* E.C.

1905 [2] Farmer, Ernest Chattock, *London, Edinburgh & Glasgow Insurance Co., Ltd., Insurance-buildings, Farringdon-st.,* E.C.

1896 [2] Featherstonehaugh, William Irwin, *Commercial Union Assurance Co.,* 24, 25 *&* 26 *Cornhill,* E.C.

1903 [2] Ferguson, Colin C., B.A., *Canada Life Assurance Co., Toronto, Canada.*

1906 [2] Fielder, William Crowhurst, *Atlas Assurance Company, Ltd.,* 92 *Cheapside,* E.C.

1905 [2] File, Lorne K., B.A., *Imperial Life Assurance Co. of Canada, Toronto, Canada.*

1897 [2] Findlay, Alexander Wynaud, LL.D., *Prudential Assurance Company, Holborn-bars,* E.C.

1902 [2] FitzGerald, Charles R., *State Mutual Life Assur. Co., Worcester, Mass., U.S.A.*

1901 [2] FitzGerald, William George, B.A., 68 *Dupont-street, Toronto, Canada.*

1890 (2) Fox, Charles Edward, F.F.A., *Standard Life Assurance Co.,* 83 *King William-street,* E.C.

1886 Fox, Morris, F.A.S., *New Zealand Government Life Insurance Dept., Wellington, New Zealand.*

1894 [2] Fraser, Thomas John, *Australian Alliance Assurance Company, Melbourne, Australia.*

1901 (2) Gaff, William Robertson, C.A., F.F.A., 53 *New Broad-street,* E.C.

1873 [2] Gage, Uriah Woodard, *North British & Mercantile Insur. Co.,* 61 *Threadneedle-st.,* E.C.

1895 [2] Galwey, Charles Edmund, *New Zealand Government Life Insurance Dept., Wellington, New Zealand.*

1893 [2] Gardiner, Robert Edward, *Sun Life Assurance Society,* 63 *Threadneedle-street,* E.C.

1885 [2] Gayford, Herbert Stannard, *Northern Assurance Co.,* 1 *Moorgate-street,* E.C.

1899 [3] Gibb, James Burnett, F.F.A., *Penn Mutual Life Insce. Co. of Philadelphia,* 923 *Chestnut-st., Philadelphia, U.S.A.*

1871 [2] Glennie, William Gordon, *Scottish Union & National Insur. Co.,* 3 *King William-street,* E.C.

1897 [2] Goggs, Frank Sidney, *Scottish Metropolitan Life Assur. Co., Ltd.,* 25 *St. Andrew-sq., Edinburgh.*

1882 Goldman, Leopold, F.S.S., *North American Life Assurance Co., North American Life Building,* 112-118 *King-street-west, Toronto, Canada.*

1904 [2] Goodman, Gilbert, *Prudential Assurance Company, Holborn-bars,* E.C.

1897 [2] Goodwyn, John, Jr., *Ocean Accident and Guarantee Corporation Ltd.,* 131 *Pitt-st., Sydney, Australia.*

1905 [2] Gould, W. H., M.A., *Annuity Company of Canada, Winnipeg, Manitoba, Canada.*

1902 [2] Gray, Robert Alexander, B.A., 324 *Markham-street, Toronto, Canada.*

1868 Greig, John Andrew, *Sun Life Assurance Society,* 60 *Charing-cross,* S.W.

1869 Griffith, E. Clifton, 4 *Carlton-chambers,* S.W.

1903 [2] Hall, John Bertram, A.A.S., *Dominion Life Assurance Co., Waterloo, Ontario, Canada.*

1893 [2] Hall, John Francis Edmund, *Eagle Insurance Company,* 79 *Pall-mall,* S.W.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1905 ²Hallman, M. S., F.A.S., *Mutual Life Assurance Company of Canada, Waterloo, Ontario, Canada.*

1905 ²Hammond, Reginald, *British Equitable Assur. Co., Ltd., 1, 2 & 3 Queen-street-place,* E.C.

1869 Hann, Robert George, F.A.S., *Equitable Life Assur. Soc. of the United States,* 120 *Broadway, New York.*

1894 ²Hardcastle, Edward Edgington, M.A., F.A.S., *Union Central Life Office, Cincinnati, Ohio, U.S.A.*

1900 ²Harding, Harry Burnard, *Commercial Union Assur. Co.,* 26 *New Bridge-street,* E.C.

1896 ³Harris, Frederick Joseph, *Australian Mutual Provident Society, Sydney, Australia.*

1904 ²Harriss, Walter James, *Law Life Assurance Society,* 187 *Fleet-street,* E.C.

1897 ²Haycraft, William Melhuish, *Prudential Assurance Company, Holborn-bars,* E.C.

1897 ²Hazell, James Stanley (Auditor), *National Provident Institution,* 48 *Gracechurch-street,* E.C.

1895 ²Heness, Leonard Thomas, *Prudential Assurance Company, Holborn-bars,* E.C.

1878 Henry, Alfred, F.C.A., *Throgmorton-house, Copthall-avenue,* E.C.

1900 ³Hicks, Arthur Joseph, *Law Life Assurance Society,* 187 *Fleet-street,* E.C.

1884 Higham, William Samuel, *Equitable Life Assurance Soc., Mansion-house-street,* E.C.

1905 ²Hitchins, William Richmond, B.A., F.A.S., 336 *Shaw-st., Toronto, Canada.*

1894 ²Hollingworth, Albert Charles, *Australian Mutual Provident Society, Sydney, Australia.*

1883 Holt, Edward Hallett, *Law Life Assurance Society,* 187 *Fleet-street,* E.C.

1898 ²Howell, Chas. Edward, B.A., LL.D., *Standard Life Assurance Compy.,* 59 *Dawson-street, Dublin.*

1899 ²Hudson, Alfred James, *Northern Assurance Company,* 60 *&* 61 *London-wall,* E.C.

1875 Hunt, Richard Aldington, F.S.S., *Wesleyan & General Assur. Soc., Steelhouse-lane, Birmingham.*

1893 (2) Hunter, Arthur, F.F.A., F.A.S., F.S.S. *New York Life Insurance Co.,* 346 *&* 348 *Broadway, New York, U.S.A.*

1902 ²Hunter, Robertson G., F.A.S., *New York Life Insurance Co., New York Life Building, Chicago, Ill., U.S.A.*

1887 ²Hunter, Samuel, 66 *St. Lawrence-road, Clontarf, Dublin.*

1904 (2) Imrie, John Hamilton, M.A., F.F.A., *Life Association of Scotland,* 82 *Princes-street, Edinburgh.*

1889 (2) Jacobs, Frederick Job, *Australian Mutual Provident Society, Sydney, Australia.*

1876 ²James, George Trevelyan, 12 *Waterloo-place,* S.W.

1905 (2) Jamieson, Charles William Steele, F.F.A., *Scottish Amicable Life Assur. Society,* 35 *St. Vincent-place, Glasgow.*

1905 ²Jefferson, John Arthur, *Britannic Ass. Co., Ltd., Broad-street-corner, Birmingham.*

1871 Jellicoe, George Rogers, *Eagle Insurance Company,* 79 *Pall-mall,* S.W.

1883 Jerman, Richard, *Commercial Union Assurance Company, Exeter.*

1896 ²Jobson, Alexander, *Equitable Life Assurance Society of the United States, Sydney, Australia.*

1894 ²Johannessen, Nikolai Mikal, *Hygea Life Assurance Company, Bergen, Norway.*

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute. Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1894 [2] Johnston, Frederick H., F.A.S., *Prudential Life Insurance Co. of America, Newark, N.J., U.S.A.*

1903 [2] Jones, Leonard Alexander Mouat, *Commercial Union Assur. Co.*, 24, 25 *&* 26 *Cornhill*, E.C.

1903 [2] Jones, Wallace Mouat, *General Reversionary & Investment Company, Limited*, 26 *Pall-mall*, S.W.

1898 [2] Kaufman, Henry N., A.A.S., *Phœnix Mutual Life Insurance Co., Hartford, Connecticut, U.S.A.*

1876 Kearry, Joseph, 44 *Charlwood-street, Belgrave-road*, S.W.

1899 [2] Kelly, John Joseph, *Citizens' Life Assurance Co., Sydney, Australia.*

1897 [2] Kemp, Julian Ernest Sandford, *Eagle Insurance Company*, 79 *Pall-mall*, S.W.

1902 [2] Kilgour, David Errett, M.A., *North American Life Assurance Co., North American Life Building*, 112-118 *King-street-west, Toronto, Canada.*

1874 King, Arthur Thomas, I.S.O., *National Debt Office*, E.C.

1882 [2] King, William Alfred, *Northern Assurance Company*, 1 *Moorgate-street*, E.C.

1902 [2] Kitchin, Frederick Harcourt, B.A., *Broad Clyst, Gloucester-road, Teddington.*

1905 [3] Laing, James Murray, F.F.A., *National Mutual Life Assocn. of Australasia*, 5 *Cheapside*, E.C.

1893 [2] Laing, William Claud, *North British and Mercantile Insurance Company*, 61 *Threadneedle-street*, E.C.

1897 [2] Lane, Arthur Vere, B.A., 17 *Park-road East, Birkenhead.*

1905 [2] Langstaff, James Miles, *Imperial Life Assurance Co. of Canada, Toronto, Canada.*

1905 [2] Latham, Bertrand, *Australian Mutual Provident Society, Melbourne, Australia.*

Date of becoming an Associate.

1906 [2] Latham, Percy James, *Inland Revenue Department, Falkirk, N.B.*

1906 (2) Latta, Alexander, F.F.A., *Guardian Assurance Company*, 28 *King-st., Covent-garden*, W.C.

1899 [2] Lawton, George Herbert, *Clerical, Medical & General Life Assurance Society*, 15 *St. James's-square*, S.W.

1905 [2] Leigh, Samuel George, *Refuge Assurance Co., Oxford-street, Manchester.*

1879 Leitch, Alexander, *Scottish Provident Institution*, 17 *King William-street*, E.C.

1897 [2] Le Maitre, Frank William, *Sun Life Assurance Society*, 63 *Threadneedle-street*, E.C.

1885 Leveaux, Arthur Michael, F.S.S., *Registry of Friendly Societies, Central Office*, 28 *Abingdon-street, Westminster*, S.W.

1868 Litchfield, Edward, 92 *St. Vincent-street, Glasgow.*

1876 [2] Lucey, Herbert, *General Assurance Company*, 103 *Cannon-street*, E.C.

1890 (2) Lugton, Hugh, F.F.A. (AUDITOR), *North British and Mercantile Insurance Co.*, 61 *Threadneedle-street*, E.C.

1900 [3] McArthur, Harry de C., *Box* 282, *Dunedin, New Zealand.*

1867 Macdonald, William Rae, F.F.A., *Scottish Metropolitan Life Assur. Co., Limited*, 25 *St. Andrew-square, Edinburgh.*

1882 [3] McDougald, Alfred, *Pelican and British Empire Life Office, Montreal, Canada.*

1905 [2] Macfarlane, James Allan, *North American Life Assurance Co., North American Life Building*, 112-118 *King-street-west, Toronto, Canada.*

1884 Mackay, Alexander, *Law Union & Crown Insur. Co.*, 126 *Chancery-lane*, W.C.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute. Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1905 [2] McKechnie, James Baldwin, *Manufacturers' Life Insurance Company, Toronto, Canada.*

1901 [3] Mackenzie, Michael Alexander, *c/o Messrs. John Mackay & Co., Investment Brokers,* 7 *&* 9 *King-street-east, Toronto, Canada.*

1896 [2] Macmillan, John Campbell, *Northern Assur. Co. and Law Union and Crown Insurance Co., Apartado Postal No.* 872, *Mexico.*

1905 [2] McPhail, Frederick Charles, *Colonial Mutual Life Assurance Soc., Ltd., Melbourne, Australia.*

1883 [2] Makeham, William Reed, *Alliance Assurance Co., Ltd.* (*Imperial Life Assurance Fund*), 47 *Chancery-lane,* W.C.

1905 [2] Makepeace, Francis Lucas, B.A., 5 *&* 6 *Clement's-inn, Strand,* W.C.

1883 Mannering, George Willsher, *London and Lancashire Life Assur. Co.,* 66 *&* 67 *Cornhill,* E.C.

1880 Manwaring, Henry, *National Debt Office,* E.C.

1896 [2] Martin, Sidney George, *National Mutual Life Assoc. of Australasia, Ltd.,* 295 *Queen-street, Brisbane, Australia.*

1897 [2] Mascall, Alfred John, *Standard Life Assurance Co.,* 3 *Pall-mall East,* S.W.

1904 [2] Maudling, Reginald G., *London and Lancashire Life Assur. Co.,* 66 *&* 67 *Cornhill,* E.C.

1900 [2] Maunder, George Harvard, *National Mutual Life Assur. Society,* 39 *King-st., Cheapside,* E.C.

1902 (2) Maxwell, Benjamin Bell, F.F.A., *Scottish Equitable Life Assur. Society,* 28 *St. Andrew-square, Edinburgh.*

1899 [2] Meade, Gerald Willoughby, *North British & Mercantile Insurance Company,* 61 *Threadneedle-street,* E.C.

Date of becoming an Associate.

1896 [2] Merfield, Percy Henry, *Law Life Assurance Society,* 187 *Fleet-street,* E.C.

1874 Miller, John W., F.S.S., *Scottish Widows' Fund and Life Assur. Soc.,* 28 *Cornhill,* E.C.

1905 [2] Monilaws, William Barrington, *Scottish Provident Institution,* 17 *King William-street,* E.C.

1879 Monilaws, William Macgeorge, *Scottish Provident Institution,* 17 *King William-street,* E.C.

1905 [2] Monkhouse, Charles Cosmo, B.A., *Clerical, Medical and General Life Assurance Society,* 15 *St. James's-square,* S.W.

1877 Moon, James, *Prudential Assurance Company,* 30 *Dale-street, Liverpool.*

1877 Moon, John, *Parkhurst, Didsbury, Manchester.*

1879 Moon, Sidney Norman Laming, 133 *West* 129*th-street, New York, U.S.A.*

1903 [2] Moore, George Cecil, *Imperial Life Assurance Co. of Canada, Toronto, Canada.*

1905 [2] Moore, George Edward, *Australian Widows' Fund Life Assurance Society, Melbourne, Australia.*

1905 [2] Moore, Gerald Leslie, A.C.A., 1 *Rosebery-gardens, Muswell-hill,* N.

1898 [2] Moore, Joseph Patrick, *Citizens' Life Assurance Co., Sydney, Australia.*

1871 [2] Moore, Roderick Mackenzie, *United Kingdom Temperance and General Provident Institution,* 1 *Adelaide-place, London-bridge,* E.C.

1893 [2] Munro, Donald Alexander, *Brook-house,* 10 *Walbrook,* E.C.

1900 [2] Nash, Alfred Charles, *Clerical, Medical and General Life Assurance Society,* 15 *St. James's-square,* S.W.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute. Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1897 [2] Newling, Sidney Wallis, B.A., *Woodleigh, South Woodford, Essex.*

1905 [2] Newnham, Ernest Whiffin, *Prudential Assurance Company, Holborn-bars,* E.C.

1903 [2] Nicholls, Arthur William, *Australian Mutual Provident Society, Brisbane, Australia.*

1884 Nicoll, John, F.F.A., *Life Association of Scotland,* 82 *Princes-street, Edinburgh.*

1883 Orr, Lewis P., F.F.A., *Scottish Life Assur. Co., Ltd.,* 19 *St. Andrew-sq., Edinburgh.*

1906 (2) Padday, Percy King, F.F.A., *Scottish Metropolitan Life Assurance Co., Ltd.,* 8 *King-st., Cheapside,* E.C.

1895 [2] Pagden, Lionel King, *Union Assurance Society,* 81 *Cornhill,* E.C.

1864 Panton, Edward Henry, 50 *Wood-vale, Forest Hill,* S.E.

1901 [3] Papps, Percy Charles Herbert, F.A.S., *Manufacturers' Life Insurance Co., Toronto, Canada.*

1895 [2] Paradice, William Henry, *Australian Mutual Provident Society, Sydney, Australia.*

1869 Park, David Francis, C.A., F.F.A., *Crédit Foncier of Mauritius, Limited,* 39 *Lombard-st.,* E.C.

1905 [2] Paton, Albert George, *London Assurance Corporation,* 7 *Royal Exchange,* E.C.

1898 (2) Pearce, Henry John, F.F.A., *Scottish Amicable Life Assurance Society,* 35 *St. Vincent-place, Glasgow.*

1899 [2] Peele, Thomas, *Universal Insur. Loan & Investment Co., Ltd.,* 77 *New Briggate, Leeds.*

1900 [2] Peters, Charles Furness, *L'pool. Victoria Legal Friendly Society, St. Andrew-street,* E.C.

1895 (2) Pierson, Israel Coriell, F.A.S., 141 *Broadway, New York, U.S.A.*

Date of becoming an Associate

1902 [2] Pigrome, George Davey, *Prudential Assurance Company, Holborn-bars,* E.C.

1899 [2] Pipe, Sidney Herbert, *London and Lancashire Life Insurance Company, Montreal, Canada.*

1883 Pitts, Thomas, *Commercial Union Assurance Company, Exeter.*

1906 [2] Portch, Albert Garfield, A.A.S., *Canada Life Assurance Co., Toronto, Canada.*

1890 [2] Powell, Alfred, *Alliance Assurance Company, Limited, Bartholomew-lane,* E.C.

1881 Price, William John, *Life Association of Scotland,* 18 *Bishopsgate-st.-Within,* E.C.

1869 Pringle, James, C.A., F.F.A., 42 *Drumsheugh-gardens, Edinburgh.*

1884 Pullar, James, F.F.A., *Colonial Mutual Life Assurance Society, Melbourne, Australia.*

1881 Purves, Thomas Peter, *New York Life Insurance Company, Sydney, Australia.*

1904 (2) Rankin, John Adam, F.F.A., *Edinburgh Life Assurance Co.,* 22 *George-street, Edinburgh.*

1867 Rattray, Patrick, C.A., 115 *St. Vincent-street, Glasgow.*

1874 [2] Ray, Charles Richard, *Commercial Union Assur. Co.,* 26 *New Bridge-street,* E.C.

1905 [2] Raynes, Harold Ernest, *Legal and General Life Assurance Society,* 10 *Fleet-street,* E.C.

1885 Rea, Charles Herbert Edmund, F.R.A.S., F.S.S., *National Standard Assurance Corporation,* 149 *Leadenhall-st.,* E.C.

1898 [2] Reid, Edward E., B.A., *London Life Insurance Co., London, Ontario, Canada.*

1901 [2] Rhodes, Francis, B.A., *Royal Insurance Co., Liverpool.*

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute. Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.

1887 Richardson, Josephus Hargreaves, F.F.A., F.A.S., *New Zealand Government Life Insurance Department, Wellington, New Zealand.*

1879 Roberts, Thomas B., *Australian Alliance Assurance Company, Collins-street, Melbourne, Australia.*

1904 3 Robertson, Frederick William, F.F.A., *Caledonian Insurance Company,* 19 *George-street, Edinburgh.*

1904 (2) Robertson, James Leask, F.F.A., *Edinburgh Life Assurance Co.,* 22 *George-street, Edinburgh.*

1878 Robertson, William, F.F.A., 29 *Stafford-street, Edinburgh.*

1876 Robinson, Andrew, *Sunningdale-park, Sunningdale, Berks.*

1885 Ronald, Thomas Robert, *Law Guarantee and Trust Soc., Ltd.,* 49 *Chancery-lane,* W.C.

1904 2 Rudd, Alfred James, *Australian Widows' Fund Life Assurance Society, Grenfell-street, Adelaide, South Australia.*

1897 2 Ryley, Edmund, *Prudential Assurance Company, Holborn-bars,* E.C.

1896 2 Sanderson, Frank, M.A., F.F.A., F.A.S., F.S.S., *Canada Life Assurance Company, Toronto, Canada.*

1904 2 Sare, Thomas Henry, *Commercial Union Assur. Co.,* 24, 25 *&* 26 *Cornhill,* E.C.

1905 2 Savery, Robert S. B., *Gresham Life Assurance Society, Giselastrasse, No.* 1, *Vienna.*

1884 Schooling, John Holt, *Fotheringay-house, Montpelier-row, Twickenham.*

1899 2 Schouten, Pieter, *Verzekering Maatschappij, "Arnhem," Stations-plein,* 17, *Arnhem, Holland.*

Date of becoming an Associate.

1906 (2) Scott, Albert George, *English and Scottish Law Life Assur. Association,* 12 *Waterloo-place,* S.W.

1873 Scott, Ernest Willem, F.A.S., *Algemeene Maatschappij van Levensverzekering en Lijfrente, Damrak,* 74, *Amsterdam.*

1904 2 Searle, Arthur Joseph, *English & Scottish Law Life Assurance Association,* 12 *Waterloo-place,* S.W.

1861 2 Searle, Thomas John, *Mansion-house-chambers, Bucklersbury,* E.C.

1900 2 Searls, Edwin Richard, *Northern Assurance Company,* 60 *&* 61 *London-wall,* E.C.

1900 2 Sharpe, Edgar Cecil Engledue, *London Life Association, Ltd.,* 81 *King William-street,* E.C.

1894 3 Sheppard, Herbert Norman, B.A., F.A.S., *Home Life Insurance Company,* 256 *Broadway, New York, U.S.A.*

1897 2 Shimmell, James Edward, *United Provident Assurance Co., Ltd.,* 96 *Oxford-rd., Manchester.*

1896 2 Shlager, Joseph, *Equitable Life Assurance Society of the United States, Mansion-house-chambers, Adderley-street, Cape Town, South Africa.*

1903 2 Shovelton, Sydney Taverner, M.A., 3 *Dynham-road, West Hampstead,* N.W.

1905 2 Shute, Oxenham Bent, *National Provincial Bank of England,* 53 *Baker-street,* W.

1864 Smith, Howard Samuel, F.F.A., F.C.A., F.S.S., *Bank-chambers,* 11 *Waterloo-street, Birmingham.*

1898 2 Smith, Robert Parker, *Royal Insurance Company, Liverpool.*

1906 2 Smither, Herbert Buxton, *University Life Assurance Soc.,* 25 *Pall-mall,* S.W.

ASSOCIATES.

Those marked 2 *or* 3 *have passed two or three of the four Examinations of the Institute.*
Those marked (2) *have been exempted under the Bye-laws from the Examinations in Parts I and II.*

Date of becoming an Associate.	
1884	Smithett, Edward Henry, *"Hillside," Fitzroy-park, Highgate,* N.
1905	2 Somerville, Walter Harold, *Mutual Life Assur. Co. of Canada, Waterloo, Ontario, Canada.*
1871	Spencer, Robert James, F.S.S., 75 *King's-road, Southsea.*
1868	Spens, William George, *Scottish Amicable Life Assur. Soc.,* 35 *St. Vincent-pl., Glasgow.*
1866	Stark, William Emery, *Chapel-walks, Manchester.*
1878	Stevenson, Charles, 9 *Albert-square, Manchester.*
1880	Stock, Edward James, *National Mutual Life Assoc. of Australasia, Melbourne, Australia.*
1906	2 Story, Cyril Lionel William Steane, *Norwich Union Life Insurance Society,* 71 *&* 72 *King William-street,* E.C.
1905	2 Strong, Allan Wilmot, *Sun Life Assurance Co. of Canada, Montreal, Canada.*
1896	2 Stuckey, Jos. James, M.A., *Salisbury Chambers,* 49*a King William-street, Adelaide, South Australia.*
1905	2 Stuckey, Reginald Robert, *Australian Mutual Provident Society, Adelaide, South Australia.*
1905	2 Sturt, Herbert Rothsay, *National Standard Assurance Corporation, Ltd.,* 149 *Leadenhall-street,* E.C.
1904	(2) Tatlock, John, M.A., F.R.A.S., F.A.S., 376 *West End Avenue, New York, U.S.A.*
1893	2 Taylor, Arthur, *Guardian Assurance Company,* 28 *King-street, Covent-garden,* W.C.
1875	Taylor, J. Wilford, *North British and Mercantile Insur. Co.,* 61 *Threadneedle-st.,* E.C.
1906	2 Thompson, John Spencer, *Mutual Life Insurance Co. of New York, New York, U.S.A.*
1906	2 Thomson, Frederick Robert T., *Kent-house, Church-end, Finchley,* N.
1904	(2) Thomson, John Walter, F.F.A., *Scottish Life Assur. Co.,* 19 *St. Andrew-square, Edinburgh.*
1883	2 Titmuss, Walter George, *Alliance Assur. Co., Ltd. (Provident Life Fund),* 50 *Regent-street,* W.
1905	2 Touzel, Philip Duncan, *Australian Mutual Provident Society, Melbourne, Australia.*
1905	2 Townley, Ebenezer William, *National Mutual Life Assurance Soc.,* 39 *King-st., Cheapside,* E.C.
1902	3 Traversi, Antonio Thomas, *New Zealand Government Life Insurance Department, Wellington, New Zealand.*
1883	Tregaskis, George Alfred, *Commercial Union Assur. Co.* 26 *New Bridge-street,* E.C.
1894	2 Trenerry, Charles Farley, B.A., *University of London, South Kensington,* S.W.
1905	2 Tully, Arthur Patrick Thomas, *Consolidated Assur. Co., Ltd.* 9 *Fleet-street,* E.C.
1891	2 Turnbull, A. D. Lindsay, C.A., F.F.A., F.C.I.S., *Scottish Widows Fund and Life Assurance Society,* 9 *St. Andrew-square, Edinburgh.*
1884	Vian, William Collett, *Railway Passengers' Assurance Company,* 64 *Cornhill,* E.C.
1884	Vincent, Frederick James, F.S.S., *London, Edinburgh & Glasgow Assurance Co., Ltd., Insurance-buildings, Farringdon-street,* E.C.
1899	2 Vokins, George Alfred, *Prudential Assurance Company, Holborn-bars,* E.C.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.	
1879	Wall, Walter George, 3 *Shrewsbury-road, Birkenhead.*
1878	Walton, William Gandy, F.F.A., *Scottish Provident Institution,* 6 *St. Andrew-square, Edinburgh.*
1905	2 Wansbrough, Thomas Percival, *English and Scottish Law Life Assurance Association,* 37 *Queen Victoria-street,* E.C.
1906	(2) Wardrop, James Charles, *Life Association of Scotland,* 18 *Bishopsgate-street-Within,* E.C.
1903	2 Watherston, Charles F., B.A., *War Office,* S.W.
1883	2 Watson, John Robertson, *British Law Fire Insurance Co.,* 105 *West George-st., Glasgow.*
1894	2 Watt, George, *Royal Insurance Co., Liverpool.*
1900	(2) Watt, James, F.F.A., 18 *Moray-place, Edinburgh.*
1883	Weall, Bertram, 16 *Waldegrave-park, Twickenham.*
1902	2 Weatherill, Charles, *Scottish Office,* S.W.
1894	(2) Weeks, Rufus Wells, F.A.S., *New York Life Insurance Co.,* 346 & 348 *Broadway, New York, U.S.A.*
1898	3 Whigham, Charles Frederick, F.F.A., C.A., *Messrs. Moncreiff & Horsbrugh,* 46 *Castle-street, Edinburgh.*
1884	Whyte, Alexander, *c/o Messrs. Lever Bros., Ltd., Cambridge, Mass., U.S.A.*
1897	2 Wickens, Charles H., *Commonwealth Bureau of Census and Statistics, Melbourne, Victoria, Australia.*
1896	2 Wilkinson, Edward Berkeley, 24 *Maxilla-gardens, N. Kensington,* W.
1903	2 Wilkinson, William Magnay, *Citizens' Life Assurance Co., Sydney, Australia.*
1904	2 Williams, Frederick Alfred, F.S.S., A.A.S., *Home Life Assoctn. of Canada, Toronto, Canada.*
1904	2 Wilson, Arthur Benjamin, *Australian Mutual Provident Soc., Wellington, New Zealand.*
1900	2 Wilson, George, *Standard Life Assurance Company,* 3 *George-st., Edinburgh.*
1870	2 Wilson, Henry Edward, *Northern Assurance Co.,* 1 *Moorgate-street,* E.C.
1873	2 Windett, Charles, *Legal & General Life Assurance Society,* 10 *Fleet-street,* E.C.
1905	2 Winstanley, Charles William, *North British & Mercantile Insurance Co.,* 61 *Threadneedle-street,* E.C.
1903	2 Wood, William Archibald Porter, B.A., *Canada Life Assurance Co., Toronto, Canada.*
1883	Woodhouse, Lister, A.C.A., F.S.S., *City Comptroller, Westminster City-hall, Charing Cross-road,* W.C.
1877	2 Woods, Arthur Biddle, *Rock Life Assurance Company,* 15 *New Bridge-street,* E.C.
1866	Woods, Bernard, *Metropolitan Life Assurance Society,* 13 *Moorgate-street,* E.C.
1879	Wornum, Thornton Selden, *Rock Life Assurance Company,* 15 *New Bridge-street,* E.C.
1903	2 Worth, Bertram Oliver, *Clerical, Medical & General Life Assurance Society,* 15 *St. James's-square,* S.W.
1871	Yardley, John, *Prudential Assurance Company, Holborn-bars,* E.C.
1873	Young, Alexander Hunter, 60 *Market-street, Melbourne, Australia.*

STUDENTS.

Those marked 1, 2, *or* 3 *have passed one, two, or three of the four Examinations of the Institute.*
Those marked (1) *have been exempted under the Bye-laws from the Examination in Part I.*

Date of becoming a Student.

1892 [1] Aaron, David Hyam,
Sun Life Assurance Society, 63 *Threadneedle-street*, E.C.

1906 [1] Abdul-Ali, Sijil,
3 *New Quebec-street, Portman-square*, W.

1903 [1] Acum, Wilfred Harry,
15 *Lordship-lane, Wood Green*, N.

1905 [1] Adam, Cyrus Cyril,
Northern Assurance Company, 7 *Westmoreland-street, Dublin.*

1904 [1] Addey, Leonard,
Clergy Mutual Assurance Soc., 2 *&* 3 *The Sanctuary*, S.W.

1905 [1] Agutter, William John,
Prudential Assurance Company, Holborn-bars, E.C.

1905 [1] Alder, Milton Cromwell,
Citizens' Life Assurance Co., Sydney, Australia.

1906 (1) Allen, Arthur Ormiston, M.A., B.Sc.,
2 *Norwood-grove, Leeds.*

1905 [1] Allen, John,
Imperial Life Assurance Co. of Canada, Brandon, Manitoba, Canada.

1904 [1] Allison, Sinclair E.,
Canada Life Assurance Co., Toronto, Canada.

1906 [1] Anderson, Robert Duncan,
Commercial Union Assurance Co., 24, 25 *&* 26 *Cornhill*, E.C.

1904 [1] Armstrong, Charles Henry,
Imperial Life Assurance Co. of Canada, Toronto, Canada.

1886 Arnold, Thomas, Jr.,
British Equitable Assur. Co., Ltd., 1, 2 & 3 *Queen-street-place*, E.C.

1902 [1] Askwith, Thomas Nowell,
London Life Association, Ltd., 81 *King William-street*, E.C.

1905 [1] Atkins, Francis Cuthbert,
Prudential Assurance Company, Holborn-bars, E.C.

1904 [1] Ayscough, Ivan,
Equity and Law Life Assurance Soc., 18 *Lincoln's-inn-fields*, W.C.

1899 [1] Baber, Walter Crosbie, A.A.S.,
Royal Victoria Life Insur. Co. of Canada, Montreal, Canada.

1903 [1] Baggs, Henry Ernest,
English and Scottish Law Life Assurance Association, 12 *Waterloo-place*, S.W.

1899 [1] Barnett, Isaac,
North British and Mercantile Insurance Co., 61 *Threadneedle-street*, E.C.

1896 [1] Barry, David,
Royal Commission on the University of Melbourne, Supreme Court Library, Melbourne, Australia.

1898 [1] Bennell, Samuel Thomas,
25 *Meath-road, Ilford.*

1906 [1] Bennett, Henry Gordon,
Australian Mutual Provident Society, Melbourne, Australia.

1903 [1] Bennett, Reginald,
Refuge Assurance Co., Oxford-street, Manchester.

1898 [1] Bennett, Samuel,
National Deposit Friendly Soc., 37 *Queen-square*, W.C.

1902 [1] Biden, Norman Frederick,
Standard Life Association, 28 *Elizabeth-st., Sydney, Australia.*

1895 [1] Bigby, Robert Frederick Mitchell,
General Assurance Company, 103 *Cannon-street*, E.C.

1900 [1] Bingeman, Milton H.,
Great-West Life Assurance Co., Winnipeg, Manitoba, Canada.

1903 [1] Binney, Charles Eardley-Wilmot,
Royal Exchange Assurance Corporation, Royal Exchange, E.C.

1891 [1] Bird, Edward William,
Northern Assurance Company, 60 & 61 *London Wall*, E.C.

1905 [1] Blackadar, E. Gordon, B.A.,
Canada Life Assurance Co., Toronto, Canada.

1902 [3] Blanchard, Norman, B.A.,
Equity & Law Life Assur. Soc., 18 *Lincoln's-inn-fields*, W.C.

1887 Blossom, James,
186 *South-view-road, Sheffield.*

STUDENTS.

Those marked 1, 2, *or* 3 *have passed one, two, or three of the four Examinations of the Institute.*
Those marked (1) *have been exempted under the Bye-laws from the Examination in Part I.*

Date of becoming a Student.

1892 [1] Boddy, Henry Mitchell,
Manufacturers' Life Insurance Co., Cape Town, South Africa.

1903 [1] Bodley, Rupert Frank,
Star Life Assurance Society, 32 *Moorgate-street,* E.C.

1906 [1] Bolt, Jan Cornelis,
's Gravendijkwal 221, *Rotterdam.*

1897 Bond, Frederic D.,
122 *South* 39*th Street, Philadelphia, U.S.A.*

1902 [1] Bowerman, Judah Philip,
472 *Lafayette-avenue, Brooklyn, New York, U.S.A.*

1897 [1] Bowles, Francis Marsh,
Pearl Life Assurance Company, London-bridge, E.C.

1891 [1] Boyd, Henry Norris,
City of Glasgow Life Assurance Company, 21 *St. Andrew-square, Edinburgh.*

1903 [1] Bradbury, Algernon Charles,
Australian Mutual Provident Society, Melbourne, Australia.

1905 [1] Bradshaw, Frank Law,
Law Guarantee and Trust Soc., Ltd., 49 *Chancery-lane,* W.C.

1899 [1] Brady, John Francis,
Citizens' Life Assurance Co., Sydney, Australia.

1906 [1] Breeds, Arthur Heywood,
Aldenham Corner, Radlett, Herts.

1902 [1] Brook, Frank A.,
Refuge Assurance Co., Oxford-street, Manchester.

1894 [1] Brough, Frank,
Federal Life Assurance Company, Hamilton, Ontario, Canada.

1904 [1] Brown, Arthur Ewart,
Scottish Widows' Fund and Life Assur. Society, 28 *Cornhill,* E.C.

1906 [1] Brown, B. G. H.,
Union Assurance Society, 81 *Cornhill,* E.C.

1906 [1] Brown, Frank, M.A.,
1163 *De Lorimier-avenue, Montreal, Canada.*

Date of becoming a Student.

1905 [1] Brown, James,
Robert-st., Marrickville, Sydney, Australia.

1906 [1] Brown, Peter Gordon,
Ecclesiastical Commission, Millbank, S.W.

1891 [1] Brown, William Heron,
Gresham Life Assur. Soc., Ltd., St. Mildred's-house, Poultry, E.C.

1905 [1] Burrows, Victor Albert,
Sun Life Office, 332 *Oxford-street,* W.

1904 [1] Canter, Harold,
National Provident Institution, 48 *Gracechurch-street,* E.C.

1903 [1] Capon, Frank Christopher,
Prudential Assurance Company, Holborn-bars, E.C.

1902 [1] Capon, Geoffrey William,
Norwich Union Life Insurance Society, Norwich.

1903 [1] Carpenter, Thomas B. Boyd,
Clergy Mutual Assur. Society, 2 *&* 3 *The Sanctuary,* S.W.

1899 [3] Carter, Norman John,
Eagle Insurance Company, 79 *Pall-mall,* S.W.

1900 [1] Chambers, John Joseph,
North Rigton, Ruby, nr. Leeds.

1902 [1] Chandler, Frederick Joseph,
Eagle Insurance Co., 79 *Pall-mall,* S.W.

1897 [1] Cherry, John Arnold,
Chamber of London, Guildhall, E.C. (*Reinstated,* 1905.)

1903 [1] Cheshire, Harold Frank,
9 *Wellington-place, Hastings.*

1903 [1] Child, Robert Harold,
North British and Mercantile Insurance Company, 61 *Threadneedle-street,* E.C.

1905 [1] Clarke, Herbert George,
Australian Widows' Fund Life Assurance Society, Melbourne, Australia.

1905 [1] Clemens, Frederic Broadbent,
Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.

STUDENTS.

Those marked 1, 2, *or* 3 *have passed one, two, or three of the four Examinations of the Institute. Those marked* (1) *have been exempted under the Bye-laws rom the Examination in Part I.*

Date of becoming a Student.

1897 [1] Clinton, George, *Prudential Assurance Company, Holborn-bars*, E.C.

1902 [3] Clinton, Louis Ernest, *Alliance Assurance Company, Ltd., Bartholomew-lane*, E.C.

1902 [2] Coates, Frederick George, *Commercial Union Assur. Co.,* 26 *New Bridge-street*, E.C.

1901 [1] Cockerton, John Leonard, *Pioneer Life Assurance Co., Ltd., Century-buildings,* 31 *North John-street, Liverpool.*

1895 [1] Cogar, William Edward, *New York Life Insurance Co., Trafalgar-square*, W.C.

1899 [1] Collins, Patrick A., *Citizens' Life Assurance Co., Sydney, Australia.*

1902 [1] Collins, William Ernest, *Assoc. Inst. Accts. S.A.,* 49*a King William-street, Adelaide, South Australia.*

1896 [1] Cook, Henry Milton, *Standard Life Assurance Company, Dalhousie-square, Calcutta, India.*

1900 [1] Cooper, Bernard Hugh, *Prudential Assurance Company, Holborn-bars*, E.C.

1906 [1] Cooper, John Lewis, *Liverpool and London and Globe Insur. Co.,* 1 *Dale-st., Liverpool.*

1902 [1] Corbett, Archibald Gladstone, *Australian Mutual Provident Society, Collins-st., Melbourne, Australia.*

1899 [1] Cotterill, William Ernest, *Mutual Life Association of Australasia, Ltd., Sydney, Australia.* (*Reinstated*, 1905.)

1903 [1] Cotton, Arthur Sparkes, *Scottish Office*, S.W.

1905 [1] Coutts, Kenneth Vawdrey, *Clergy Mutual Assurance Soc.,* 2 *&* 3 *The Sanctuary*, S.W.

1901 [1] Coventry, Cameron H., *Australasian Temperance and General Mutual Life Assurance Society, Melbourne, Australia.*

1906 (1) Coward, Charles Ernest, B.A., *Estate Duty Office, Somerset House*, W.C.

Date becoming a Student.

1904 [1] Cowdy, Henry Leslie, *Scottish Union & National Insur. Co.,* 3 *King William-street*, E.C.

1894 Cox, Edward William, *Canada Life Assurance Co., Toronto, Canada.*

1894 Cox, Herbert Coplin, *Canada Life Assurance Co., Toronto, Canada.*

1905 [1] Cox, Stanley Nelson, *Prudential Assurance Company, Holborn-bars*, E.C.

1887 [1] Cross, Henry John, 3 *Park-rd., Wandsworth-common*, S.W.

1897 [2] Crump, Percy C., *Prudential Assurance Company, Holborn-bars*, E.C.

1904 [1] Cushing, Robertson Macaulay, *Sun Life Assurance Company of Canada, Montreal, Canada.*

1904 [1] Dalrymple, Alfred George, *Canada Life Assurance Company, Toronto, Canada.*

1897 [1] Dalton, John, *London Life Association, Ltd.,* 81 *King William-street*, E.C.

1905 [1] Dark, Thomas Arthur, *Excelsior Life Insurance Co., Toronto, Canada.*

1889 [1] Davies, Hugh Myddleton, *Royal Insurance Co., Liverpool.*

1900 [1] Davies, William Allison, *Borough Treasurer's Office, Town Hall, Birkenhead.*

1906 [1] Davis, Archibald Percy, *Sydenham-road, Marrickville, Sydney, Australia.*

1899 [1] Davison, Horace Williams, 7 *North-street, Toronto, Canada.*

1891 [1] Dawson, Frank Aubrey, *Ecclesiastical Insurance Office, Limited,* 11 *Norfolk-street, Strand*, W.C.

1902 [1] Deck, James Gilbert, *National Provident Institution,* 48 *Gracechurch-street*, E.C.

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1902 [1] Denmark, Robert John, *Norwich Union Life Insurance Society, Norwich.*

1901 [1] Dent, Ernest Edward, *London and Lancashire Life Assurance Company,* 66 *&* 67 *Cornhill,* E.C.

1905 [1] Derrick, Victor Percival Augustine, 43 *Falmouth-road,* S.E.

1896 [1] de Ville, Francis, *Clergy Pensions Institution,* 11 *Norfolk-street, Strand,* W.C.

1906 [1] Dobbie, John Albert, *Provincial Normal School, Ottawa, Canada.*

1890 [1] Docker, Leslie, *North British and Mercantile Insurance Co.,* 61 *Threadneedle-street,* E.C.

1897 [1] Dorrian, John Christopher, *Citizens' Life Assurance Company, Sydney, Australia.*

1906 [1] Doucet, Gerald Danby, *Rock Life Assurance Company,* 15 *New Bridge-street,* E.C.

1906 [1] Doyle, Joseph Patrick, *Citizens' Life Assurance Co., Sydney, Australia.*

1904 [1] Drake, Charles Clifford Hall, *Prudential Assurance Company, Holborn-bars,* E.C.

1906 [1] Drake, John William, Jr., *Fern Bank, Wisewood, Sheffield.*

1905 [1] Dulley, John Francis, *Prudential Assurance Company, Holborn-bars,* E.C.

1905 [1] Eastcott, William Merrill, *Sun Life Assur. Co. of Canada, Montreal, Canada.*

1892 [1] Edwards, Edward Samuel, M.A., *Australian Mutual Provident Society, Sydney, Australia.*

1905 [1] Edwards, Herbert Alfred, *British Homes Assurance Corp.,* 6 *Paul-street, Finsbury,* E.C.

1905 [1] Edwards, Herbert Horace, 31 *Haverstock-road,* N.W.

Date of becoming a Student.

1902 [1] Edwards, Thomas Baker, *Comptroller's Dept., London County Council, Spring-gardens,* S.W.

1892 [1] Eedy, Arthur Malcolm, *Citizens' Life Assurance Company, Sydney, Australia.*

1901 [1] Egleton, Harold Edward, *Prudential Assurance Company, Holborn-bars,* E.C.

1904 [1] Eldridge, Ernest Edward Booth, *Atlas Assurance Company, Ltd.,* 92 *Cheapside,* E.C.

1893 [1] Emery, John M., *Des Moines Life Insurance Co., Des Moines, Iowa, U.S.A.*

1906 [1] Emery, Walter Sydney, *Australian Widows' Fund Life Assurance Society, Melbourne, Australia.*

1906 [1] Emmerson, Walter Hector Ross, *London and Lancashire Life Assurance Company,* 66 *&* 67 *Cornhill,* E.C.

1904 [1] Esler, John, *Crown Life Insurance Company, Toronto, Canada.*

1892 [1] Farrell, John, *Citizens' Life Assurance Co., Sydney, Australia.*

1902 [1] Farrow, Alfred Ellis, *Flaxton-rectory, York.*

1906 [1] Fender, William Martin, *Australian Mutual Provident Society, Melbourne, Australia.*

1904 [2] Fippard, Richard Clift, *Prudential Assurance Company, Holborn-bars,* E.C.

1901 [1] Fisher, John William, B.A., A.A.S., *Crown Life Insurance Co., Toronto, Canada.*

1896 [1] Fisk, George William Victor, F.S.S., *Prudential Assurance Company, Holborn-bars,* E.C.

1904 [1] Fletcher, Andrew W. A. C., *Standard Life Assurance Co.,* 3 *George-street, Edinburgh.*

STUDENTS.

Those marked 1, 2, *or* 3 *have passed one, two, or three of the four Examinations of the Institute. Those marked* (1) *have been exempted under the Bye-laws from the Examination in Part I.*

Date of becoming a Student.

1905 [1] Flynn, Benedict Devine, F.A.S., *Travelers' Insurance Company, Hartford, Conn., U.S.A.*

1904 [1] Foot, Alfred Helsdon, *Law Accident Insurance Society, Limited,* 215 *Strand,* W.C.

1905 [1] Forbes, James, *Great-West Life Assurance Co., Winnipeg, Manitoba, Canada.*

1906 [1] Foster, Joseph, 33 *Westwood-street, Moss Side, Manchester.*

1906 [1] Foster, Wilfred Justus, *Prudential Assurance Company, Holborn-bars,* E.C.

1901 [1] Franklin, Herbert Dare, *Australian Mutual Provident Society, Melbourne, Australia.*

1906 [1] Frost, Charles Frederick, *Prudential Assurance Company, Holborn-bars,* E.C.

1903 [1] Fulford, William John, *Prudential Assurance Company, Holborn-bars,* E.C.

1900 [1] Garner, James, 6 *St. Stephen's-avenue, Shepherd's Bush,* W.

1901 (1) Gerrish, Frank Wilfred, B.A., *Minerva-villa, Albert-rd.-south, Buckhurst-hill, Essex.*

1899 [1] Giles, Hylton Lloyd, *Pelican & British Empire Life Office,* 70 *Lombard-street,* E.C.

1895 [1] Gill, James Stewart, *Australian Widows' Fund Life Assur. Soc., Sydney, Australia.*

1901 [1] Glassford, David Murray, *Mutual Life Association of Australasia, Sydney, Australia.*

1893 Glasson, George Cornish, *Economic Life Assurance Soc.,* 4 *St. Stephen's-chbrs., Baldwin-street, Bristol.*

1902 [1] Gleave, Charles Sheldon, *Refuge Assurance Co., Oxford-street, Manchester.*

1902 [1] Godsill, Richard Collis, *Liverpool Victoria Legal Friendly Soc.,* 18 *St. Andrew-street,* E.C.

Date of becoming a Student.

1894 [1] Golding, Arthur, 40 *Allerton-road, Stoke Newington,* N.

1905 [1] Goodall, Ernest Victor, 51 *Ardgowan-road, Hither-green,* S.E.

1888 [1] Gooding, Harold John, *Law Guarantee and Trust Soc., Ltd.,* 70 *Cornhill,* E.C.

1903 [1] Gopp, John Ive, 14 *Church-hill-road, Walthamstow,* E.

1902 [1] Gordon, Walter Hamilton, 45 *Braydon-road, Stamford-hill,* N.

1897 [2] Gosset, Thorold, 16 *Durham-rd., Wimbledon,* S.W.

1886 Gover, Frederick Field, F.S.S., 10 *Lee-park, Blackheath,* S.E.

1886 Greening, Herbert Joseph, *Abstainers' & General Insur. Co., City-buildings, Birmingham.*

1906 [1] Gunningham, Sidney Joseph, *Ecclesiastical Commission, Millbank,* S.W.

1901 [1] Hall, Arthur F., *North American Life Assurance Co., North American Life Building,* 112 - 118 *King-street-west, Toronto, Canada.*

1906 [1] Hall, John Vaughan Lewis, *Equity and Law Life Assurance Society,* 18 *Lincoln's-inn-Fields,* W.C.

1902 [2] Hallett, William Sebastian, B.A., *Equitable Life Assurance Soc., Mansion-house-street,* E.C.

1901 [1] Hamilton, George Powell, *North American Life Assurance Co., McLean Block,* 6 *Douglas-street, Guelph, Ontario, Canada.*

1905 [1] Hamley, Ernest Fountain, *Australasian Temperance and General Mutual Life Assurance Society, Melbourne, Australia.*

1902 [1] Hammant, Francis Clive, *Prudential Assurance Company, Holborn-bars,* E.C.

1905 [1] Hammond, Harry Pierson, B.A., A.A.S., *Mutual Life Insurance Co. of New York, New York, U.S.A.*

STUDENTS.

Those marked 1, 2, *or* 3 *have passed one, two, or three of the four Examinations of the Institute. Those marked* (1) *have been exempted under the Bye-laws from the Examination in Part I.*

Date of becoming a Student.	
1892	Hancock, Arthur Tom, *Clerical, Medical & General Life Assurance Society,* 15 *St. James's-square,* S.W.
1903	[2] Hancock, Edwin J., 72 *Tredegar-road, Bow,* E.
1906	[1] Handford, John James William, *Scottish Office,* S.W.
1902	[1] Hardy, Reginald Herbert, 32 *Highfield-street, Leicester.*
1903	[1] Harley, Brian, *Guardian Assurance Co.,* 11 *Lombard-street,* E.C.
1905	[1] Harnack, Frederick William, *Sceptre Life Association, Ltd.,* 40 *Finsbury-pavement,* E.C.
1901	[1] Harper, Henry, 103 *Waverley-road, Small Heath, Birmingham.*
1905	[1] Harrington, Eustace Woods, *Northern Assurance Company,* 1 *Moorgate-street,* E.C.
1903	[1] Harris, Ernest Arthur, 40 *Lambert-rd., Brixton-hill,* S.W.
1889	[1] Harris, Henry, *Friends' Provident Institution,* 17 *Gracechurch-street,* E.C.
1905	[1] Harrison, Launcelot, *Citizens' Life Assurance Co., Sydney, Australia.*
1906	[1] Harrison, Robert James, 19 *Tintern-street, Clapham,* S.W.
1896	Haskins, George Frederick, A.C.A., 18 *Walbrook,* E.C.
1894	[1] Hatten, David Leslie, *Standard Life Assurance Co.,* 83 *King William-street,* E.C.
1905	(1) Heron, David, M.A., *Viewbank, New Scone, Perth, N.B.*
1906	[1] Hilbery, Reginald William, *Clerical, Medical & General Life Assurance Society,* 15 *St. James's-square,* S.W.
1903	[1] Hill, Frank Wilson, *Norwich Union Life Insurance Society, Norwich.*
1896	[2] Hines, Walter Robert, *Norwich Union Life Insurance Society, Norwich.*
1902	[1] Hodge, Cecil Wilfred, *Star Life Assurance Society,* 32 *Moorgate-street,* E.C.
1896	[1] Hogg, Charles, *Ecclesiastical Commission, Millbank,* S.W.
1906	[1] Holness, Archibald Stephen, *Pelican and British Empire Life Office,* 70 *Lombard-street,* E.C.
1905	[1] Homan, Russell Charles, 3 *The Terrace, Camden-sq.,* N.W.
1898	[2] Hooper, George Duncan, *Prudential Assurance Company, Holborn-bars,* E.C.
1895	[2] Horn, Ernest Frederick, *Equity & Law Life Assur. Soc.,* 18 *Lincoln's Inn Fields,* W.C.
1902	[1] Houston, Charles Cornelius, *Metropolitan Asylums Board, Victoria-embankment,* E.C.
1901	[1] Howell, Archibald Rennie, B.A., *Royal Insurance Co., Montreal, Canada.*
1898	Hughes, Arthur J., *China Mutual Life Insur. Co., Shanghai, China.*
1902	[1] Hughes, Charles, A.A.S., *Crown Life Insurance Company, Toronto, Canada.*
1906	(1) Hughes, Tom, M.A., *Commercial Union Assur. Co.,* 26 *New Bridge-street,* E.C.
1902	[1] Hugill, Herbert, *"Briarfield," Keighley.*
1904	[1] Humphreys, Harry Lewis, *Pelican and British Empire Life Office,* 70 *Lombard-street,* E.C.
1902	[1] Humphreys, John A., *National Mutual Life Assurance Society,* 39 *King-street, Cheapside,* E.C.
1902	[1] Humphry, Edmund William, *Life Association of Scotland,* 18 *Bishopsgate-st.-Within,* E.C.
1891	Hunt, Arthur Leonard, *Wesleyan and General Assur. Soc.,* 101 *Finsbury-pavement,* E.C.
1906	[1] Hustwitt, William Edmund, *Prudential Assurance Company, Holborn-bars,* E.C.

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Those marked 1, 2, *or* 3 *have passed one, two, or three of the four Examinations of the Institute. Those marked* (1) *have been exempted under the Bye-laws from the Examination in Part I.*

Date of becoming a Student.

1902 (1) Jackson, Charles William, M.A., *c/o* M. M. Dawson, Esq., 11 *Broadway, New York, U.S.A.*

1902 [2] Jackson, Herbert Moore, *Australian Mutual Provident Society, Sydney, Australia.*

1890 [2] Jackson, Samuel, F.F.A., *Scottish Widows' Fund and Life Assurance Society, Liverpool.*

1896 [1] Jepps, John Blacklee, *English and Scottish Law Life Assurance Assoc.,* 12 *Waterloo-place,* S.W.

1906 [1] Jerrold, Allan Laman, 22 *Rue du Champ de Mars, Paris.*

1905 [1] Johns, Arthur Humphreys, *Colonial Mutual Life Assurance Society, Melbourne, Australia.*

1904 [1] Johnson, Frank Henry, *Law Life Assurance Society,* 187 *Fleet-street,* E.C.

1898 [1] Johnston, Arthur Edward, 3 *Cumnor-road, Sutton.*

1903 [1] Jones, Ernest Stephens, *National Debt Office,* E.C.

1896 [1] Jones, Richard Foxley, *Refuge Assurance Co., Oxford-street, Manchester.*

1906 Kearns, William Norman, *Royal Insurance Co., Liverpool.*

1905 [1] Keevil, Norman Alexander Clement, *Blagdon, Park-road, Watford, Herts.*

1902 [1] Kemper, J. M. de Bosch, *La Mutuelle Hollandaise Compagnie d'Assurance,* 21 *Avenue de l'Opera, Paris.*

1905 [1] Kenchington, Frank, *North British and Mercantile Insurance Company,* 61 *Threadneedle-street,* E.C.

1906 [1] Kidd, Alan Bruce, *North British and Mercantile Insurance Company,* 1 *Dawson-street, Dublin.*

1905 [1] Kidd, Hubert D., *Mutual Life Insur. Co. of New York,* 16, 17 *&* 18 *Cornhill,* E.C.

1906 [1] Kime, Virgil Morrison, 624 *Packard-street, Ann Arbor, Michigan, U.S.A.*

1905 [1] King, Albert Edward, *Provident Clerks' and General Mutual Life Assur. Association,* 27 *&* 29 *Moorgate-street,* E.C.

1894 [2] Kingsbury, James William, *Australian Mutual Provident Society, Sydney, Australia.*

1903 [1] Kirsopp, Frederick, *Liverpool Victoria Legal Friendly Society, St. Andrew-street,* E.C.

1895 [1] Knight, Alfred Murray, *Bank-house, Chapel-st., Devonport.*

1905 [1] Lafford, Harry George, *Legal and General Life Assur. Society,* 10 *Fleet-street,* E.C.

1905 [1] Laing, John Morrison, *Mutual Life Assurance Co. of Canada, Waterloo, Ontario, Canada.*

1902 [1] Lang, Frederick John, *Royal London Friendly Society, Finsbury-square,* E.C.

1905 [1] Langstaff, Milton Palmer, *Imperial Life Assurance Co. of Canada, Toronto, Canada.*

1904 [1] Lee, Frank Sidney, *Ocean Accident and Guarantee Corporation,* 36-44 *Moorgate-street,* E.C.

1904 [1] Lee, Frederick, *Ecclesiastical Insurance Office,* 11 *Norfolk-street, Strand,* W.C.

1906 [1] Leigh, Walter Lewis, 58 *Lichfield-road, Bow,* E.

1894 Leonard, Maurice, *Frith Hill Cottage, Great Missenden, Bucks.*

1906 [1] Le Rossignol, Leonard F., *English and Scottish Law Life Assur. Association,* 12 *Waterloo-place,* S.W.

1904 [1] Levey, Ralph, *Prudential Assurance Company, Holborn-bars,* E.C.

1906 [1] Lewis, David Hugh, *Refuge Assurance Company, Oxford-street, Manchester.*

STUDENTS.

Those marked 1, 2, *or* 3 *have passed one, two, or three of the four Examinations of the Institute. Those marked* (1) *have been exempted under the Bye-laws from the Examination in Part I.*

Date of becoming a Student.

1904 [1] Lewty, Francis Arthur, *Equity and Law Life Ass. Soc.*, 18 *Lincoln's-inn-fields*, W.C.

1896 [1] Ley, James, *Office of the Actuary for Friendly Societies, Melbourne, Australia.*

1889 [1] Lighton, Harold John, *Law Union & Crown Insurance Co.*, 126 *Chancery-lane*, W.C.

1904 [1] Linzmeyer, Louis, F.A.S., *Manhattan Life Insurance Co.*, 64-70 *Broadway, New York, U.S.A.*

1895 [1] Littell, Lewis Lloyd, *Standard Life Assurance Co.*, 83 *King William-street*, E.C.

1904 [1] Littlefair, James Taylor, *Refuge Assurance Co., Oxford-street, Manchester.*

1906 [1] Lohan, John Joseph, *National Mutual Life Association of Australasia, Melbourne, Australia.*

1906 [1] Lolley, Clement Francis, *Universal Insurance Loan and Investment Co., Ltd.*, 77 *New Briggate, Leeds.*

1890 Love, Robert, *Ecclesiastical Insurance Office*, 11 *Norfolk-street, Strand*, W.C.

1906 [1] McCall, Robert, 58 *Lichfield-road, Bow*, E.

1888 [1] McConway, James Robert, 15 *Henthorn-road, New Ferry, Cheshire.*

1906 [1] McCulloch, James Arthur, *Ecclesiastical Commission, Millbank*, S.W.

1903 [1] McDonald, Charles Joseph Angus, *Australian Mutual Provident Society, Wellington, New Zealand.*

1903 [1] Macdonald, Charles Strange, M.A., *Confederation Life Association, Toronto, Canada.*

1904 [1] Macfarlane, Edmond Scales, *Manufacturers' Life Insurance Company*, 23 *Water Street, Yokohama, Japan.*

1902 [1] McGee, Cyril H., *Box* 981, *St. Thomas, Ontario, Canada.*

1905 [1] McKechnie, John Henry, *Manufacturers' Life Ins. Co., Toronto, Canada.*

1902 [1] McKellar, John A., *Equitable Life Assur. Society of the United States*, 120 *Broadway, New York, U.S.A.*

1906 [1] Mackenzie, Donald Rae, *Manufacturers' Life Insurance Company, Toronto, Canada.*

1903 [2] Maltby, Charles Hugh, *Scottish Widows' Fund and Life Assurance Society*, 28 *Cornhill*, E.C.

1903 [1] Manly, George William, B.A., *Clerical, Medical & General Life Assurance Society*, 15 *St. James's-square*, S.W.

1904 [1] Marlin, James Harold, *Ocean Accident and Guarantee Corporation*, 36-44 *Moorgate-street*, E.C.

1905 [1] Marshall, Arthur William, *Consolidated Assur. Co., Ltd., Thanet House*, 231 & 232 *Strand* (*opposite the Law Courts*), W.C.

1905 [1] Marshall, John Edwin, *Prudential Assurance Company*, 47 *Earl-street, Coventry.*

1903 [1] Martin, Frederick Charles, *Prudential Assurance Company, Holborn-bars*, E.C.

1906 [1] Martin, William Alexander, *National Mutual Life Association of Australasia, Melbourne, Australia.*

1904 [1] Matheson, Donald, *Imperial Life Assurance Co. of Canada, Toronto, Canada.*

1906 [1] Maunder, Henry Ernest, 86 *Tyrwhitt-road, St. John's*, S.E.

1895 [1] Mayhew, Percy Craske, 4 *Princess-road, Selhurst*, S.E.

1890 [1] Meikle, Henry George Watson, F.F.A., *Oriental Government Security Life Assurance Co., Limited, Bombay, India.*

1901 [2] Melville, Henry Edward, *Alliance Assurance Company, Ltd., Bartholomew-lane*, E.C.

STUDENTS.

Those marked 1, 2, *or* 3 *have passed one, two, or three of the four Examinations of the Institute. Those marked* (1) *have been exempted under the Bye-laws from the Examination in Part I.*

Date of becoming a Student.

1892 [1] Meyers, Henry Wilson, *National Mutual Life Association of Australasia,* 5 *Cheapside,* E.C.

1899 [2] Minns, Ernest Edwin, *Norwich Union Life Insurance Society, Norwich.*

1902 [1] Moore, Hubert Fred, *London Assurance Corporation,* 7 *Royal Exchange,* E.C.

1903 [1] Moore, Roderick John, 102, *Caithness-road, Mitcham, Surrey.*

1898 [1] Moore, Stanley, *Prudential Assurance Company, Holborn-bars,* E.C.

1904 [1] Moran, Albert James, *Sun Life Assurance Society,* 63 *Threadneedle-street,* E.C.

1902 [1] Morton, Francis, *Commercial Union Assur. Co.,* 24, 25 *&* 26 *Cornhill,* E.C.

1904 [1] Mulcahy, Francis Benedict, *Citizens' Life Assurance Co., Sydney, Australia.*

1903 [1] Myers, Harry Duxbury, A.S.A.A., 64 *Devonshire-street, Keighley.*

1906 [1] Naismith, Keith Errol, *Refuge Assurance Co., Oxford-street, Manchester.*

1906 [1] Nathan, Eric Burnett, *Norwich Union Life Insur. Soc., Finsbury-pavement-house,* E.C.

1896 [1] Neale, Maurice Baldwin, *Alliance Assurance Company, Ltd., Bartholomew-lane,* E.C.

1906 [1] Needell, Brian, *Alliance Ass. Co., Ltd.* (*Provident Life Fund*), 50 *Regent-street,* W.

1903 [1] Neill, William Adam Hoyes, *Scottish Widows' Fund & Life Assur. Soc.,* 9, *St. Andrew-sq., Edinburgh.*

1905 (1) Nicholl, Charles Carlyon, B.A., F.F.A., *Royal Exchange Assur. Corp., Royal Exchange,* E.C.

1904 [1] Norris, Isaac Taylor, *Collegiate Institute, Ottawa, Canada.*

1903 [1] Northcott, John Arthur, *St. Andrew's College, Toronto, Canada.*

1901 [1] Nugent, James, *Cornwall, Ontario, Canada.*

1903 [1] Oates, Percy Tuckfield, 30 *High-street, Wimbledon,* S.W.

1902 (1) O'Connor, William, M.A., M.D., *Mutual Life Insurance Co. of New York, Toronto, Canada.*

1892 [1] O'Reilly, Anthony James, *Government Insurance Department, Ottawa, Canada.*

1897 [1] Osborn, Nathaniel Banner Francis, 11 *Bruce-grove, Tottenham,* N.

1905 [1] Osborne, William Arthur, *Ocean Accident & Guar. Corp., Ltd.,* 36-44 *Moorgate-street,* E.C.

1905 (1) Owen, David John, B.A., *Commercial Union Assur. Co.,* 26 *New Bridge-street,* E.C.

1893 [1] Owen, Edgar Theodore, F.S.S., *Registrar of Friendly Societies and Government Actuary, Perth, West Australia.*

1901 [1] Papworth, Frederick William, A.S.A.A., *St. David's, Longley-road, Tooting,* S.W.

1904 [1] Parker, John G., *Imperial Life Assurance Co. of Canada,* 24 *King-street East, Toronto, Canada.*

1904 [1] Parker, Walter Montgomery, *Prudential Assurance Company, Holborn-bars,* E.C.

1895 [1] Pascoe, William Yeoman Bennett, *Prudential Assurance Company, Holborn-bars,* E.C.

1897 [1] Paton, Harry Arthur, *Royal Exchange Assurance Corporation, Royal Exchange,* E.C.

1897 [1] Patrick, James, *Audit Office, Town-hall, Birkenhead.* (*Reinstated,* 1905.)

1906 [1] Patrick, Walter S., *Britannic Assurance Company, Limited, Broad-street Corner, Birmingham.*

STUDENTS.

Those marked 1, 2, *or* 3 *have passed one, two, or three of the four Examinations of the Institute. Those marked* (1) *have been exempted under the Bye-laws from the Examination in Part I.*

Date of becoming a Student.

1896 [2] Penny, Charles Augustus, *Prudential Assurance Company, Holborn-bars,* E.C.

1905 [1] Perry, Sidney James, *Northern Assurance Company,* 60 *&* 61 *London-wall,* E.C.

1906 Peter, James Calthorpe, *London and Lancashire Life Assurance Company,* 66 *&* 67 *Cornhill,* E.C.

1901 [1] Petter, Herbert, *Britannic Assurance Co., Ltd., Broad-st.-corner, Birmingham.*

1905 [1] Phillips, Thomas Ashley, *New York Life Insurance Co.,* 346 *&* 348 *Broadway, New York, U.S.A.*

1904 [1] Phillips, Walter, A.C.I.S., A.S.A.A., 8 *Riverdale-terrace, Richmond, Surrey.*

1905 [1] Pickup, John Richardson, *National Provident Institution,* 48 *Gracechurch-street,* E.C.

1905 [1] Pollard, Edward Cecil, *The Castle, Rochester.*

1898 Poort, Willem Anthonie, Phil. Nat. Doct., *Algemeene Friesche Levensverzekerings Maatschappij Leeuwarden, Leeuwarden, Holland.*

1903 (1) Porter, Frank, M.A., *Mansfield House, Canning Town,* E.

1893 [1] Pownall, Herbert Wilfred, *Australian Mutual Provident Society, Adelaide, Australia.*

1906 [1] Priestman, Basil, 23 *Highfield-road, Edgbaston, Birmingham.*

1901 [1] Ramsay, Cecil Byron, *Mutual Life Insur. Co. of New York,* 16, 17 *&* 18 *Cornhill,* E.C.

1905 [1] Reeve, Gilfrid Montier, *Newlands, Prospect-hill, Walthamstow,* E.

1898 [1] Reynell, Guy Courtenay, *National Mutual Life Assurance Society,* 26 *Charles-street, St. James's,* S.W.

1904 [1] Reyner, Harry Fane, *Refuge Assurance Company, Oxford-street, Manchester.*

1903 [1] Reynolds, William Daniel, *Prudential Assurance Company, Holborn-bars,* E.C.

1894 [1] Richards, Gilbert P. A., *Oak Cottage, Bulwer-road, New Barnet.*

1904 [1] Ridgway, Wulfric, *Sun Life Assurance Society,* 63 *Threadneedle-street,* E.C.

1902 [1] Robertson, Aubrey Charles, *London Assurance Corporation,* 7 *Royal Exchange,* E.C.

1903 [1] Robertson, Bernard, *Prudential Assurance Company, Holborn-bars,* E.C.

1903 [1] Robinson, Ernest William, *Standard Life Association, Ltd.,* 28 *Elizabeth-street, Sydney, Australia.*

1896 [1] Robinson, Frederick Charles, *Royal Exchange Assur. Corporation, Royal Exchange,* E.C.

1893 [1] Roll, Frederick James, *Pearl Life Assurance Company, London-bridge,* E.C.

1893 [1] Roodenburch, Bartholomeus Adrianus, *Verzekeringsbank Victoria,* 126 *Keizersgracht, Amsterdam.*

1906 [1] Rose, Gilbert Melville, *General Accident, Fire and Life Assurance Corporation,* 13 *Pall Mall,* S.W.

1895 [1] Ross, Christopher Watson, *c/o Messrs. M. Moss & Co., Flinders-lane, Melbourne, Australia.*

1901 [1] Rountree, Arthur FitzGerald, *The Rectory, Stretford, near Manchester.*

1905 [1] Rowland, Stanley Jackson, *Clerical, Medical and General Life Assurance Society,* 15 *St. James's-square,* S.W.

1895 Rowley, James Edward, A.C.A., 7 *Waterloo-street, Birmingham.*

1906 [1] Ruddle, Francis, 31 *Esmond-road, Bedford Park,* W.

1899 [1] Rutter, Edward Valentine, *Pelican & British Empire Life Office,* 70 *Lombard-street,* E.C.

STUDENTS.

Those marked 1, 2, *or* 3 *have passed one, two, or three of the four Examinations of the Institute. Those marked* (1) *have been exempted under the Bye-laws from the Examination in Part I.*

Date of becoming a Student	
1904	[1] Sadler, Percy, *Prudential Assurance Company, Holborn-bars,* E.C.
1894	Salter, George Ferry, M.S., F.A.S., 123 *N.* 16*th-street, E. Orange, N.J., U.S.A.*
1905	[1] Schooling, Terence Holt, 40B *Lavender Sweep,* S.W.
1897	[1] Scott, Alexander Lewis, *Australian Mutual Provident Society, Melbourne, Australia.*
1905	[1] Sharp, Harold Gregory, *Friends' Provident Institution,* 17 *Gracechurch-street,* E.C.
1906	[1] Simmonds, Reginald Claud, *Alliance Assurance Co., Ltd., Bartholomew-lane,* E.C.
1892	[1] Simpson, William Murray, *North British and Mercantile Insurance Company,* 61 *Threadneedle-street,* E.C.
1905	[1] Sinclair, Coll Claude, B.A., *Great-West Life Assurance Co., Vancouver, B.C., Canada.*
1904	[1] Sinclair, William Alexander, *Canada Life Assurance Company, Toronto, Canada.*
1891	[1] Sindall, Alfred John, *London and Lancashire Life Assur. Co.,* 66 *&* 67 *Cornhill,* E.C.
1888	[2] Slimon, William James, F.F.A., 10 *Mayfield-terrace, Edinburgh.*
1905	[1] Sloan, Joseph James Eastwood, *Corporation Audit Department, Liverpool.*
1902	[1] Smith, Septimus Wontner, *Equitable Life Assurance Soc., Mansion-house-street,* E.C.
1903	[1] Smith, William, *Standard Life Association, Ltd.,* 28 *Elizabeth-street, Sydney, Australia.*
1903	[2] Sneddon, Andrew William, *Australian Mutual Provident Society, Sydney, Australia.*
1904	[1] Spring, Stanley Harold, *London Guarantee and Accident Company,* 61 *Moorgate-st.,* E.C.

Date of becoming a Student.	
1904	[1] Sprules, Alfred M., *"Trelawne," Sutton, Surrey.*
1901	[1] Steffensen, Johan F., *Forsikringsraadet,* 1 *Christiansgade, Copenhagen.*
1906	[1] Stephenson, Herbert Roy, *Manufacturers' Life Insurance Company, Toronto, Canada.*
1886	[2] Stirling, James, F.F.A., *Law Union and Crown Insur. Co.,* 126 *Chancery-lane,* W.C.
1905	[1] Stone, Mark, *Merton College, Oxford.*
1888	[1] Stott, Walter, *Royal Insurance Co., Liverpool.*
1893	[1] Streeter, Theodore Edward, *Lesser Slave Lake, via Athabasca Landing, Alberta, Canada.*
1904	[1] Strong, Gordon Gilbert, *Sun Life Assurance Society,* 63 *Threadneedle-street,* E.C.
1902	[1] Strong, William Boughton, *Prudential Assurance Company, Holborn-bars,* E.C.
1904	[1] Stuart, Arthur William, *National Provident Institution,* 48 *Gracechurch-street,* E.C.
1905	[1] Stuart, C. J. S., *Canada Life Assurance Co., Toronto, Canada.*
1904	[1] Sturt, Arthur James, *Pelican and British Empire Life Office,* 70 *Lombard-street,* E.C.
1903	[1] Suddaby, William A., *Liverpool Victoria Legal Friendly Society, St. Andrew-street,* E.C.
1906	[1] Sutton, Maurice William, 1 *Foxbourne-road, Balham,* S.W.
1904	[1] Tamkin, Walter Ellis, *Prudential Assurance Company, Holborn-bars,* E.C.
1905	[1] Thompson, John Henry Reginald, *Prudential Assurance Company, Holborn-bars,* E.C.
1905	[1] Thompson, Joseph William, *Norwich Union Life Insurance Society, Norwich.*

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.	
1904	[1] Thompson, William George, *Commercial Union Assurance Co.*, 24, 25 *&* 26 *Cornhill*, E.C.
1906	[1] Thomson, Ernest H. W., *London and Lancashire Fire Insurance Company, Dale-street, Liverpool.*
1902	[1] Thwaites, Frederick George, *Norwich Union Life Insurance Society, Norwich.*
1897	[1] Tipping, Oswald, *Trustees, Executors and Agency Co., Limited*, 412 *Collins-street, Melbourne, Australia.*
1897	[1] Townshend, Edward Villiers, *Scottish Widows' Fund and Life Assurance Society*, 28 *Baldwin-street, Bristol.*
1901	[1] Tregaskis, George, *Sun Insur. Office*, 40 *Chancery-lane*, W.C.
1905	[1] Tutill, Hubert Linzee, *English & Scottish Law Life Assur. Association*, 12 *Waterloo-place*, S.W.
1891	Tyler, Edgar Alfred, F.S.S., 9 *Old Jewry-chambers, Bank*, E.C.
1906	[1] Tyler, Victor William, *Alliance Assur. Co., Ltd.* (*Provident Life Fund*), 50 *Regent-street*, W.
1904	[1] Underwood, Reginald, *Guardian Assurance Company*, 11 *Lombard-street*, E.C.
1906	[1] Vaughan, Hubert, *Citizens' Life Assurance Co., Sydney, Australia.*
1905	[1] Walker, Dwight A., *Equitable Life Assurance Soc. of the United States*, 120 *Broadway, New York, U.S.A.*
1906	Warhurst, James, *Alliance Assur. Co., Ltd.* (*Provident Life Fund*), 68 *Fountain-street, Manchester.*
1904	[1] Warnock-Fielden, Francis Hugh, *Prudential Assurance Company, Holborn-bars*, E.C.
1904	[1] Warren, Cyril Ferdinand, *Prudential Assurance Company, Holborn-bars*, E.C.

Date of becoming a Student.	
1903	[1] Watson, Alexander R. D., 89 *Queen-street, Auckland, New Zealand.*
1906	[1] Watson, Andrew Daniel, *Government Insurance Department, Ottawa, Canada.*
1906	[1] Watson, John A., *Law Guarantee and Trust Soc., Ltd.*, 49 *Chancery-lane*, W.C.
1900	[1] Watt, Arthur W., *Sun Life Assur. Co. of Canada, Montreal, Canada.*
1906	(1) Webb, Herbert Anthony, B.A., *Trinity College, Cambridge.*
1898	[1] Webb, Lloyd, *Commercial Union Assur. Co.*, 26 *New-bridge-street*, E.C.
1905	[1] Wellington, Frank, *Australian Mutual Provident Society, Melbourne, Australia.*
1902	[1] Wellisch, Frederick, *Australian Mutual Provident Society, Sydney, Australia.*
1893	[1] Welman, Arthur Joseph, *Legal & General Life Assurance Soc.*, 15 *Tithebarn-st., Liverpool.*
1905	[1] Welsh, Willis, *Prudential Assurance Company, Holborn-bars*, E.C.
1904	[1] Wenn, Albert Edward *Prudential Assurance Company, Holborn-bars*, E.C.
1905	[1] White, Osborn Denyer, *Commercial Union Assurance Co.*, 24, 25 *&* 26 *Cornhill*, E.C.
1902	[1] White, Wilfred Clare, *Federal Life Assurance Co., Hamilton, Ontario, Canada.*
1886	[1] Williams, David, 181 *Queen Victoria-street*, E.C.
1905	[1] Williams, Hugh Corden, *Australian Mutual Provident Society, Melbourne, Australia.*
1895	[1] Williams, Henry Samuel Walter, *Liverpool & London & Globe Insurance Company, Melbourne, Australia.*

STUDENTS.

Those marked 1, 2, or 3, have passed one, two, or three of the four Examinations of the Institute. Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.

1900 (1) Williams, Lewis, B.A., *Commercial Union Assur. Co., 24, 25 & 26 Cornhill,* E.C.

1906 1 Williamson, Wallace White, *Norwich Union Life Insurance Society, Norwich.*

1901 2 Wilton, Herbert George, *Norwich Union Life Insurance Society, Norwich.*

1894 1 Windett, Sydney V., *Eagle Insurance Company,* 79 *Pall-mall,* S.W.

1905 1 Wisdom, Sidney Herbert, *Estate Duty Office, Somerset-house,* W.C.

1903 2 Wolfenden, Edgar Sydney, *Australian Mutual Provident Society, Sydney, Australia.*

1895 1 Wood, David James, *Commercial Union Assurance Co.,* 24, 25 *&* 26 *Cornhill,* E.C.

1901 1 Wood, Roland Stuart, *Liverpool & London & Globe Insurance Co.,* 1 *Cornhill,* E.C.

1906 1 Woodall, Edward Arthur, *National Mutual Life Assurance Society,* 39 *King-street, Cheapside,* E.C.

Date of becoming a Student.

1902 1 Woodhouse, David Alfred, *Refuge Assurance Co., Oxford-street, Manchester.*

1905 1 Woodward, James Howard, 29 *Newhall-street, Birmingham.*

1900 1 Woolston, Paul Livingston, B.S., 50 *Maine-avenue, Ocean-grove, New Jersey, U.S.A.*

1894 1 Wyatt, George Matthew, *Central Insurance Company,* 12 & 13 *Nicholas-lane,* E.C.

1886 Yeatman, Alexander Alfred, 2 *Coleman-street,* E.C.

1895 1 Yeldham, William James, *Prudential Assurance Company, Holborn-bars,* E.C.

1906 1 Yeomans, Ernest Charles, *Australian Widows' Fund Life Assurance Society, Melbourne, Australia.*

1903 1 Young, Henry J., *Prudential Assurance Company, Holborn-bars,* E.C.

1897 1 Younger, R. H., *Liverpool & London & Globe Insurance Co.,* 1 *Dale-street, Liverpool.*

1904 1 Zumstein, Herbert Christian, *Australian Mutual Provident Society, Melbourne, Australia.*

CORRESPONDING MEMBERS.

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Prof. Thorvald Nicolai Thiele, Dr. Phil.,
Astronomical Observatory.

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Président de l'Institut des Actuaires Français. Député du Morbihan; 2 Rue Dante.

M. Hermann Laurent, Dr. Sc.,
Professeur à l'Institut Agronomique. Vice-Président du cercle des Actuaires Français; 18 *Rue Denfert-Rochereau.*

M. Léon Marie, F.A.S.,
Sous-Directeur, Le Phénix Compagnie d'Assurances sur la vie. Membre Correspondant de l'Association des Actuaires Belges. Membre Agrégé de l'Institut des Actuaires Français; 28 *Rue de Châteaudun* (*IX*e).

M. Albert Quiquet, F.A.S.,
Actuaire, La Nationale Compagnie d'Assurances sur la vie. Secrétaire de l'Institut des Actuaires Français; 17 *Rue Laffitte* (*IX*e).

M. Alfred Thomereau,
8 *Rue le Peletier.*

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Dr. Johannes Karup,
Actuary of the Gotha Life Assurance Company.

Dr. Karl Samwer,
Manager of the Gotha Life Assurance Company, Ohrdruferstrasse, 4.

Hungary.

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M. Julius Altenburger,
Consulting Actuary. Corresponding Member of the "Institut des Actuaires Français," and of the "Association des Actuaires Belges." Member of the "Versicherungs-Wissenschaftliche Vereinigung of Vienna"; vii. *Hajtsár ut,* 20.

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M. Guido Toja,
Condirecteur Général des Compagnies Italiennes d'Assurances sur la Vie et Contre L'Incendie "La Fondiaria," Piazza Vittorio Emanuele.

Russia.

ST. PETERSBURG.

M. Serge de Savitch,
Professor of Mathematics in the Electro-Technical Institute and in the University of St. Petersburg. Manager of the Tariffs' Service of the United Russian Insurance Companies; Nikolaewskaia, 35.

Spain.

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Dr. José Maluquer y Salvador,
Insurance Counsellor (Ex-Actuary at the Home Office). Member of the Institute of Social Reforms; calle de Campomanes, 10.

Sweden.

STOCKHOLM.

M. Anders Lindstedt, Dr. Phil.,
Professor of Mathematics and Director of the Polytechnicum. Counsellor of the Regency for Insurance Questions. Member of the Royal Swedish Academy of Science; etc.

Switzerland.

ZURICH.

Dr. Gottfried Schaertlin,
Manager of the "Schweizerischen Lebensversicherungs-und Rentenanstalt."

United States.

NEW YORK.

Mr. David Parks Fackler, M.A., F.A.S.,
Ex-President of the Actuarial Society of America (1891-93). *Consulting Actuary;* 35 *Nassau-street.*

*** *It is requested that any inaccuracy in the foregoing list may be pointed out to the* ASSISTANT SECRETARY.